Report 23

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Wind and Temperature Deformations of the 300-ft Homologous Telescope

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Summary

The optical pointing system will eliminate all slow deformations occurring between ground and back-up structure of the dish. With a servo bandwidth of .3 cps, only deformations up to 3 sec wavelength remain. The longest (one-sided) deformation then is of 1.5 sec duration.

Wind measurements were taken at Green Bank at 100 ft above ground, with a time resolution of 0.5 sec. The rms pressure difference, between neighboring averages of 1.5 sec duration, is .38 of the mean pressure. A further reduction arises from the fact that gusts of 1.5 sec duration, at v = 18 mph, have a size of only 40 ft; they will average out (to a high degree) over the much larger telescope area.

The combined rms pointing error (towers, dish, feed legs) is calculated (for 18 mph) as $\Delta\theta = 2.83$ arcsec. The rms surface deformation is $\Delta z = .0135 = .34$ mm. The pointing error would be 12 times larger without the optical pointing system.

Measurements at Green Bank gave: (1) $\Delta T \leq 5\ ^{o}C$ for the difference between sunshine and shadow on calm sunny days, with white protective paint; (2) $\Delta T \leq 6.1\ ^{o}C$ per inch of wall thickness from the time-lag of heavy members during the steepest daily temperature changes (on 3/4 of all calm days where $v \leq 5$ mph); and (3) $\Delta T = 1.5\ ^{o}C$ during nights, cloudy days, or sunny windy days.

For the new telescope we suggest (a) to blow ambient air (with 20 mph) through the pipes of the feed legs, reducing T for sunshine from 5 ^{o}C to 2 ^{o}C; and (b) to build the 8 heavy back-up members from open shapes instead of pipes, reducing ΔT for time-lag by a factor two.

Thermal deformations then result in an rms pointing error of $\Delta\theta = 6.2$ arcsec, and an rms surface deformation of $\Delta z = .031$ inch $= .79$ mm on calm sunny days. For all other times we find $\Delta\theta = 3.8$ arcsec, and $\Delta z = .0093 = .24$ mm. Without the optical pointing system, the pointing error would be 4 times larger.

Comparison of wind deformation, with both thermal deformation and observational demands, shows that telescope and towers are stiffer than needed for the original goal of $\lambda = 2$ cm.

I. Wind Deformations

1. High-Resolution Wind Data

Measurements were taken (by Bruce Draine) with a heated-wire wind gauge, on our wind tower at 100 ft above ground; the instrumental time constant is 0.5 sec. Data were recorded graphically, then read off and punched on cards for computer analysis.

The program calculates, with respect to the telescope application, the relative pressure fluctuation

$$P(\tau) \;=\; (1/\overline{p})\ \mathrm{rms}(p_{i+1} - p_i) \tag{1}$$

where p_{i+1} and p_i are neighboring time-averages (of the pressure) of duration τ:

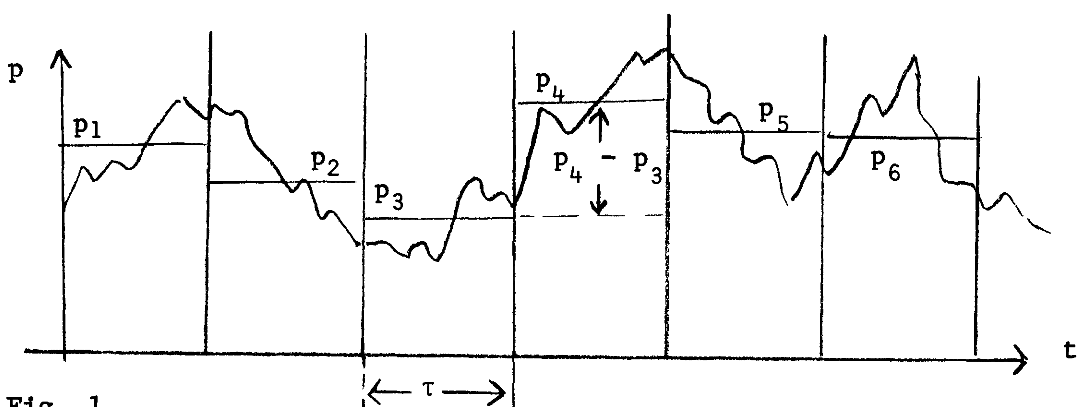


Fig. 1.

We call τ_o the instrumental time constant, and $\tau_1 = A/v$, with A = size of largest turbulence elements, and v = average wind velocity. For the fluctuation function $P(\tau)$ we then expect:

$P(\tau) \sim \tau$, for $\tau \ll \tau_o$ (linear, for smooth p(t));

$P(\tau) \sim \tau^{1/3}$, for $\tau_o \ll \tau \ll \tau_1$ (Kolmogoroff's spectrum of turbulence);

$P(\tau) \sim \tau^{-1/2}$, for $\tau \gg \tau_1$ (average over many uncorrelated large elements);

$P(\tau) \leq 1/\sqrt{2}$, for any τ.

The measurements confirm these expectations quite well, see Fig. 2. The results are given in Table 1.

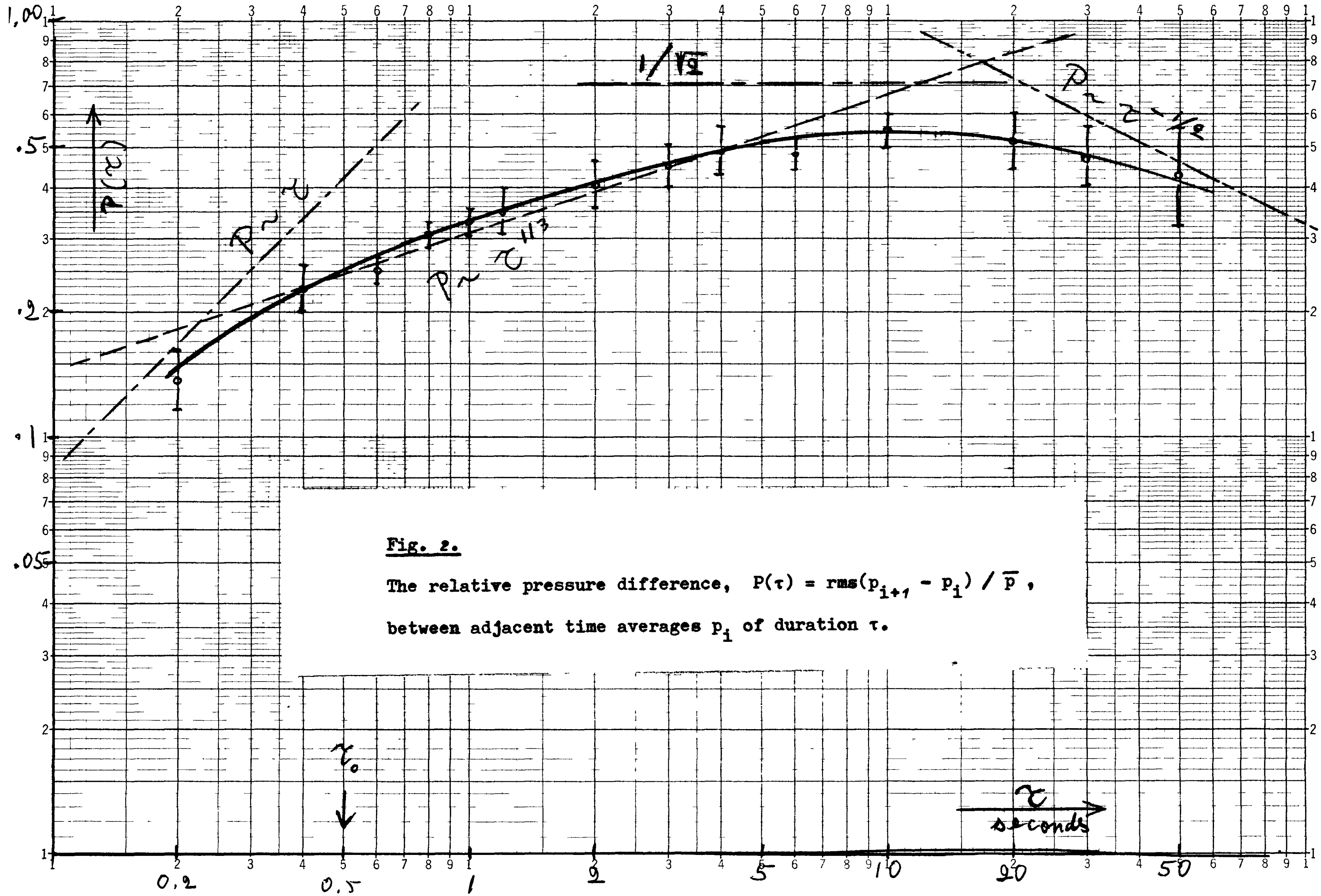


Fig. 2.

The relative pressure difference, $P(\tau) = \mathrm{rms}(p_{i+1} - p_i) / \overline{p}$, between adjacent time averages p_i of duration τ.

Table 1. Relative pressure fluctuation $P(\tau)$, as a function of the averaging time τ.

sec	$P(\tau)$
.5	.240
1	.248
2	.405
3	.450
5	.508
7	.537
10	.550
20	.518

2) The Cut-Off Time

The combined structure (telescope, drives, towers, foundation, ground) has a lowest dynamical mode of about 1.0 cps. In order to avoid oscillations and hysteresis, the sero system bandwidth is taken as 0.3 cps, corresponding to a time constant of about 3 seconds.

The combined servo and drive system then will correct all pointing errors much slower than 3 seconds, but it will leave unchanged all errors much faster than 3 seconds, with a transition region in between. As an approximation, we assume in the following that we have a sharp cut-off at 3 seconds. All slower pointing errors are completely eliminated, all faster ones are completely unaffected. Since 3 seconds is the longest duration of a full error cycle, the one-sided error itself has a longest duration of

$$\tau = 1.5 \text{ sec}, \tag{2}$$

with

$$P(\tau) = .375, \tag{3}$$

which then is the fraction of the total (wind induced) pointing error which cannot be eliminated by the optical pointing system. Actually, we should use only $P/\sqrt{2}$, the rms error with respect to the average. But since our assumption of a sharp cut-off might reduce the results somewhat, we'd better leave this $\sqrt{2}$ as a safety factor, using $P(\tau)$ as given (3).

3) Pointing Error

a) From Tower Deformation

The deformation of the towers was calculated, assuming the worst case for pointing errors: wind from front, telescope looking at 60^o elevation, which gives the highest torque; the effect of asymmetric wind gusts was added. In agreement with our own analysis, Simpson, Gumpertz and Heger find a maximum pointing error of 26 arcsec. This holds for wind with v = 18 mph (third quartile of wind distribution), acting on the telescope and the tower members, with torques from gusts and elevation tilt adding up, calculating the deformations of tower tops and elevation drive point.

If the telescope were smaller than the gust size (lateral size of most effective turbulence elements), the pointing error would be that fraction of the deformation which is faster than 1.5 sec, or $\Delta\theta$ = 26 P(1.5 sec) = 9.8 arcsec. Actually, the most effective lateral gust size (for isotropic turbulence) is only $v\tau$ = 16 mph x 1.5 sec = 39.6 ft, and the dish then is simultaneously hit by $n = (300/39.6)^2 = 57.5$ such gusts. With respect to pointing, these gusts are uncorrelated since all slower gusts are eliminated by the optical pointing system. The pointing error thus is reduced by a factor $1/\sqrt{n} = 7.5$. In general, the pointing error is

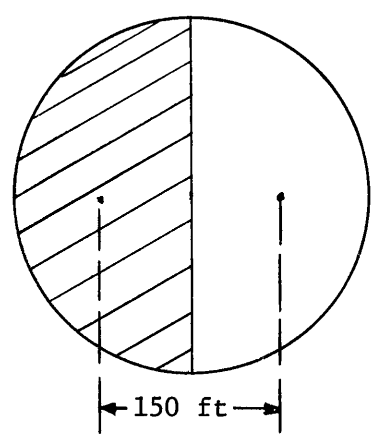
$$\Delta\theta \quad = \quad 26 \text{ arcsec} \frac{v\ \tau}{D} P(\tau), \qquad (4)$$

and in our case

$$\Delta\theta \quad = \quad 1.29 \text{ arcsec}. \qquad (5)$$

b) Dish Deformation

We divide the surface into two halves, assume different gusts on them, and ask for the resulting pointing error, if the elevation bearings are fixed.



150 ft/18 mph = 5.7 sec.

Fig. 3

In our computer analysis, we apply a constant wind of 18 mph face-on, and we obtain an average deformation of .080 inch at the surface from design (no best-fit applied). The antifocus, point 57, deforms by 0.34 inch. Since both halves of the dish meet at the antifocus, we might subtract its deformation, but for being on the safe side we subtract only 1/2 of it, leaving .080 - .017 = .063 inch.

With a distance of 150 ft between the two halves, we would obtain an angle of 7.2 arcsec, if we had a wind difference of 18 mph on both sides. Actually, this is reduced by P(150 ft) = P(5.7 sec) = .51. Furthermore, this tilt is also present at point 38 where the optical pointing system is mounted, which means that only the fraction P(1.5 sec) remains. In summary, we have

$$\Delta\theta = 7.2 \text{ arcsec } P(5.7 \text{ sec})\, P(1.5 \text{ sec}), \tag{6}$$

or

$$\Delta\theta = 1.37 \text{ arcsec}. \tag{7}$$

c) Focal Structure

From the focal cabin (10 x 13 ft, shape factor $C_s = 1.4$), the focal structure surrounding the cabin, and the four feed legs, we obtain a wind force at the focus of 755 lb. The resulting deformation was calculated as Δx = .0090 inch. With a focal length of 128 ft, and a beam-tilt factor of .844 (J. Baars, NRAO Report 57, 1966), we obtain a pointing error, for 18 mph, of

$$\Delta\theta = 1.02 \text{ arcsec}. \tag{8}$$

d) Combined Pointing Error from Wind

The deformations from towers and dish are partly correlated, and we add their errors linear. Since this combination results only from gusts, while the focal contribution is due to the wind average, we add both quadratically. The total pointing error from 18 mph wind then is (beamwidth β = 52 arcsec, for λ = 2 cm):

$$\Delta\theta = 2.83 \text{ arcsec} = .054\, \beta. \tag{9}$$

Without the optical pointing system, using decoders at the axes, we obtain a pointing error of 34 arcsec instead of (9), which means:

The optical pointing system reduces the wind-induced pointing error by a factor of 12. (10)

Value (9) is much better than actually needed, since (9) is the error for observations with 1.5 sec time constant or shorter, whereas observations of accurate positions will require much longer integration times where the short-term errors are averaged. This means our present telescope design is too stiff, its weight should be reduced.

4) Defocussing (surface deformation)

We divide the surface into three parts, with different gusts acting on them. Defocussing (non-rigid deformation) is due to the pressure difference $(p_1 + p_3)/2 - p_2$. We call the resulting pressure fraction P_d, and from the identity

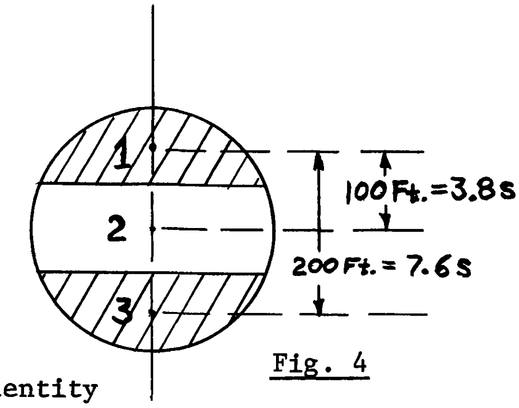


Fig. 4

$$\left(\frac{p_1 + p_3}{2} - p_2\right)^2 = \tfrac{1}{2}(p_1 - p_2)^2 + \tfrac{1}{2}(p_2 - p_3)^2 - \tfrac{1}{4}(p_1 - p_3)^2 \qquad (11)$$

we derive

$$P_d = \left\{\left[P(3.8 \text{ sec})\right]^2 - \left[\tfrac{1}{2} P(7.6 \text{ sec})\right]^2\right\}^{\frac{1}{2}} = 0.415. \qquad (12)$$

For this type of deformation, point 57 certainly is a neutral point. We thus subtract its deformation from that of the surface and obtain .080 - .034 = .046 inch deformation, if part 1 and 3 have a wind of 18 mph while part 2 has none. This deformation must be multiplied with P_d from (12). Furthermore, the parts of Fig. 4 are about twice as long as they are wide and thus will contain about two independent turbulence elements each, giving a reduction factor $1/\sqrt{2}$. Altogether, the rms surface deformation is

$$\Delta z = .046 \text{ inch } P_d/\sqrt{2} = .0135 \text{ inch}. \qquad (13)$$

Or, with $\lambda = 2$ cm,

$$\Delta z = .0172\ \lambda. \qquad (14)$$

This deformation again is smaller than actually needed. Demanding $\Delta z = \lambda/32$, say, we find

Dish structure is too stiff
by a factor of 1.78. (15)

Although a good deal of the dish strength is needed for survival, the total weight could still be reduced by a significant fraction. It might be considered worth-while to change the present design.

II. Thermal Deformations

1) Time-Lag of Thick Members

During a fast change of the ambient air temperature, thick-walled structural members need a longer time for adaption than thin members, which results in temperature differences in the structure. This question is treated in detail in Report 17 (Jan. 1967). From our own experiments we find a time-lag of 1.73 hours per inch of wall-thickness, for steel pipes with white protective paint, in the absence of wind ($v < 5$ mph). From measurements at Sugar Grove and Green Bank we find a steepest daily temperature change $\leq 3.5\ ^{\circ}$C/hour on 3/4 of all days. Combining both values we find that on 3/4 of all calm days the maximum temperature difference between thick and thin members is less than

$$6.1\ ^{\circ}\mathrm{C}\ /\ \text{inch wall thickness.} \qquad (16)$$

Members with more than 1 inch thickness occur only in the towers and in the telescope suspension, and both their deformations are eliminated by the optical pointing system. The heaviest remaining members are in the cone (from antifocus to basic octagon) and have a wall thickness of .500 inch, yielding $\Delta T = 3.05\ ^{\circ}$C. We suggest designing these cone members from open shapes instead of pipes, which reduces their time lag by a factor 2, yielding $\Delta T = 1.53\ ^{\circ}$C. Almost all other members are below .25 inch thickness. We thus adopt

$$\begin{array}{c} \Delta T = 1.5\ ^{\circ}\mathrm{C} \text{ from time-lag} \\ \text{(if cone members have open shape)} \end{array} \qquad (17)$$

2) Sun and Shadow

Experiments were done at Green Bank with a 140-ft spare panel, pointed at the noon sun. With white protective paint we found the difference between the surface and a member in its shadow, on clear sunny summer days, without wind, as

$$\Delta T \leq 5^{\circ}\ \mathrm{C} \text{ from sunshine.} \qquad (18)$$

(The maximum ever measured was 9° C on an exceptionally clear and calm day.) Values (16) and (18) also agree with more recent measurements on 12 different points in the structure of the 140-foot.

3) Deformation of Cone Members

The optical pointing system completely eliminates the thermal deformations of foundations, towers, and telescope suspension. It will also eliminate the major part of the deformation of the cone members, but to be on the safe side we adopt only a factor of two.

The cone members are 1605 inch long. If they deform by ΔL, the surface moves up by 1.36 ΔL for geometrical reasons. With ΔT in ^{o}C, we obtain a surface deformation of

$$\Delta z = .0263 \, \Delta T \text{ (inch)}. \tag{19}$$

We consider the case that one side of the telescope is in sunshine, while center and other side are in shadow. The cone deformation then results, first in a rotation of the main dish structure including the feed system, and the resulting pointing error is

$$\Delta\theta = 1.24 \, \Delta T \text{ (arcsec)}. \tag{20}$$

Second, we obtain a non-rigid surface deformation, with an rms of $(1/6)\sqrt{2}$ times (19), or

$$\Delta z = .0062 \, \Delta T \text{ (inch)}. \tag{21}$$

4) Feed Legs, Vertical

Let all four feed legs be ΔT degrees warmer than the rest of the structure. If they expand by ΔL, the focal cabin moves up by $\Delta z = 1.14 \, \Delta L$. For L we use only that part of the legs which is above the surface (L = 1700 inch). We obtain

$$\Delta z = .0233 \, \Delta T \text{ (inch)}. \tag{22}$$

Using formula (17) of J. Baars (NRAO Report 57), this axial movement results for λ = 2 cm in a gain change of

$$\Delta G/G = 4.80 \times 10^{-4} \, (\Delta T)^2 \tag{23}$$

which is negligible for $\Delta T \leq 5^{o}$ C.

5) Feed Legs, Horizontal

Let one leg be $\Delta T/2$ degrees warmer, the opposite leg $\Delta T/2$ cooler, than the rest of the structure. If the first leg expands by ΔL, the focal cabin moves horizontally by $\Delta x = 2.22 \, \Delta L$. We use again L = 1700 inch and obtain for the cabin movement

$$\Delta x = .0226 \, \Delta T \text{ (inch)}. \tag{24}$$

For $\Delta T = 5^{\circ}$ C, gain and coma problems can be neglected, but not the pointing error. With a beam deviation factor of .844 we obtain

$$\Delta\theta = 2.51 \, \Delta T \text{ (arcsec)}. \tag{25}$$

6) Summary and Conclusions

Going through equations (17) to (25), we find, first, that the effect of sunshine is more severe than that of time-lag (if the cone members are open shapes). Second, the pointing error is more severe than the surface deformation. Third, the largest error comes from the feed legs. With $\Delta T = 5^{\circ}$ C for sunshine, the pointing error due to the feed legs is

$$12.6 \text{ arcsec} = .24 \, \beta \text{ (for } \lambda = 2 \text{ cm).} \tag{26}$$

One could call this pointing error acceptable after comparison with good existing telescopes. The pointing error of the 140-ft telescope, for example, goes up to 25 arcsec on calm sunny days or during fast temperature changes, which is .224 β at λ = 2 cm. On the other side, observations of weak sources could be done much faster if pointing error (26) could be reduced by a factor of two or more.

In Report 17 (Jan. 1967) I have already suggested to blow ambient air through long hollow members; a rough estimate showed that their thermal deformation could be reduced by a factor of 3 if the air is blown with 15 - 20 mph. These blowers should be used on sunny calm days, but could be turned off all other time. According to Report 22 (Feb. 1969), Table 1, the chords of the feed legs are standard steel pipe of nominal diameter 4 inch; the outer diameter then is 4.5 inch, the inner diameter 4.03 inch, and the wall thickness is .237 inch. Since a cooling system of this type seems to be no problem, we will suggest to apply it, and we will assume that ΔT then is reduced from 5 to 2° C.

$\Delta T = 2^{\circ}$ C in sunshine for feed legs, with ambient air blown (20 mph) through all chord pipes. (27)

Next, we consider four different observing conditions.

a) Sunny calm days (10 am - 4 pm, say, wind $\leq$ 5 mph). We use $\Delta T = 5^{\circ}$ C for the cone, and 2° C for the feed legs. The pointing error then is 6.2 arcsec from cone members, and 5.0 arcsec from feed legs. In no telescope orientation do both errors add up with their full amount, frequently they even counteract each other. We thus adopt the maximum, $\Delta\theta = 6.2$ arcsec. The rms surface deformation from cone members is $\Delta z = .031$ inch.

b) Fast emperature changes (mostly at sunrise or sunset). We use $\Delta T = 1.5^{\circ}$ C from (18). The feed legs have no effect. The cone members yield an rms surface deformation of .0093 inch. No pointing error because of symmetry.

c) Clouds, or sun plus wind ($v > 7$ mph). Present measurements at the 140-foot show temperature differences of 1 - 2° C, part of which will be measuring error; we adopt $\Delta T = 1.5^{\circ}$ C. The pointing error then is 1.86 arcsec from the cone, and 3.77 from the feed legs. Again, they cannot add up, and we adopt the maximum, $\Delta\theta = 3.8$ arcsec. The rms surface deformation is .0093 inch.

d) Nights. Since the telescope receives different radiations from sky and ground, and since the air temperature is not completely homogeneous, small temperature differences in the structure result. From our measurements we find $\Delta T = 1.5^{\circ}$ C, as for item (c) and therefore giving the same errors.

We thus have only two different conditions: sunny calm days, and all other time. The results are summarized in Table 2.

Table 2. Thermal deformations

	pointing error		rms surface deformation	
	arcsec	fraction of beam	inch	fraction of λ (2 cm)
sunny calm days	6.2	.119	.031	.039
else	3.8	.073	.0093	.012

For a comparison, we divide the maximum thermal error by the maximum wind error. We find a factor of 2.2 for the pointing error, and a factor of 2.3 for the rms surface deformation. This shows again that the telescope weight could be reduced.

Without the optical pointing system, with decoders at the axes, the worst case occurs if one tower is $\Delta T = 5^{\circ}$ C warmer than the other, which tilts the elevation axis by 13.3 arcsec. The cone deformation, without optical pointing, gives twice the value of (20), or 12.4 arcsec. Both will add up with full amounts to $\Delta\theta$ = 25.7 arcsec. Comparison with Table 2 shows:

The optical pointing system reduces the thermal pointing error by a factor of 4.1. (28)