# Homologous 300-ft Telescope:
# Data for Analysis of Towers and Built-up Members

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The following contains the design data for the azimuth towers and their built-up members, to be given to Simpson, Gumpertz and Heger for a complete analysis of stress, stability and deformations.

The telescope is fully steerable, with an alt-azimuth mount: two towers sit on wheels on a circular railway track; all lateral forces are taken up by a strong, central pintle bearing. The rear view of telescope and towers is shown in Fig.1, and the tower geometry is explained in Fig.2. Points 4 and 4a carry the full weight of the dish in two elevation bearings; the elevation drive is at point 6. Four azimuth drives are at points 2, 2a, 3, and 3a. Each member of the towers is a tapered, pin-ended built-up member, of the same type as the members used for the dish structure. The members are optimized with respect to a compromise between maximum stiffness/weight and maximum axial load/weight.

## I. Built-up Members

### 1. Design Data

All members are tapered, with a parabolic shape, each consisting of 12 panels of equal length. Definitions are given in Fig.3, and the parabolic shape is shown in Fig.4. Note that the crossing point of the diagonals is always at 1/2 the panel length. In the tower design, each member is defined by its length L, its nominal bar area A, and its typt t. There are two types with different slenderness, see Fig.4. We call

$$b_c = \text{length of center batten} \tag{1}$$

and

| | = bar area of | | = bar area of |
|---|---|---|---|
| $A_p$ | pyramid bars | $A_d$ | diagonals |
| $A_c$ | chords | $A_t$ | triangle bars |
| $A_b$ | battens | | |

(2)

The values for both types of members are given in Table 1. The single pieces are standard pipe, all connections are welded. The material is steel with

$$\begin{aligned} &\text{density} && \rho = 0.283\ \mathrm{lb/in^3} \\ &\text{elasticity} && E = 29\ 000\ \mathrm{ksi} \\ &\text{yield stress} && Y = 36\ \mathrm{ksi} \end{aligned} \qquad (3)$$

Table 1. Design of built-up members.

| | $b_c/L$ | $A_p/A$ | $A_c/A$ | $A_b/A$ | $A_d/A$ | $A_t/A$ |
|---|---|---|---|---|---|---|
| type 1 | .0557 | .430 | .2736 | .0428 | .0466 | .0191 |
| type 2 | .0808 | .455 | .2906 | .0589 | .0474 | .0203 |

## 2. Load conditions

Each member is exposed to three loads simultaneously:

1) Dead load (its own weight), depending on tilt angle $\varphi$; (4)

2) Survival wind force, 85 mph = 18.5 lb/ft$^2$; assume always perpendicular to member and parallel to ground; use shape factor $C_s = 1.00$ for cylindrical pipes; use diameter = 2.154(area)$^{2/3}$ for standard pipes; (5)

3) Axial force F, applied at end points. Assume compression. (6)

## 3. Tasks

### a. Equivalent Values

Take one member each of type 1 and 2. (1) Calculate total weight W and total wind force f (neglect shadowing and geometrical projections). (2) Apply dead load plus wind; find $S_{dw}$ = maximum chord stress. (3) Apply force F only; find axial deformation $\Delta L$, and maximum chord stress $S_F$. Calculate

$$\text{equivalent area} \qquad A_{eq} = \frac{F\,L}{E\,\Delta L} \quad \text{in}\ \ \mathrm{in}^2, \qquad (7)$$

$$\text{equivalent density} \qquad \rho_{eq} = \frac{W}{L\,A_{eq}} \quad \text{in}\ \ \mathrm{lb/in^3}. \qquad (8)$$

Check whether or not the following 6 statements are correct:

1. $A_{eq} = A$ for both types. (9)

2. $\rho_{eq} = 0.332$ for type 1, and 0.370 for type 2. (10)

3. $f = W\,(6.26/A_{eq})^{1/3}$ for both types. (11)

4. $S_{dw} = S_o\,(L/1000\text{ in})\sqrt{\cos^2\varphi + (6.26/A_{eq})^{2/3}}$; (12)

   with $S_o = 3.637$ ksi for type 1, and 2.702 ksi for type 2.

5. $S_F = 1.04\,F/A_{eq}$ for both types. (13)

6. The analysis of the towers then can be performed by assuming each member to be a solid rod of bar area A, density $\rho_{eq}$ and elasticity E, picking up a wind force f. If $S_c$ is the stress resulting from this analysis, the actual chord stress, from axial forces only, then is

$$S_F = 1.04\,S_c\,. \qquad (14)$$

b. Maximum Axial Force

Let a member be exposed simultaneously to all three loads (4), (5) and (6). The first two loads will cause an original excentricity or sag, which is further increased by the third one. If the axial force F increases beyond a critical value $F_{cr}$, the whole member will buckle because one of its single pipes will buckle or yield.

The task is to find $F_{cr}$ for each one of the tower members, either by application of some general formula, or by individual numerical analysis. The result should be expressed in terms of a critical stress

$$S_{cr} = F_{cr}/A \qquad (15)$$

to be compared later on in the tower analysis with $S_F$ from (14). For the stability of the single pipes, use the conventional safety factors.

c. Balance of Bar Areas

At $F = F_{cr}$ at least one of the single pipes reaches its limit of stability. The bar area ratios given in Table 1 are chosen such that this limit should be reached simultaneously for a chord, a batten and a diagonal (optimum ratios for maximum force /weight).

Once the ratios are chosen, this optimum condition can exactly be fulfilled only for some value of A (depending on L), but it should be approximately fulfilled also for other values of A. Question: How much do the tower members actually deviate from this optimum?

Define $S_m$ as the maximum allowed stress of each pipe; for the radius of gyration, use

$$r = 0.702\ (\text{area})^{2/3}, \text{ for standard pipe.} \tag{16}$$

Call S the actually prevailing stress, and define a stress ratio

$$Q = S/S_m\ . \tag{17}$$

At the limit $F = F_{cr}$, one of the pipes will have reached $Q = 1$; if this happens for a chord, give also the Q values for the most critical batten and diagonal, and vice versa.

d. Torsional buckling

Find the critical axial load $F_t$ for which torsional buckling would occur. Since according to my estimates $F_t \gg F_{cr}$ for all members, the application of some rough approximation formula should be satisfactory.

## II. The Towers

### 1. Design Data

The geometry of the towers is shown in Figures 1 and 2, and the coordinates are given in Table 2. Nominal bar areas, as defined in Section I,1 and (9), are given in Table 3, together with the legth L and type t, see Fig.4. Since the azimuth structure is symmetrical with respect to the plane $y = 0$, points and members of only one hemisphere are given in the tables.

**Table 2.** Coordinates (inch)

| point | x | y | z |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | -1039 | 1800 | 0 |
| 3 | 1039 | 1800 | 0 |
| 4 | 0 | 1200 | 1990 |
| 5 | 0 | 810 | 1000 |
| 6 | 0 | 0 | 890 |
| 7 | -2078 | 0 | 0 |
| 8 | 2078 | 0 | 0 |

**Table 3.** Members; A = area (in$^2$), L = length (inch), t = type.

| points | A | L | t |
|---|---|---|---|
| 1-2 | 75 | 2078 | 2 |
| 1-3 | 70 | 2078 | 2 |
| 1-5 | 130 | 1286 | 1 |
| 1-6 | 20 | 890 | 1 |
| 1-7 | 70 | 2078 | 1 |
| 1-8 | 30 | 2078 | 2 |
| 2-3 | 30 | 2078 | 2 |
| 2-4 | 140 | 2324 | 1 |
| 2-5 | 50 | 1749 | 1 |
| 2-7 | 40 | 2078 | 2 |
| 3-4 | 140 | 2324 | 1 |
| 3-5 | 50 | 1749 | 1 |
| 3-8 | 30 | 2078 | 2 |
| 4-5 | 140 | 1064 | 1 |
| 5-6 | 30 | 817 | 1 |
| 6-7 | 70 | 2261 | 1 |

## 2. Load Conditions

We consider three different load conditions: snow, survival storm, and winds during observation. For the first two, we want to know stresses and reactions, for the third one the deformations. The conditions are the following.

(a) Dead load, plus weight of dish (2430 kip), plus maximum snow load (20 lb/ft$^2$ = 1454 kip), both to be taken up by the elevation bearings at points 4 and 4a.

(b) Dead load, plus weight of dish, plus survival storm (85 mph) from any direction, dish in stow position. With full side area of the dish, and a shape factor of $C_s = 1.30$, the storm load on the dish then is 1050 kip. Furthermore, the asymmetry of the dish plus asymmetric gusts will result in a torque, yielding 334 kip on bearings and elevation drive. We thus have $1/2\ (1050 + 334) = 692$ kip on each bearing in wind direction, points 4 and 4a, and 334 kip on point 6 in opposite direction. In addition, the wind pressure on the tower members is 18.5 lb/ft$^2$, with $C_s = 1.00$ for pipes, to be calculated from (11).

(c) Winds during observation, of 18 mph, from any direction. If from x-direction, we take the full face-on area, with $C_s = 1.56$ for open parabola. If from y-direction, we take the side area with $C_s = 1.30$. The forces from torque must be added, too. The wind pressure on the tower members is 0.83 lb/ft$^2$, with $C_s = 1.00$ for pipes, where instead of (11) we now have

$$f_w = W\ (6.26/A)^{1/3}\ (0.83/18.5) = 0.0826\ W\ A^{-1/3} . \qquad (18)$$

For the numerical analysis, it seems best to break these conditions down into their single contributions, as shown in Table 4.

Table 4. Load conditions.

| condition | dead load | points 4, 4a kip, each | point 6 kip | wind on tower lb/ft² | wanted stress | deform. |
|---|---|---|---|---|---|---|
| A. Dish + snow | yes | 1942, z | - | - | $S_A$ | - |
| B. Dish | yes | 1215, z | - | - | $S_B$ | - |
| C. Storm, x | - | 692, x | 334, -x | 18.5, x | $S_C$ | - |
| D. Storm, y | - | 692, y | 334, -y | 18.5, y | $S_D$ | - |
| E. Wind, x | - | 64.9, x | 31.3, -x | 0.83, x | - | $\alpha_x$ |
| F. Wind, y | - | 31.0, y | 15.0, -y | 0.83, y | - | $\alpha_y$ |

For each tower member, the stresses then are combined as

$$S_s = \left|S_B\right| + \sqrt{S_C^2 + S_D^2} \,, \tag{19}$$

in order to yield the maximum stress for any storm direction. The maximum survival stress then is

$$S_c = \max\left\{S_A,\ S_s\right\} . \tag{20}$$

The vertical distance bewteen points 4 and 6 is 1100 inch. The deformations Δ of these two points are combined in order to yield an angle α (pointing deviation) according to

$$\begin{aligned} \alpha_x &= (\Delta_{4xE} - \Delta_{6xE})/1100 \,, \\ \alpha_y &= (\Delta_{4yF} - \Delta_{6yF})/1100 \,, \end{aligned} \tag{21}$$

and the maximum possible angle then is simply

$$\alpha_c = \max\left\{\alpha_x,\ \alpha_y\right\} . \tag{22}$$

**Task:**

(a) Find angle $\alpha_c$, in arcsec.

(b) Find $S_c$ for each tower member, and with (14) and (15) the stress ratio

$$Q = S_F / S_{cr} . \tag{23}$$

## 3. Restraints

Point 1 is a strong pintle bearing, restrained in all directions for all load conditions. Points 2, 2a, 3, 3a, 7, 8 move on wheels on a circular railway track, and points 2, 2a, 3, 3a have one drive unit each. Their wheels are assumed to be fixed in condition A (vertical load only), and gliding in conditions B, C and D (survival storm); but in conditions E and F (observation, wind) they are fixed along the tracks while bending or gliding perpendicular to the tracks.

Some additional restraints have been added to prevent free rotation. All restraints are summarized in Table 5. If only one tower is analysed because of the symmetry, some more constraints are needed to simulate the reaction of the second tower.

Table 5. Restraints.

| point | A | | | B,C | | | D | | | E | | | F | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | x | y | z | x | y | z | x | y | z | x | y | z | x | y | z |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | T | T | 1 | T | T | 1 |
| 3 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | T | T | 1 | T | T | 1 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 8 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |

0 = free

1 = restrained

T = restrained along track, free perpendicular

## 4. Reactions

Calculate all reactions for load conditions A through D. The maximum downward and horizontal forces on the pintle bearing then are

$$R_{down} = R_A, \text{ point } 1, \tag{24}$$

$$R_{hor} = R_D, \text{ point } 1. \tag{25}$$

The maximum downward and uplifting forces at the tower legs, on points 2 and 3, and on point 7, are obtained for any storm direction as

$$R_{max} = R_B + \sqrt{R_C^2 + R_D^2}, \quad \text{points 2, 3 and 7;} \tag{26}$$

$$R_{min} = R_B - \sqrt{R_C^2 + R_D^2}, \quad \text{points 2, 3 and 7.} \tag{27}$$

If $R_{min}$ is negative, counterweights of this amount are needed at the tower legs, and at point 7.

**Task:**

Calculate all forces according to (24) through (27).

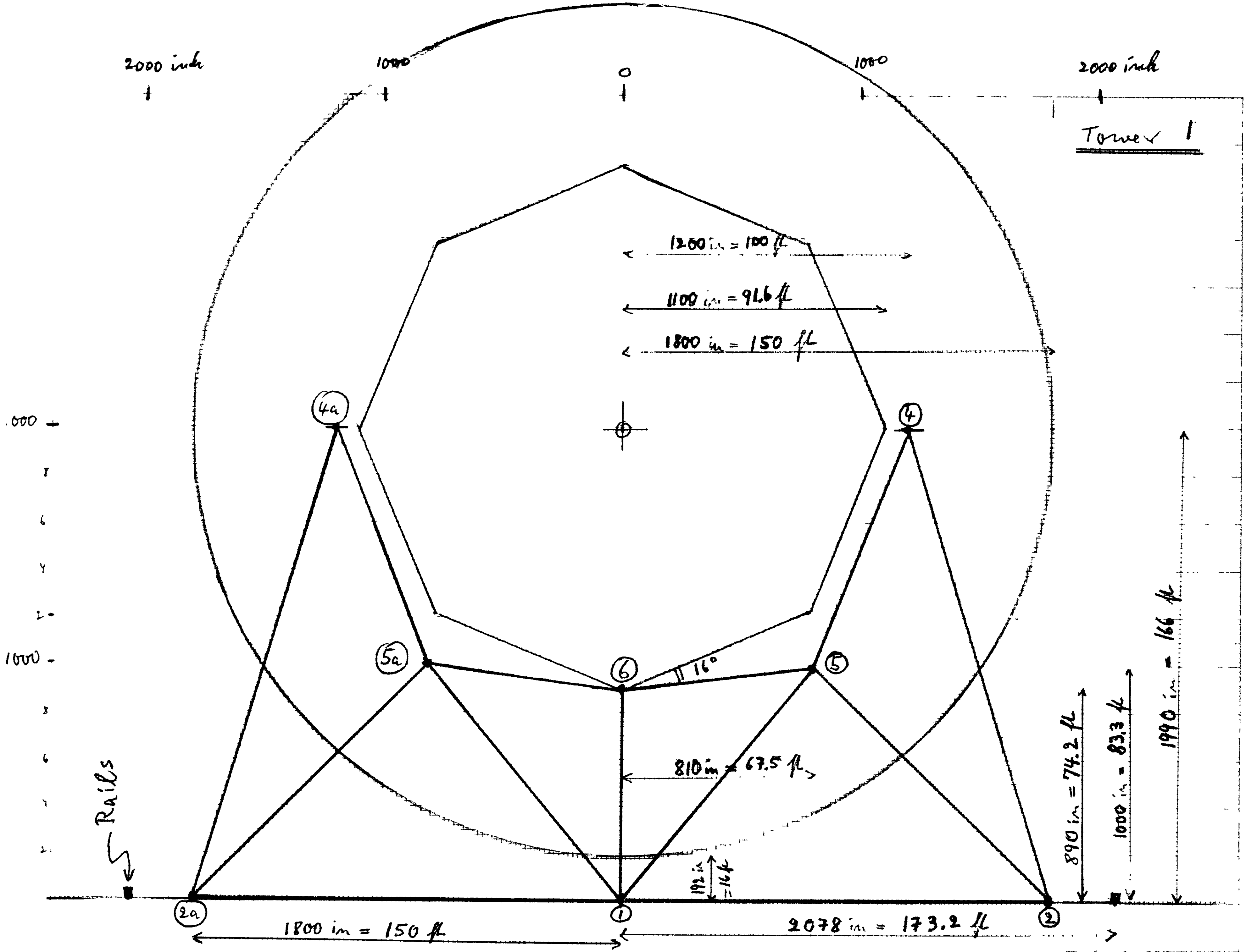
## 5. Dynamical Behaviour

Find the dynamical frequencies for the following oscillations (towers plus weight of dish):

1. Rotation about z-axis.
2. Rotation about elevation axis.
3. Parallel shift, x-direction.
4. Parallel shift, y-direction.

Data: The radius of gyration of the dish is

$$r = 77 \text{ ft about z-axis,} \qquad r = 65 \text{ ft about elevation axis.} \tag{28}$$



2000 inch
1000
0
1000
2000 inch
Tower 1
1200 in = 100 ft
1100 in = 91.6 ft
1800 in = 150 ft
4a
4
5a
6
5
16°
1000
810 in = 67.5 ft
890 in = 74.2 ft
1000 in = 83.3 ft
1990 in = 166 ft
Rails
192 in = 16 ft
2a
1
2
1800 in = 150 ft
2078 in = 173.2 ft

Fig. 1 : Sizes

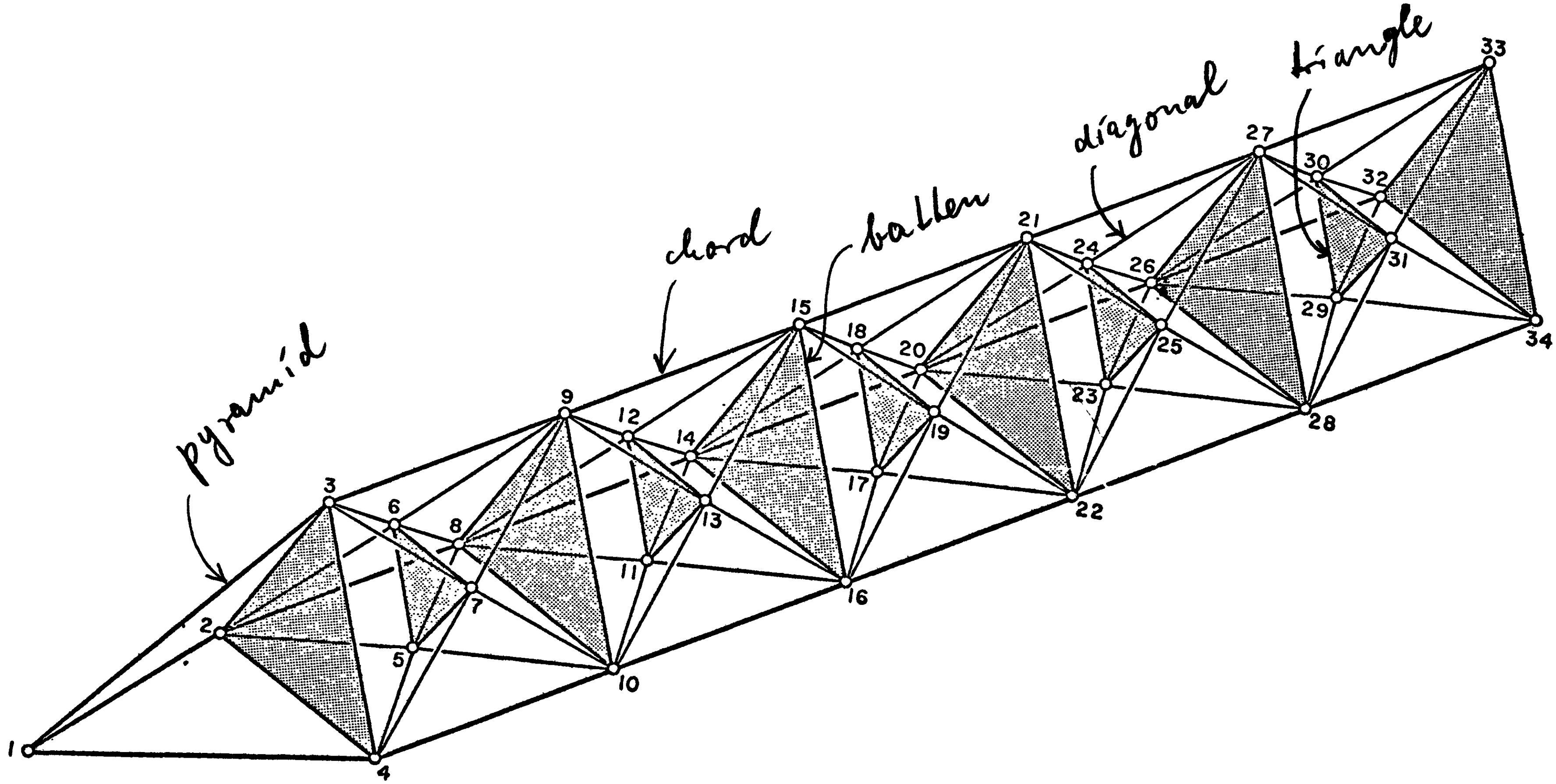
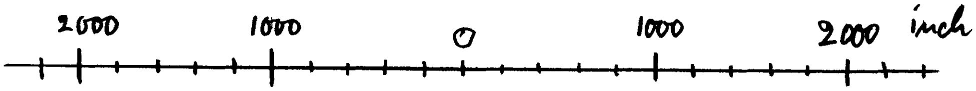
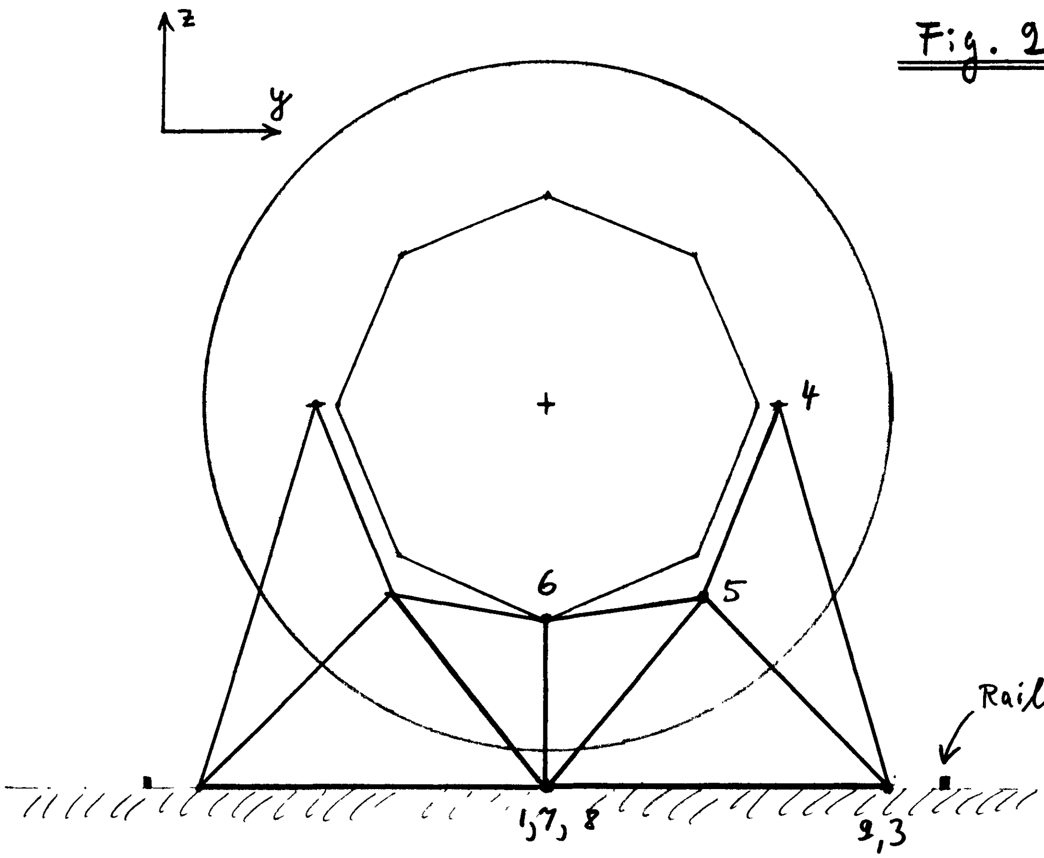



Fig. 3

points and connections of ½ member.

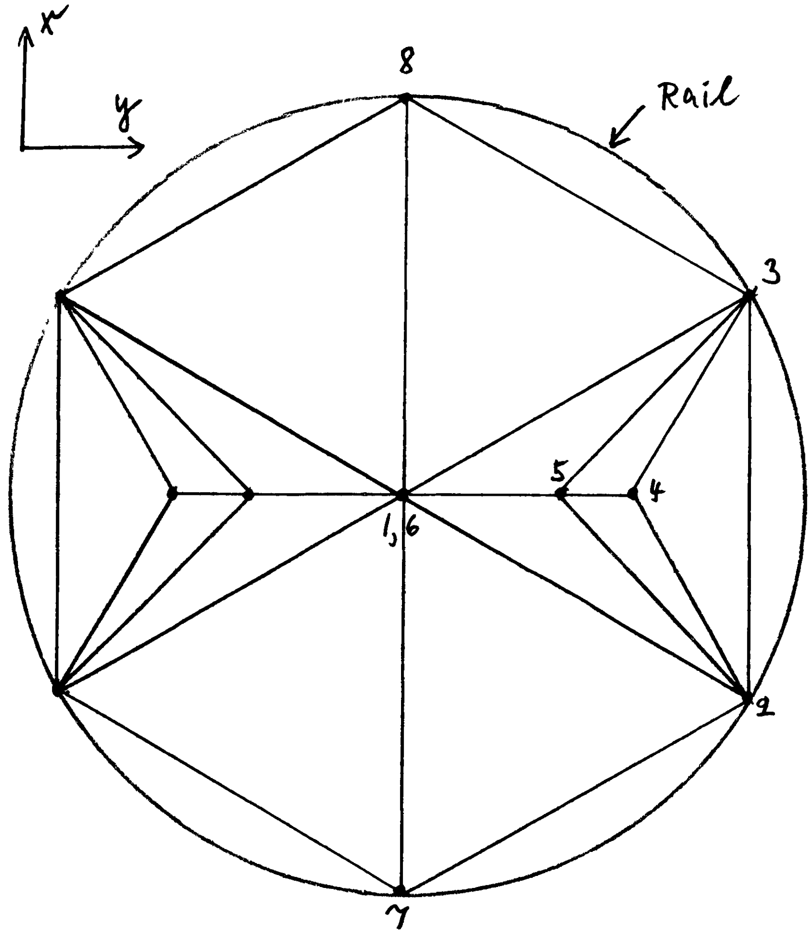


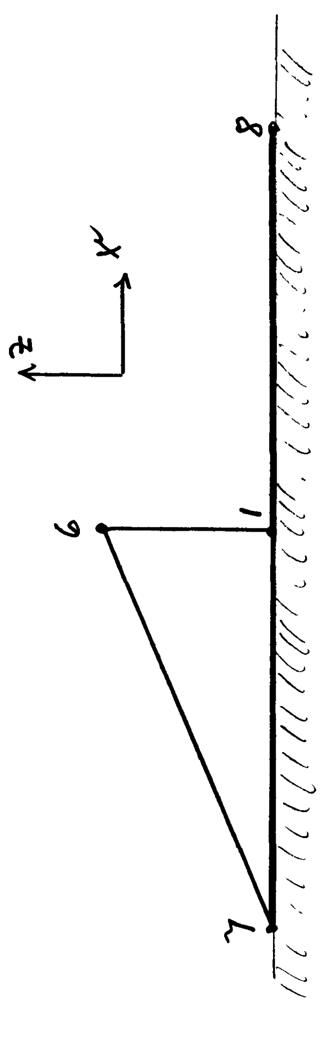

# Fig. 2 ; Towers




Restraints:

| point | x y z |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |



## Parabolic Shape, $n = 12$

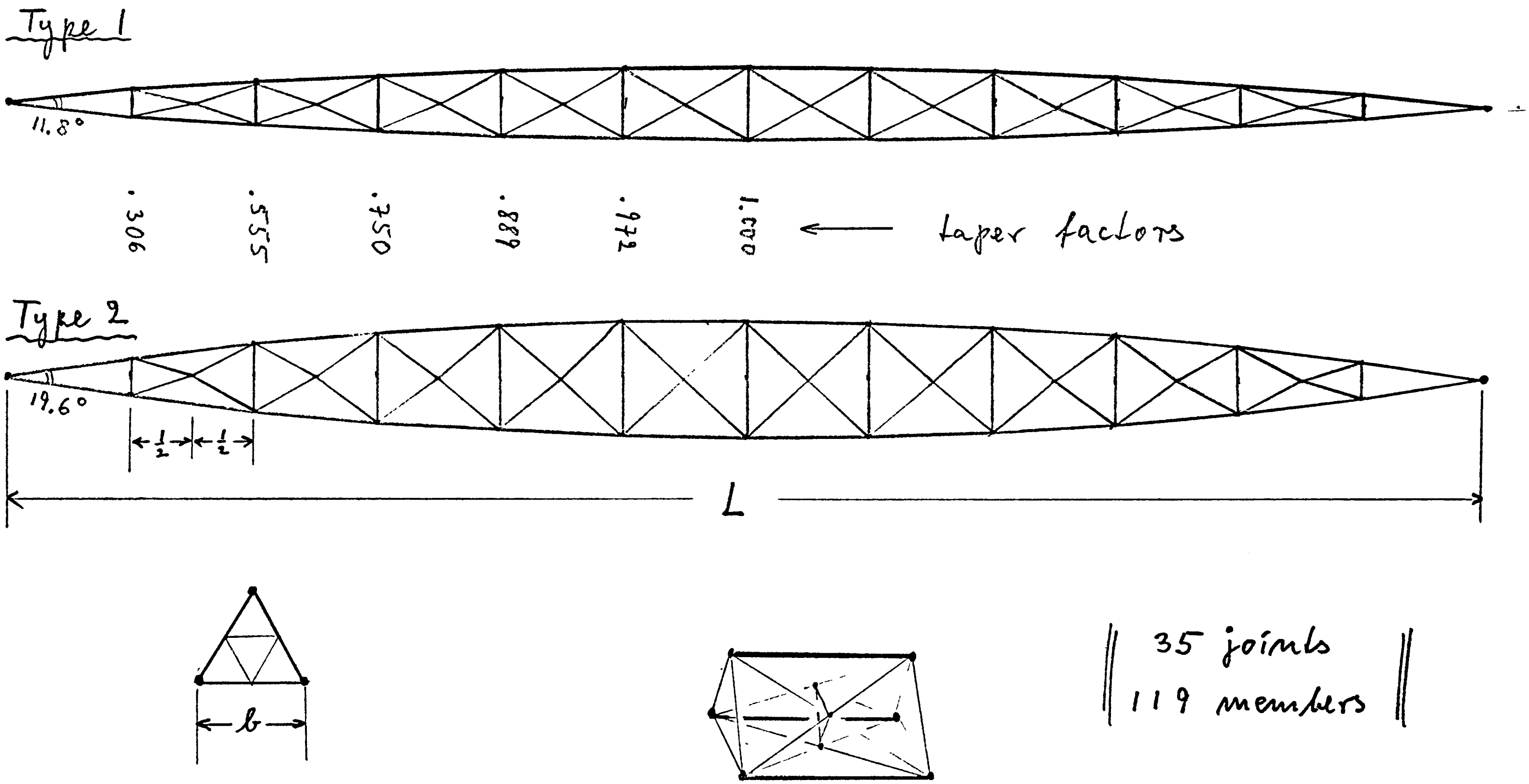



**Fig. 4:** Geometry of built-up members

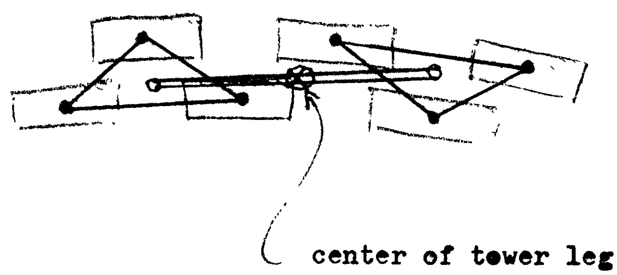



Fig. 5: Support from 6 gondolas for one tower leg.