NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

REPORT NO. 17
CONTRACT NO.
PAGE 1 OF
DATE Jan. 3, 1967

PROJECT: LFST

SUBJECT:

Thermal Deformations of Telescopes

Sebastian von Hoerner

Summary

The time scale τ_i of internal heat conduction is so small that all points of a cross section of a member always have practically the same temperature.

If the ambient temperature changes suddenly, convection and radiation cause each member to adapt slowly to the new temperature; for hollow members with white paint in winds below 5 mph, the time scale τ for this adaption is 1.14 hours/inch wall thickness for aluminum, and 1.73 hours/inch wall thickness for steel. The time scales are half these values for T and L shapes and solid rods. The time scales of unpainted aluminum or galvanized steel are 1.8 times longer.

If the ambient air changes constantly by $\dot{T}$ (°C/hour), a member of time scale τ lags behind with a temperature difference of $\Delta T = - \tau\dot{T}$. For 1/4 of all days, the measured maximum change is $\dot{T} \gtrsim 3.5$ °C/hour.

A second temperature difference is caused by sunshine and shadow. Some measurements showed that, with white protective paint, the difference between members in sunshine and those in shadow is about $\Delta T = 5$ °C on the middle of clear days.

With these data, predictions were made for the 140-foot telescope. On about 1/4 of all days, the maximum thermal deformations should be as follows or larger: change of focal length (relative to feed supports) = 5.9 mm; pointing error = 30 arcsec; rms surface deviation = 1.6 mm. It might be possible to reduce these deformations, by about a factor 3, by blowing ambient air through all heavy, hollow members.

I. Theory and Formulae

1. Internal Heat Conduction

We assume a long, hollow member of wall thickness w, which originally is at temperature T_o; then we cool its outside down to $T = 0$. How long does it take for its interior to cool down, too?

We call

$$
\begin{aligned}
h &= \text{coeff. of heat conduction} \\
k &= \text{heat capacity} \\
\rho &= \text{density}
\end{aligned}
\tag{1}
$$

and

$$m = \frac{h}{k\rho} . \tag{2}$$

If the temperature T is a function of one coordinate x and of time t, the general equation of heat conduction is

$$\frac{\partial}{\partial t} T(x,t) = m \frac{\partial^2}{\partial x^2} T(x,t) \quad \text{or} \quad \dot{T} = m\, T''. \tag{3}$$

As a simplification, we regard an infinite plate of thickness w, with the boundary conditions

$$
\begin{aligned}
T(0,t) &= 0, \quad \text{cooled surface;} \\
\frac{\partial}{\partial x} T(w,t) &= 0, \text{ insulated surface.}
\end{aligned}
\tag{4}
$$

Next, we ask only for a simple type of solution, separable in x and t (called a homology solution because of keeping its shape):

$$T(x,t) = F(t)\, G(x). \tag{5}$$

One then can show that equation (3) has only one solution of type (5) with boundary conditions (4); this solution reads:

$$T(x,t) = T_o\, e^{-t/\tau_i} \sin(\pi x/2w). \tag{6}$$

This is an exponential decay, with a time scale

$$\tau_i = \frac{4}{\pi^2 m}\, w^2. \tag{7}$$

Some examples are given in Table 1. We see that these time scales of internal heat balance are very short for a thickness below one inch, and we will find later that they may be completely neglected for all practical purposes.

Table 1. Time scale τ_i of internal heat exchange for a member of wall thickness w.

	h	k	ρ	$4/(\pi^2 m)$	τ_i		
	$\frac{\text{cal}}{°\text{ C cm sec}}$	$\frac{\text{cal}}{°\text{C g}}$	$\frac{\text{g}}{\text{cm}^3}$	$\frac{\text{sec}}{\text{cm}^2}$	w = 1/8 inch	w=1 inch	w=10 inch
aluminum	0.48	0.209	2.70	.477	0.05 sec	3.1 sec	5.2 min
steel	0.11	.107	7.86	3.09	.31 sec	20.0 sec	33 min

2. Heat Radiation at Surface

If a surface has temperature T (in °K) and if the surrounding is at absolute zero, the amount of heat radiated per cm^2 and per second is proportional to T^4, and the coefficient of proportionality is known as the Stefan-Boltzmann factor σ:

$$\sigma = 5.7 \times 10^{-5} \frac{\text{erg}}{\text{sec cm}^2\ (°\text{K})^4} . \tag{8}$$

But if the surrounding is at Temperature T, and the surface is only slightly warmer, at $T + \Delta T$, then the

$$\text{heat flow by radiation} = 4\sigma T^3 \Delta T. \tag{9}$$

From (9) we define r_r, the coefficient of heat flow by radiation, as

$$r_r = 4\sigma T^3. \tag{10}$$

Table 2. The coefficient r_r of heat flow through a black surface by radiation.

T		r_r
°C	°K	$10^{-4} \frac{\text{cal}}{°\text{C sec cm}^2}$
-20	253	0.91
0	273	1.14
+20	293	1.41
+40	313	1.72

The values of Table 2 hold for a black surface, while for an actual surface the heat flow can be much smaller; it is zero for a perfect mirror. But what counts is not whether the surface appears black in the visible light; the maximum of radiation occurs at a wavelength given by Wien's law as

$$\lambda_{max} = \frac{0.289}{T} \tag{11}$$

and with T = 20°C = 293 °K we have

$$\lambda_{max} = 10 \text{ micron.} \tag{12}$$

What matters, then, is whether the surface is "black" at 10 micron wavelength.

A good protective paint for telescopes should absorb only little sun radiation, and should radiate away fast whatever it absorbed. It thus should be white at visible light (1/2 micron), and should be black around 10 micron.

3. Heat Convection at Surface

The amount of heat per cm^2 and per second, transported away from a surface into the surrounding air by convection, is proportional to the temperature difference ΔT between surface and air:

$$\text{heat flow by convection} = r_c \, \Delta T \, . \tag{13}$$

This coefficient of heat flow by convection, r_c, is usually of the same magnitude or larger than the values of Table 2 for radiation. It depends to a large extent on the surface conditions (smooth or rough, paint, ...) and on the wind velocity; and for an application to telescopes we best obtain it by measurements.

Convection is larger for laminar flow of air than for turbulent flow, and the flow is turbulent if the Reynold number is above 3000. A rough estimate showed that this is the case, for a velocity of 3 mph, for members with diameters above 3 cm. We thus expect a drop of r_c with increasing diameter around 1 inch diameter, but a constant value of r_c for normal, larger diameter members.

4. Cooling time scale τ

We assume a long member of any given cross section, with

$$\begin{aligned} A &= \text{area of cross section,} \\ C &= \text{circumference of cross section.} \end{aligned} \tag{14}$$

For example, a pipe of outer diameter d and wall thickness w has

$$\left.\begin{aligned} A &= \pi\, w\, (d-w) \\ C &= \pi\, d \end{aligned}\right\} \frac{A}{C} = w\,(1 - \frac{w}{d}); \quad \text{for pipes.} \tag{15}$$

We assume that the internal heat exchange of Table 1 is so fast that all parts of a cross section have practically the same temperature T; we call T_o the air temperature and call $\Delta T = T - T_o$ the difference. The heat content per unit length of the member then is

$$H = k\rho A\, T, \tag{16}$$

and the heat flow through the surface is

$$\frac{dH}{dt} = - rC\, \Delta T. \tag{17}$$

This yields

$$\frac{d\Delta T}{dt} = \frac{r}{k\rho}\frac{C}{A}\,\Delta T \tag{18}$$

with the solution

$$\Delta T(t) = \Delta T(o)\; e^{-t/\tau}. \tag{19}$$

This is an exponential decay again, with a time scale

$$\tau = \frac{k\rho}{r}\frac{A}{C} \quad . \tag{20}$$

The coefficient of heat flow is now the sum of radiation plus convection,

$$r = r_r + r_c \tag{21}$$

and if we want to obtain it, by measuring the time scale τ, we have

$$r = \frac{k\rho}{\tau}\frac{A}{C}\,. \tag{22}$$

The assumption of fast internal exchange is valid as long as τ_i from (7) is small as compared to τ from (20), which is the case, for radiation only, as long as the wall thickness is $w \ll 80$ m for aluminum, and $w \ll 20$ m for steel.

5. Temperature Difference between Member and Air

If a member has a cooling time scale τ and is surrounded by air of varying temperature $T_a(t)$, what then is the difference $\Delta T(t)$ between member and air?

The derivation is very similar to the one of the previous section and shall be omitted. The result is

$$\Delta T(t) = - \int_0^\infty e^{-\vartheta/\tau} \dot{T}_a(t-\vartheta)\, d\vartheta . \tag{23}$$

This is an exponentially-weighted average over the past time-derivative of the air temperature. If the air temperature showed a constant rise or drop ($\dot{T}$ = const.) for a time longer than a few τ, we obtain

$$\Delta T = - \tau \dot{T}_a . \tag{24}$$

One could roughly say that the member lags behind the air with distance τ.

II. Measurements

1. Cooling Time

A sample of 14 pipes or solid rods was selected; 11 from iron and 3 from aluminum. Originally, the aluminum pipes had a blank surface; 4 steel pipes were black, 1 blank, 3 quite rusty, and 3 were galvanized. Diameter and wall thickness varied over the range:

$$0.75 \text{ inch} = 19 \text{ mm} \leq d \leq 154 \text{ mm} = 6.1 \text{ inch};$$

$$0.12 \text{ inch} = 3.1 \text{ mm} \leq w \leq 38 \text{ mm} = 1.5 \text{ inch}.$$

In each member a hole was drilled with 1 mm diameter down to 1/2 of the wall thickness, in which a thermo-couple was inserted and sealed with masking tape. The length of each member was always more than 4 times its diameter; both ends were closed and carefully insulated, for imitating very long members.

Each member was heated to 70 °C, then put on an open holder with very little contact. Measurement began after the member had cooled below 50 °C, and was stopped about 5 °C above ambient temperature. Three points were used to determine τ, in some cases a log-log plot was used.

Measurements were taken inside a room (25 °C), and outside in the open (-10 to +5 °C) on two very calm days (wind below 5 mph). Six members then were painted with the same protective paint as used on the 140-foot, and measurements repeated indoors and outside. The time scales measured varied over the wide range

$$9 \text{ min} \leq \tau \leq 2 \text{ hours}$$

but the coefficients of heat flow, obtained from (22), varied over the narrow range

$$1.3 \times 10^{-4} \leq r \leq 4.4 \times 10^{-4}.$$

In detail, the results are as follows.

a. Same r for all d and w? This can be checked only for the same type of surface and surrounding, for which we take the painted members, measured indoors. Table 3 and Figure 1 show the result.

We see that r is remarkably constant for diameters above 1 inch, and increases for the smaller diameters, as was expected in section I.3 because of laminar air flow. There is no effect of the wall thickness nor of the material.

Table 3. Coefficient r of heat flow for 6 painted members, measured indoors. (S = steel, A = aluminum)

d	w		r
mm	mm		$10^{-4}\ \frac{\text{cal}}{°\text{C sec cm}^2}$
19	9.5	S	4.14
21	3.1	S	3.11
38	3.2	A	2.76
50	25.0	S	2.69
63	6.3	A	2.64
115	5.7	S	2.62

Since the diameter of telescope members mostly will be much more than an inch, and since we are most concerned with the heaviest ones, we adopt

$$r = 2.60 \times 10^{-4}\ \frac{\text{cal}}{°\text{C sec cm}^2}\ ;\ \text{painted, wind} = 0. \qquad (25)$$

b. Outdoors, calm. The wind on these measurements was between 1 and 5 mph, mostly about 3 mph. These calm days are of importance, since the wind at Green Bank is 1/4 of all time below 4.9 mph*). For the painted members we obtain

$$r = 3.48 \times 10^{-4}\ \frac{\text{cal}}{°\text{C sec cm}^2}\ ;\ \text{painted, wind} = 3\ \text{mph}. \qquad (26)$$

c. The influence of paint. The paint made little difference for black surfaces, but a considerable one for aluminum and galvanized surfaces, as seen in Table 4.

---

*) See Report 16

Table 4. Coefficient of heat flow for various surfaces, measured indoors. (Average over all diameters).

Material	Surface	r 10^{-4} $\frac{\text{cal}}{\text{°C sec cm}^2}$
steel	black	2.90
	rusty	2.78
	blank	2.52
	galvan.	1.91
aluminum	blank	1.60
both	painted	2.99

Furthermore, it turns out that by painting a galvanized or aluminum surface, the cooling time is decreased by a considerable factor:

$$\text{painting decreases cooling time by factor } 1.60. \tag{27}$$

d. Cooling time τ. The time scale is given in (20) and (15) as a function of d and w. But since in telescope structures mostly $w \ll d$, we have

$$\tau = \frac{k\rho}{r} w \tag{28}$$

and with r from (26) we obtain

$$\left.\begin{aligned} \tau\ (\text{alum.}) &= 1.14 \text{ hours} \\ \tau\ (\text{steel}) &= 1.73 \text{ hours} \end{aligned}\right\} \text{ per inch wall thickness.} \tag{29}$$

These values hold for painted members on calm days up to, say, 5 mph. Since the effect on telescope deformation will be largest on calm days and small in high winds, no measurements in higher winds were taken. Values (29) hold for hollow members; they will be smaller (about 1/2) for open T and L shapes, and for solid rods.

e. Convection and radiation. The largest difference in r occurred between paint and a newly galvanized surface; it amounted to $\Delta r = 1.42 \times 10^{-4}$. This was measured indoors, where the average temperature of member and air was about 35 °C. If this difference is due to radiation, then the largest possible difference (Table 2) is 1.64×10^{-4}. Since both values are so close together, we may conclude that, at

10 micron, the paint is almost completely black, while the newly galvanized surface is almost completely white. Under this assumption we can derive that the painted members, outdoors and on calm days, cool down by the following proportion:

$$\frac{\text{convection}}{\text{radiation}} = 1.75 \quad \text{(painted, 3 mph, 10 °C).} \tag{30}$$

2. Steep Changes of Air Temperature

Since the temperature of heavy members lags behind the air temperature with a delay between 1/2 and 2 hours according to (29), we ask for the steepest temperature change per hour of each day, $\dot{T}$, which is to be inserted into (24).

a. At Green Bank

The air temperature has been measured on 1204 days during March 1959 through February 1964, but unfortunately only twice a day, at 8:00 a.m. and at 4:30 p.m. The distribution of the difference ΔT is shown in Figure 2. We see, for example, that on 1/4 of all days the temperature rise is 11.8 °C or larger. The largest rise measured was 30 °C (54 °F).

But since the time interval of 8.5 hours is too long, and 4:30 p.m. is too late after the daily maximum, it would be too uncertain to estimate the maximum change per hour from these measurements. Instead of, we use measurements taken at Sugar Grove, which is only 35 miles from Green Bank and about the same elevation.

b. At Sugar Grove

During 1962, the air temperature was measured each hour. We take the differences, $\dot{T}$ (in °C/hour), of all consecutive hours, and we ask for the maximum rise and the maximum drop of each day. Their distribution is shown in Fig. 3. We see, for example, that the temperature mostly rises more steeply and drops more slowly (the median rise is 2.7 °C/hour, the median drop is -2.0 °C/hour); but occasionally we have drops more steep than any rise (the steepest drop measured is -10.6 °C/hour, the steepest rise +6.7 °C/hour.

Taking from both rise or drop the steepest one of each day, we find the distribution of Table 5.

Table 5. Steepest temperature change per hour of each day. For the fraction F of all days, the steepest change is $\dot{T}$ or larger.

F	$\dot{T}$ (°C/hour)
3/4	1.9
1/2	2.7
1/4	3.5
1/10	4.0
1/20	4.7
1/50	5.9
1/100	7.1

For further application, we select from Table 5 the last quartile:

$$\dot{T} \geq 3.5 \text{ °C/hour, for 1/4 of all days.} \tag{31}$$

It should be noted that the maximum rise occurs mostly around or after sunrise, and the maximum drop around and after sunset, but many steep changes also are due to a sudden change in cloudiness any time of the day. The largest changes occur on sunny, calm days.

3. Sunshine and Shadow

A series of temperature measurements at the surface and at various members of the 140-foot is planned for the near future. But as of now, only some few measurements at a 140-foot spare panel are available. This panel is mounted at concrete pillars on a slope, pointing south.

Thermo-couples were used at the surface, and at the panel structure below (always in shadow). Originally, the panel had a blank aluminum surface, and measurements were taken on clear, sunny winter days in December 1964. Calling ΔT the difference between surface temperature and shadow temperature, we obtained, for maximum and average: ΔT_{max} = 20 °C and ΔT_{av} = 13 °C. After a white protective paint was applied (same as on 140-foot), some measurements were taken on clear, sunny, summer days, and the result is ΔT_{max} = 9 °C and ΔT_{av} = 5°C. We thus adopt:

$$\Delta T \text{ (sun - shadow)} = 5 \text{ °C (clear day, paint).} \tag{32}$$

III. Application to the 140-Foot Telescope

1. Predictions

We now must combine the temperature differences arising from the change of air temperature in members of different wall thickness, with the temperature differences between members in sunshine and those in shadow.

As to the first effect, we call Δw the difference in wall thickness. For painted aluminum members on calm days, we have from formulae (24), (29) and (31):

$$\Delta T = 4.0\ °C\ \frac{\Delta w}{\text{inch}} . \tag{33}$$

As to the second effect, we have $\Delta T = 5°C$ from (32) for the middle of the day, but we adopt only 4 °C for those parts of the day when $\dot{T}$ is large, too.

Feed support legs and surface panels have a smaller wall thickness than the main back-up members, and also the former mostly catch more sunshine than the latter. Thus, both effects will add up, at least in the morning. But for light and heavy members of the back-up structure, we assume no correlation and thus add both effects quadratically.

The larger surface panels have a length of $\ell = 9$ m, and their structure has a depth of $d = 0.9$ m. If the surface is ΔT degrees warmer than the panel structure, if the panel is held at constant height at both ends, and if C_{th} is the coefficient of thermal expansion, one can show that the center of the panel will move up by the amount

$$\Delta z = C_{th} \frac{\ell^2\ \Delta T}{8d} ; \tag{34}$$

in our case (and for aluminum) we obtain

$$\frac{\Delta z}{\Delta T} = 0.3\ \frac{\text{mm}}{°C} , \tag{35}$$

and we use $\Delta T = 4$ °C for the difference between surface and panel structure. But the same formula (35) also holds if the ends of the panels are rigidly mounted at constant distance from each other, and if the panel as a whole then is warmed up by ΔT degrees. (The curvature of the panels is small as compared to their depth.)

For this case we use $\Delta T = 2°C$ as an average for the panel as a whole, and we add only 1/2 of the resulting deformation since the panels are mounted somewhat flexible.

The results are shown in Table 6 for four different cases. Deformations of this amount or larger are to be expected on about 1/4 of all days.

Table 6. Combined thermal effects estimated for the 140-foot.

	w	τ	ΔT air change	ΔT combined	length	deformation
	mm	hours	°C	°C	m	mm
1. feed support legs	10	0.45	3.5	7.5	23	5.9
back-up structure	32	1.44				
2. heavy back-up	32	1.44	4.0	5.7	10	2.0
light back-up	6	.27				
3. heavy back-up	32	1.44	4.4	6.4	9	1.0
panels	4	.18				
4. panel surface	3	.13	.6	4.6	9	1.4
panel structure	5	.22				

The actual deformations are best measured by observing strong, small radio sources. What we measure this way is the change of: focal length, pointing correction, beamwidth and gain (from the two latter ones the rms deformation of the surface can be obtained). We thus use Table 6 for estimating the expected values for these observational quantities, and the results are shown in Table 7. The change of focal length is meant relative to the feed support (= axial focal adjustment). For the pointing error, we assume that one feed support leg is about perpendicular to the sunshine, while the opposite leg is more parallel to it. For the surface deformation, we add item 3 and 4 from Table 6 linearly, and add the result quadratically to item 2, which yields 3.12 mm for the deformation of the panel center, and then we divide by 2 for the rms deviation from the best-fit parabola.

Table 7. Predicted thermal deformations of 140-foot telescope. On about 1/4 of all days, the maximum deformation will be as shown or larger.

change of focal length	5.9 mm
pointing error	30 sec of arc
rms surface deviation	1.6 mm

2. Measurements

As to the available measurements and calibrations at the 140-foot, the observed changes are due to the combined effects of gravitational and thermal deformations. In principle, both effects could be separated by repeated calibrations: in the same telescope position but different weather conditions, and in different positions but same weather. Actually, this separation is rather time-consuming and troublesome. A more thorough investigation is planned; at present (mostly without data for temperature and sunshine) one can only say that the scatter of the calibrations, if assumed to be of thermal origin, has just the amount as predicted in Table 7.

3. Conclusions

The thermal deformations of the 140-foot are rather high as compared to the accuracy this telescope has otherwise (pointing 5"; surface rms .9 mm).

Since back-up structure, surface, and feed supports all are made from the same material (aluminum), the temperature as such should not matter at all; only temperature differences will count. These differences amount to 4 - 8 °C or more on 1/4 of all days, resulting from two causes: sunshine against shadow, and heavy against light members in changing air temperature.

This result leads to a suggestion, which does not look too expensive and which might be discussed. Since feed supports and all heavy back-up members are hollow, one could mount fans at one of their ends and could blow a constant stream of ambient air through them. Furthermore, a fan might be mounted behind the center of each panel. These fans would be operated during sunny, calm days and during sudden temperature changes, but could be turned off otherwise. A rough estimate showed that in this way all thermal deformations would be reduced by a factor of 3, if the air flow through the longest, hollow members is maintained with a speed of 15 - 20 mph.

For future telescopes, one should keep in mind that the thermal expansion of aluminum is exactly twice that of steel; also, one might replace very heavy members by a number of lighter ones.

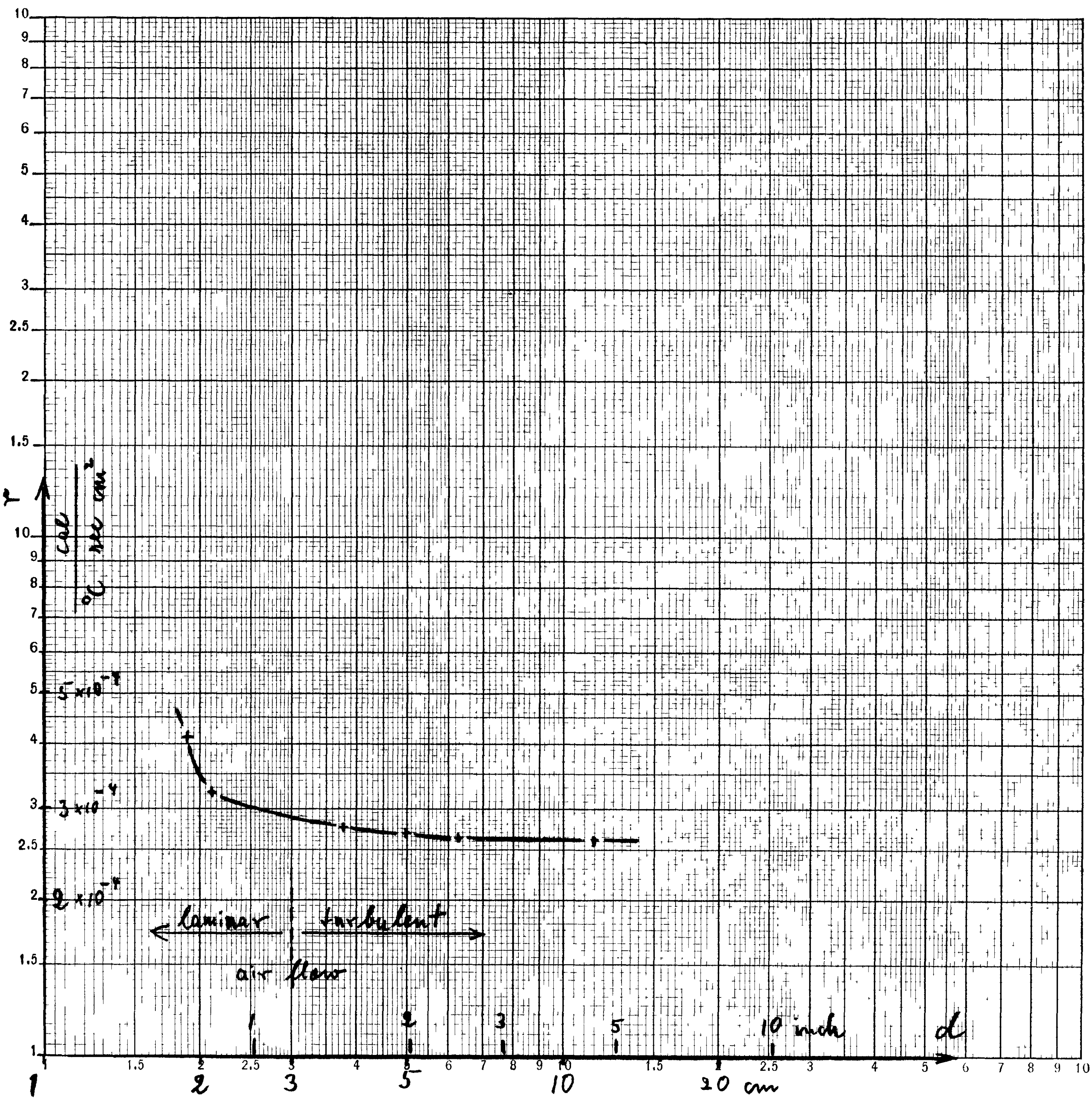

Fig. 1. Coefficient of surface heat flow, r, as a function of outer diameter, d. (Painted, wind = o)

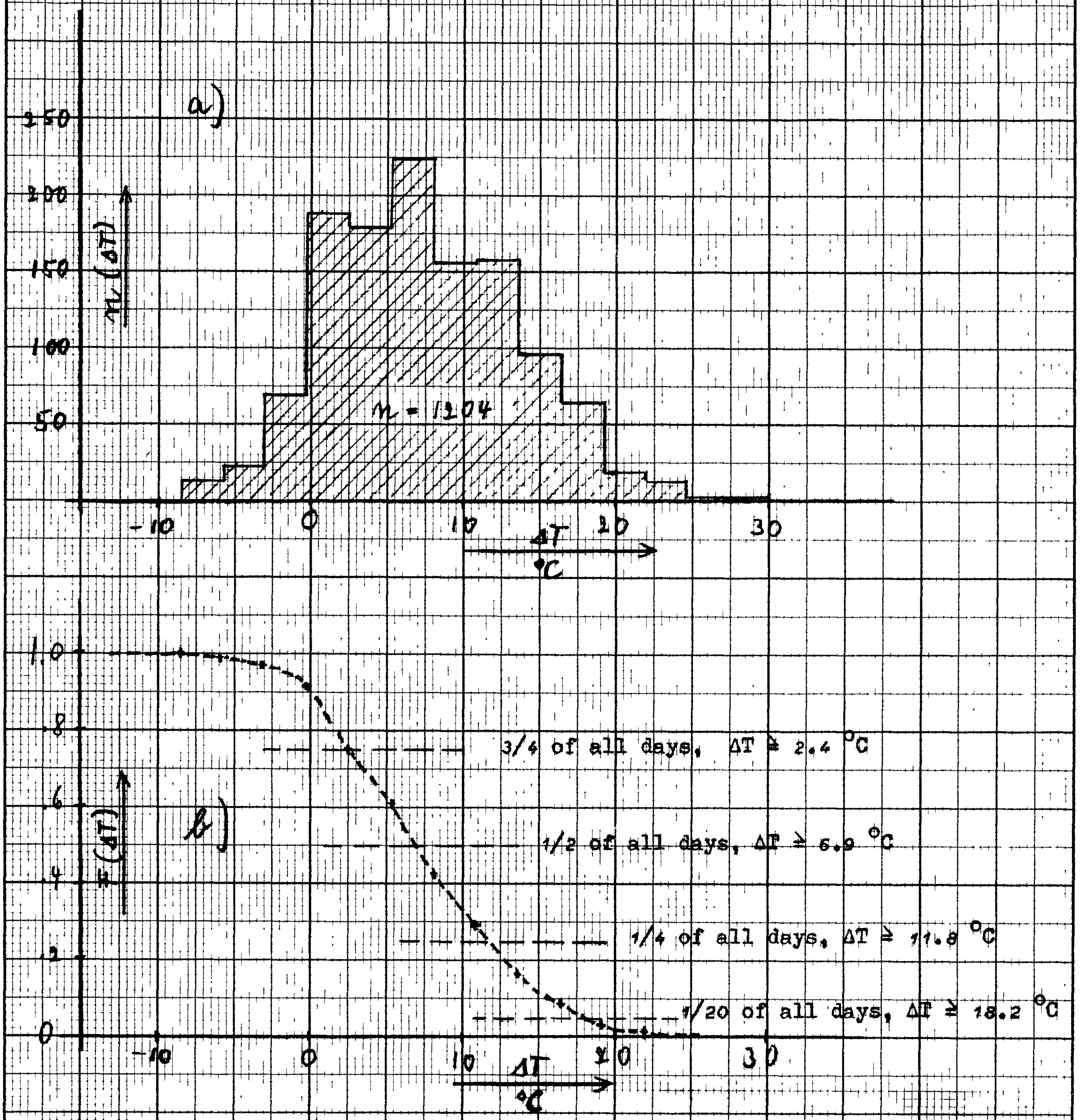

Fig. 2. Daily temperature rise at Green Bank, from 8:00 am to 4:30 pm.

a) Histogram,

b) Cumulative distribution.

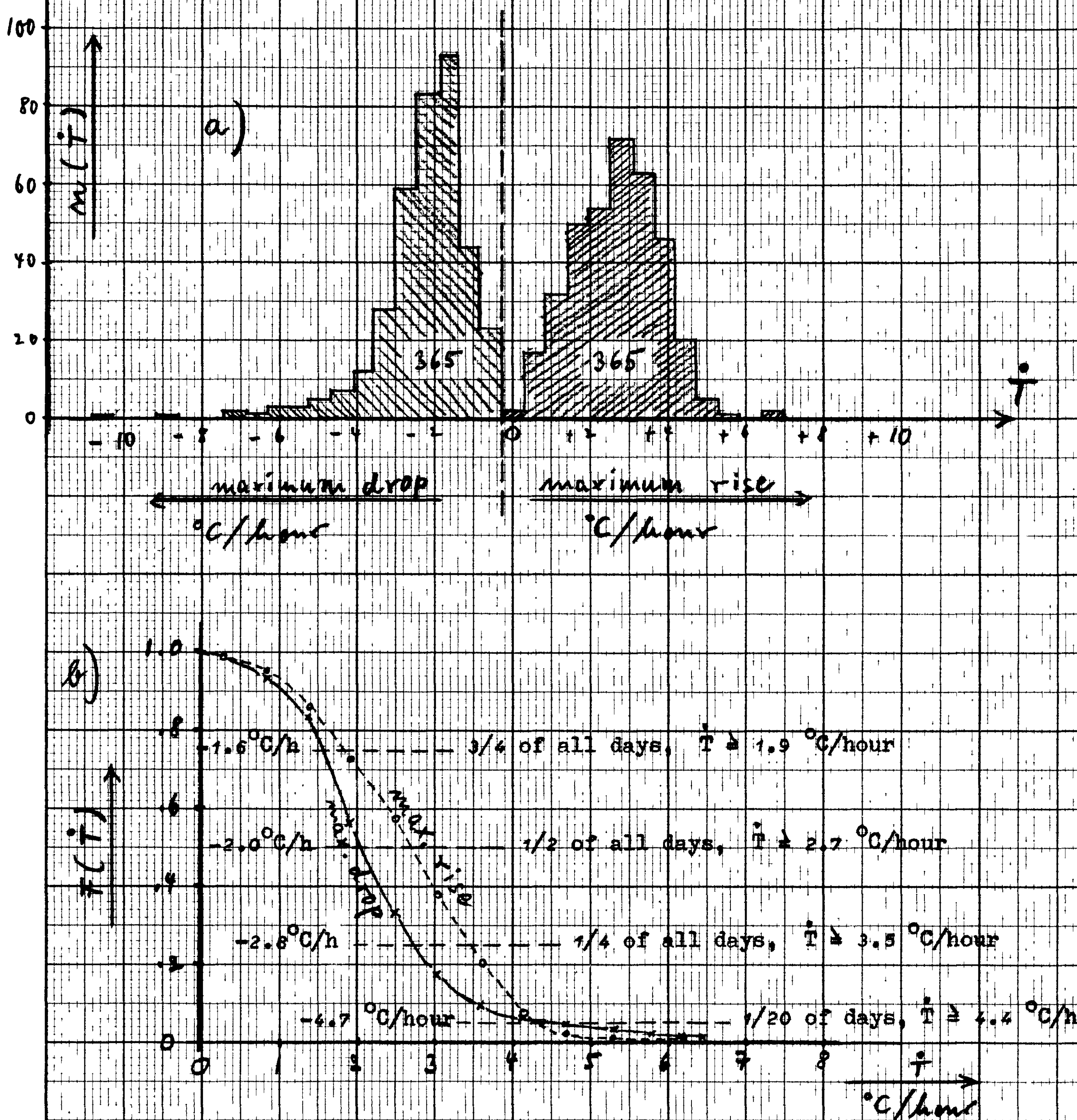

Fig.3. Temperature change per hour, at Sugar Grove in 1962; maximum rise and maximum drop of each day.