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WIND-INDUCED VIBRATIONS OF PIPES

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I. Basic Formulas

A problem which was not incorporated in the original design study is given by the wind-induced vibrations of long, thin pipes. O. Heine mentioned the problem, suggested the use of high strength steel, and provided the following formulas. Vibrations occur if the Reynold number

$$R_e = 780\ Vd \tag{1}$$

is

$$R_e < 2\times10^5 \text{(laminar flow)} \tag{2}$$

with V = wind velocity (mph) and d = outer pipe diameter (inch). The lateral wind lift (perpendicular to both wind and pipe axis) then is, for a pipe of length L (inch), in pounds:

$$F_w = 8.40\times10^{-6} V^2 Ld \tag{3}$$

which is the wind force parallel to the wind, multiplied by a v. Karman factor of 0.5 (for $10^2 < R_e < 10^5$; fast decreasing for larger R_e).

Vibrations are important only close to resonance, where the wind-induced vibration frequency

$$f_w = 3.52\ V/d \tag{4}$$

equals the natural frequency of the pipe

$$f_n = 1.75\times10^5 d/L^2 \tag{5}$$

(where half-clamped pipes are assumed).

Resonance, then, occurs at a critical wind velocity of

$$V_{cr} = 4.97\times10^4 (d/L)^2. \tag{6}$$

Assuming a damping of 2%, the dynamical force is, at resonance,

$$F_d = \frac{\pi}{.02} F_w = .00140\ V^2 Ld, \tag{7}$$

and gives a stress in the extreme fiber of

$$S_v = 1/2 \frac{F_d L}{dA}; \qquad (8)$$

where A = bar area (inch^2), for thin-walled pipes.

II. Three Critical Limits

Using these formulas, we derive for general use three critical limits. Since the bar area A, and the ℓ/r ratio

$$\Lambda = L/r = \text{slenderness ratio} \qquad (9)$$

are the most directly used design parameters, we express the three limits in these terms, eliminating L and d. Our adopted survival velocity yields the first limit; from (6) we find

$$\Lambda \leqslant 68.5, \text{ for } V_{cr} \geqslant 85 \text{ mph}; \qquad (10)$$

pipes below this slenderness do not vibrate in resonance within our survival range. Second, from (1), (2), and (6) we derive

$$\Lambda \leqslant 57.6\ A^{1/3}, \text{ for } R_e\ (V_{cr}) \geqslant 2\times10^5. \qquad (11)$$

Pipes below this limit do not vibrate in resonance because of laminar flow. Third, the stress in the extreme fiber, at resonance, can be written from (7), (8), and (6), in lb/inch^2, as

$$S_v = 6.45\times10^7\ A^{1/3} / \Lambda^2. \qquad (12)$$

Or, if $S_v < S_0$ is demanded, with some adopted stress limit S_0, we have

$$\Lambda > 100\left(\frac{6.45 \text{ ksi}}{S_0}\right)^{1/2} A^{1/6}. \qquad (13)$$

Pipes above this limit do vibrate in resonance, but the resulting stress stays below the adopted limit. If no other stress were present, we may adopt a safety factor of 1.5, say, and an endurance factor of 2, meaning that $1.5\times2\ S_0$ shall be below the yield point, S_y, or

$$S_0 = (1/3)\ S_y \qquad (14)$$

In the presence of an additional axial stress S_a, we suggest the following procedure. We call $Q = S_a / S_m$ the stress ratio, with S_m = maximum allowed axial stress according to the ℓ/r ratio and the type of steel used. If Q = 1, all allowed stress is already used up, and none is left for vibrations; for Q = 0, (14) holds. We thus use

$$S_0 = (1/3)\ (1-Q)\ S_y. \qquad (15)$$

III. The Critical Range

All three limits are shown in Fig. 1. For (15) we have taken A36 steel, $S_y = 36$, and Q = .50, giving $S_0 = 6.0$ ksi. Some other limits are also drawn, for Q = .75 with $S_0 = 3$ ksi, and Q = .88 with $S_0 = 1.5$ ksi, representing limit (13) for pipes under heavy axial loads.

We obtain an elongated unstable triangular range, surrounded by stability. The thickness of the unstable range depends on axial loads and steel type. If all pipes of a structure originally are designed with $Q \leq 1.0$ for A36 steel, the unstable range will disappear completely if we change to steel of higher yield strength, such that the stress limit (13) coincides with the most initial point, A = 1.68 inch^2 and $\Lambda = 68.5$. We find

$$\text{No critical range, if } S_y \geq 85 \text{ ksi}. \qquad (16)$$

This type of steel would be needed for a pipe of the given structure which happens to be at the most critical point and has already Q = 1.0. If not, some value smaller than (16) would be enough.

IV. Application to Homologous Telescope Design

In our present telescope structure, we have taken the longest diagonal from each member (except the surface bars which actually are panels and would demand a separate treatment). Their values of A and are plotted in Fig. 1, with division of Q in three classes. The result is the following.

33 diagonals do not vibrate;
77 diagonals vibrate, but uncritical for A36 steel; (17)
30 diagonals vibrate critical for A36 steel.

The 30 critical cases all become uncritical with regard to (13) and (15) if a steel is chosen with

$$S_y = 60 \text{ ksi}. \qquad (18)$$

It thus might be recommended to build 30 of the build-up members from high-strength steel meeting (18), and all other members from normal A36 steel.

All pieces of the build-up members can be made from standard-weight pipes. The diagonals are checked in the present calculations, and the battens of all telescope members have the same L, A and Q as the diagonals. The chords and pyramids are thicker and thus less critical. And the small triangle bars (secondary bracing) have practically no loads, Q = 0, and their small bar area and high ℓ/r ratio places almost all of them above the critical range. Whether this holds for all triangles, should be checked individually.

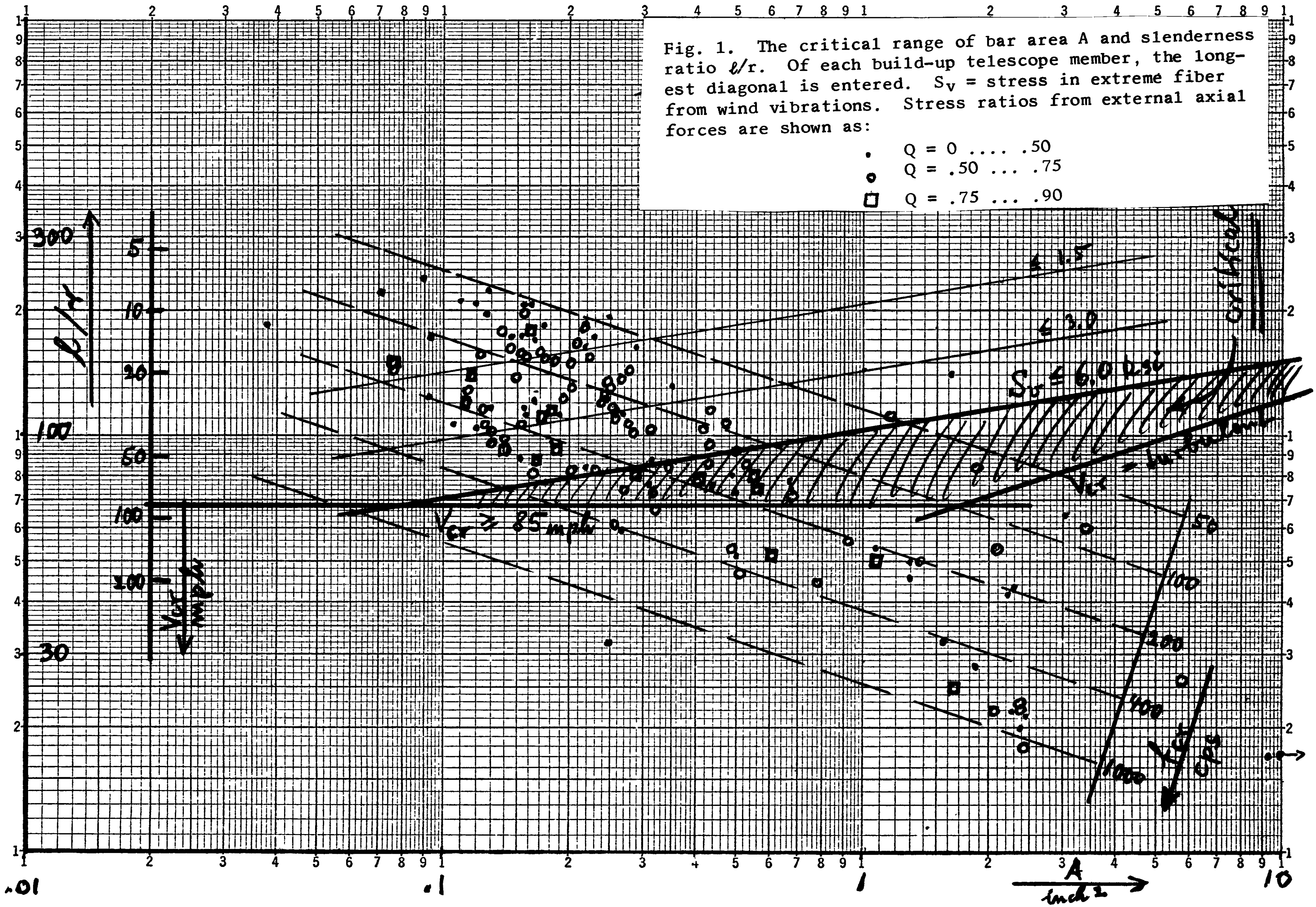

Fig. 1. The critical range of bar area A and slenderness ratio ℓ/r. Of each build-up telescope member, the longest diagonal is entered. S_V = stress in extreme fiber from wind vibrations. Stress ratios from external axial forces are shown as:

- • Q = 0 .... .50
- o Q = .50 ... .75
- □ Q = .75 ... .90