Report 20

December 16, 1968

FINAL TOWER DATA

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I. Stiffer Tower for Dynamics

The tower system of Report 19 was designed for specified wind deformations, and for stability in survival conditions. Both demands were met with a total weight of 580 tons. Meanwhile, several discussions with O. Heine resulted in a third demand, on the dynamical behavior. For the stability of the servo system, the lowest dynamical mode of the combined dish-tower system should be at least $\nu_{dt} \geq 0.70$ cps., and, if possible

$$\nu_{dt} \geq 1.00 \text{ cps.} \tag{1}$$

First, this demand needed some changes in the dish structure, and the dish then again had to be made homologous and survival-stable. This is now solved, with a dish weight of 980 tons. Second, the stiffness of this dish, and demand (1), then define the stiffness needed for the towers. After some optimization of the tower structure and of the built-up members, the stiffness requirement finally has been met, as well as the two other demands, with a total tower weight of only

$$W = 559 \text{ tons} = 1233 \text{ kip.} \tag{2}$$

II. Design Data

The tower coordinates are given in Table 1, in inch. The zero level, z = 0, will be a few feet above ground, to allow for azimuth trucks and wheels.

The dynamical analysis was made with the following restraints:

points	x	y	z
1, 2 and 3	r	r	r
7 and 8			r

Table 1. Coordinates

	x	y	z
1	0	0	0
2	-1039	1800	0
3	1039	1800	0
4	0	1200	1880
5	0	650	875
6	0	0	780
7	-2078	0	0
8	2078	0	0

Table 2. Tower Members

	A	t	L
1-2	75	2	2078
1-3	70	2	2078
1-5	140	1	1090
1-6	30	1	780
1-7	100	1	2078
1-8	30	2	2078
2-3	30	2	2078
2-4	130	1	2230

	A	t	L
2-5	35	1	1780
2-7	40	2	2078
3-4	130	1	2230
3-5	35	1	1780
3-8	30	2	2078
4-5	140	1	1146
5-6	30	1	657
6-7	100	1	2220

Table 2 gives nominal area A, type t, and Length L for all tower members. In the tower analysis, each bar is represented by a single rod of bar area A, elasticity E = 29,000 ksi, and density ρ as given for both types in Table 3.

The other columns of Table 3 show the design data for the built-up members, with the same definitions as given in Report 19. (Short and thick members, like member 1-5, can be a single column.)

Table 3. Data for built-up member design

type	ρ	b_c/L	A_p/A	A_c/A	A_b/A	A_d/A	A_t/A
1	.325	.0557	.343	.287	.0341	.0372	.0200
2	.381	.0808	.364	.283	.0684	.0540	.0197

The stiffness of this tower is given in Table 4, where K = force/deformation. The first line, for example, means that the top of one tower will deform by 1 inch in x-direction if a force of 734 kip is acting on it in x-direction.

Table 4. Tower stiffness K

		K (kip/inch)
point 4	x	734
	y	648
	z	3690
point 6	x	584
	y	1230

III. Lateral Vibrations in Members

O. Heine mentioned the lateral vibrations of the long, built-up members and asked for their frequencies.

For a prismatic, pin-supported member (see Fig. 1) we use for the lowest mode

$$\nu = \frac{\pi}{2}\sqrt{g\,\frac{E\,I}{w\,L^4}} \quad \text{cps.} \tag{3}$$

where g = 386 inch/sec^2 is the constant of gravitation, E the modulus of elasticity, I the moment of inertia, w the weight per unit length, and L the length. The sag s under dead load is at the center

$$s = \frac{5}{384}\,\frac{w\,L^4}{E\,I}\,, \tag{4}$$

and from (3) and (4) we find:

$$\nu_m = \frac{3.52}{\sqrt{s}} \qquad \begin{cases} \nu \text{ in cps,} \\ s \text{ in inch.} \end{cases} \tag{5}$$

Since the distributions of mass and stiffness influence the sag in about the same way as they do the lateral vibration, formula (5) should be a good approximation for <u>any</u> type of long member. I have used the center sag as obtained from our "Member Analysis" program for our built-up members, calculated for horizontal position of the member. The results are given in Table 5 for all of the most critical members of towers and dish.

Table 5. Lowest modes within long members

Tower	ν_m(cps)
2-4	1.86
6-7	1.87
1-7	2.00
2-5	2.33
1-2	2.60
1-5	3.81

Dish	ν_m(cps)
feed supports	1.90
long suspension	2.48
cone bars	2.56
center bar	3.94
all others	>4.0
average	6.6

IV. Dynamics of the Combined System Towers-Dish-Members

A rigorous computer analysis of the dynamical behavior will soon be done by Simpson, Gumpertz and Heger. A preliminary analysis is described in the following, based on our computer values of stiffness and of sag under dead loads. The deformations of gears, trucks, rails, and ground are neglected, but should be small as compared to the combined deformation of towers, dish, and single members.

1. Parallel Translations

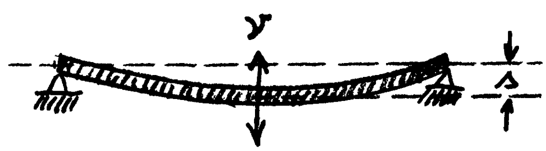

Fig. 1

$$\nu = \frac{3.52}{\sqrt{s}} \qquad (5)$$

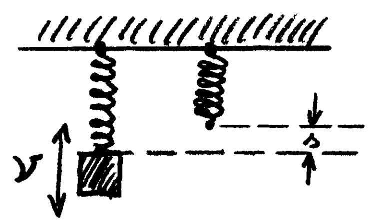

Fig. 2

$$\nu = \frac{3.13}{\sqrt{s}} \qquad (6)$$

If mass and stiffness are distributed the same way through a structure, then formula (5) can be used, see Fig. 1. The other extreme, where mass and stiffness are completely separated, is shown in Fig. 2, and formula (6) holds. Our combined system is assumed to be somewhere in between these two extremes, and we adopt

$$\nu_p = \frac{3.30}{\sqrt{s}} \quad . \qquad (7)$$

If we use for s the maximum dead load deformation of any point in the combined system, then formula (7) gives a value for the lowest mode of the z-oscillations which certainly should be on the safe side (meaning too low a value). Since we can easily apply fictitious dead load forces in x and y directions as well, we obtain the lowest modes in these directions in the same way. Actually, towers, dish, and single members are analyzed separately by different computer programs. Calling

s_d = maximum dead load sag of any dish point, with fixed elevation bearings;

s_t = deformation of tower top, under dead load plus weight of dish;

s_m = 3.57 inch = center sag of longest member, horizontal position;

we use in (7):

$$s = s_d + s_t + s_m \quad . \tag{8}$$

2. Rotations

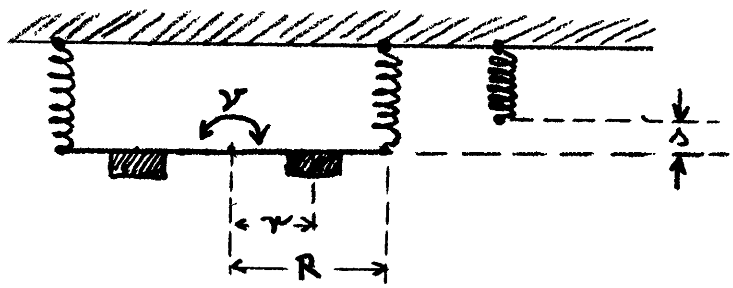

Fig. 3

$$\nu = \frac{3.13}{\sqrt{s}} \; \frac{R}{r}$$

Fig. 3 shows the case where mass and stiffness are separate; in case of a mass distribution, r is the radius of gyration, which for our present dish is 825 inch about the elevation axis and 903 inch about the z-axis. Similar to formula (7) we adopt now

$$\nu_r = \frac{3.30}{\sqrt{s_o}} \tag{9}$$

with

$$s_o = s(r/R)^2 \tag{10}$$

The lowest rotational mode is rotation in elevation, with the telescope pointed at horizon; for this case we calculate s_d with the telescope looking at zenith, but fixed at the two upper ends of the elevation wheel. We take the maximum dead load deformation of any dish point (occuring at a rim point of the surface), and we multiply this deformation by a factor of two since the elevation drive actually holds the telescope at only one point. For obtaining s_t, we apply the full weight of the dish at point 6 of the tower in x-direction, plus the dead load of the tower in x-direction. Since it is not quite clear to me how the vibrations of the single members enter, I have left $s_m = 3.57$ inch unreduced, using

$$s_o = (s_d + s_t)\ (r/R)^2 + s_m \tag{11}$$

instead of (10) and (8), to be inserted into (9), for both elevation and azimuth rotation. This procedure, again, will be on the safe side.

3. Results

Table 6 gives the resulting dynamical frequencies for the combined system of dish, towers and single members. Since I think that all of these numbers are on the safe side, I suggest that O. Heine uses in his design of the servo system $\nu \geq 1.2$ cps. Or, including the deformation of gears, trucks, rails and ground:

$$\text{all combined} \quad \nu \geq 1.1 \text{ cycles per second.} \tag{12}$$

The only somewhat uncertain case may be the elevation rotation. But if the rigorous analysis of Simpson, Gumpertz and Heger indeed should yield a value lower than (12), this could easily be corrected by some stiffening of the tower bars 1-7 and 6-7, and of the elevation wheel.

Table 6. Lowest Modes of Combined System

Telescope pointed at	ν (cps)				
	Parallel Translations			Rotations	
	x	y	z	Azimuth	Elevation
Zenith	1.28	1.26	1.48	1.39	1.32
Horizon	1.32	1.26	1.41	1.42	1.23