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DESIGN AND PERFORMANCE OF THE 300 FT HOMOLOGOUS TELESCOPE

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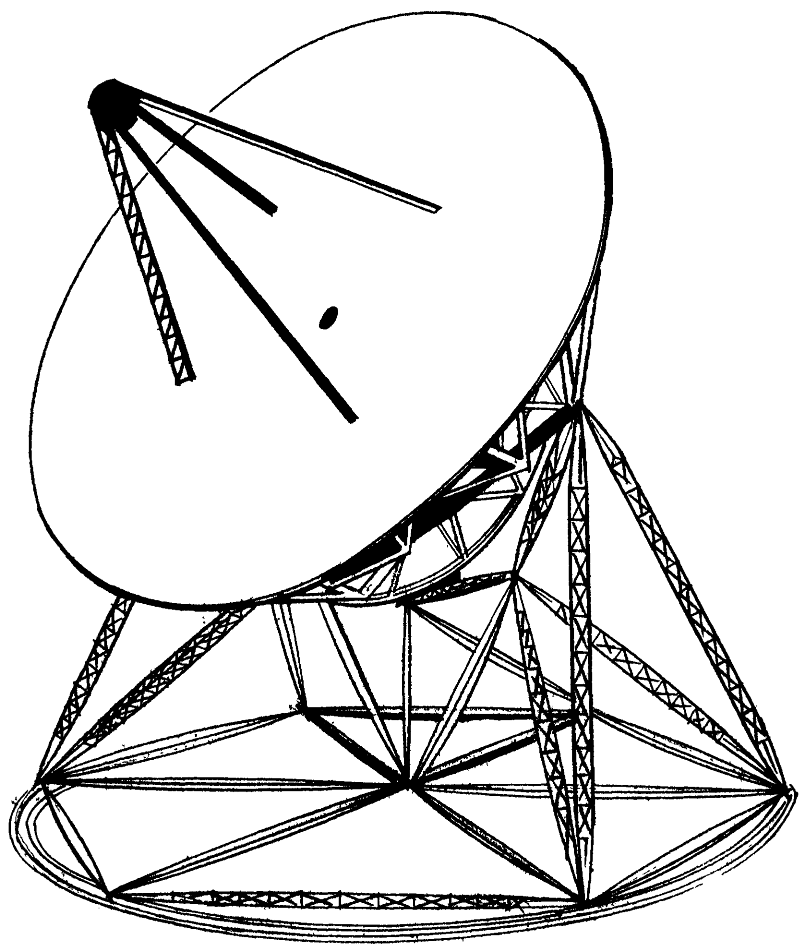

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Summary

The proposed telescope tries to provide a breakthrough in telescope design, regarding both performance and cost. First, the effect of gravity is omitted by homologous deformations, yielding a perfectly focussing mirror for any angle of tilt. This problem can be solved exactly, without any extra cost; but replacing the calculated bar areas by commercially available ones leaves an rms surface deviation of .015 inch = .39 mm for 90° tilt. Second, the optical pointing system (servoed platform, light beacons on ground) eliminates any need for accurate foundations; also, it cuts down the wind-induced pointing error by a factor of 12, and the thermal one by a factor of 4. The combined pointing error (thermal, wind, instrumental) is 7.9 arcsec on sunny calm days ($v \leq 5$ mph), and 6.2 arcsec during all other times ($v \leq 20$ mph), for an instrumental error of 5 arcsec. If this can be reduced to 3 arcsec, which seems possible for not too much extra cost, the combined pointing error is 6.9 arcsec on sunny calm days, and 4.9 arcsec else. Third, a highly automated manufacturing procedure is suggested where all details are delivered by NRAO in the form of punched cards, which should reduce the cost considerably.

The telescope is held by two elevation bearings on top of two towers which rotate 360° in azimuth on standard railway. The combined weight of the telescope and towers is 1800 tons.

The surface consists of triangular flat aluminum plates of 3 ft. length, mounted on easily adjustable studs, and floating with respect to thermal expansion. The maximum deviation from the plane defined by the three corners shall be $0 \leq \Delta z \leq .12$ inch (down only). The rms surface deviation from a best-fit paraboloid is calculated for 11 different items; the combined rms deviation, multiplied by 16, yields a shortest wavelength of $\lambda = 1.5$ cm for calm sunny days, and $\lambda = 1.0$ cm for all other time.

I. Basic Principles

1) Homologous Deformations

If a telescope is tilted in elevation angle, it must deform under its own weight, and this deformation sets a lower limit to the shortest observational wavelength, see Fig. 1 and Appendix 1. For a 300 ft. telescope, this "gravitational limit" is about $\lambda = 8$ cm, for any good conventional design. For other diameters, $\lambda \sim D^2$. There are three ways of passing this limit:

1. Servo motors in the structure or at the surface panels, as tried at Sugar Grove.
2. Surface floating on a system of levers and counterweights, as used with large optical telescopes.
3. Designing a structure which deforms completely unhindered if tilted, but which deforms the surface from one paraboloid of revolution into another one, thus yielding an exactly focussing mirror for any angle of tilt. It is called a homologous deformation, since it deforms one member of a family of surfaces into another member of the same family.

The principle underlying homologous deformations is not new. Several telescopes have been designed with the specification that the rms deformations must be below $\lambda/16$; but after the telescope was built and used, it turned out that it actually worked better than specified. The structural deformations themselves do not matter for observations; only the deviations of the surface from a best-fit paraboloid of revolution are significant, and these deviations are always less than the structural deformations, since any least-squares fit must diminish the residuals. The concept of "homologous deformations" was introduced and was given a definition in an LFST Report (June, 1965), see also Appendix 1. As a first approach to homology, the concept of an "equal-softness structure" was developed; then it was shown that exact solutions of the homology problem are possible, and two exact solutions for two-dimensional structures were given.

The task of making the deformations of a telescope more and more homologous can be attacked with a trial-and-error method by gradually changing and improving a design, and good results can be achieved this way (Rohlfs, K., "Das Bonner 90-m-Radioscop," Sterne und Weltraum, Vol. 5, Mannheim, Germany, 1966, p. 104.). The aim of our investigation was to present a rigorous mathematical method for obtaining exact homology solutions within 1 hr or 2 hr on a computer.

The parameters defining the deformation from one paraboloid to the other will be called "homology parameters" -like parallel translations, change of focal length, and change of axial direction. With regard to observation, these changes do not matter; all existing tiltable telescopes show changes of this kind, and mostly have means to correct for them, i.e., a focal adjustment.

If a telescope is to be designed to have homologous deformations, the design must have four different types of degrees of freedom: (1) the geometrical shape of the structure (coordinates of all joints); (2) the topological description (which of the joints are connected by members, which points are considered surface points, and at which points is the structure held); (3) the cross sections (bar areas) of all members; and (4) the homology parameters. Our method solves simultaneously for cross sections and homology parameters, but considers geometry and topology as given.

The conditions of homologous deformations lead to algebraic equations of very high order. A direct solution seems impossible, and a linearized iterative method was chosen; one starts with a "first guess" (initial values) for all cross sections, and in each iteration step all cross sections are changed simultaneously in such a way that the deviations of the surface from a best-fit paraboloid of revolution become zero. Since this is a linearized method for a nonlinear task, several iteration steps are required to achieve a given accuracy. To make the task uniquely defined, the method selects (out of all possible homology solutions) that solution which is most similar to the first guess. If desired, it selects a solution where the homology parameters are as small as possible.

We use Newton's method for finding the zero point of a given function, generalized to n variables (n = number of structural bars plus homology parameters). The function whose zero is wanted is ΔH, the rms deviation of N structural surface points from the best-fit paraboloid of revolution. Newton's method then needs the derivatives of ΔH with respect to all bar areas, $\partial\Delta H/\partial A$. Generally, the deformations Δx are found from the forces F with the help of the inverse, K^{-1}, of the stiffness matrix, K, as

$$\Delta x = K^{-1} F \tag{1}$$

and what we need for obtaining $\partial\Delta H/\partial A$ is the derivative of the inverse stiffness matrix with respect to all bar areas, $T = \partial K^{-1}/\partial A$, which is a tensor of three dimensions and can be obtained as

$$T = \partial K^{-1}/\partial A = -K^{-1}(\partial K/\partial A)K^{-1} \tag{2}$$

from the derivatives of the stiffness matrix. What makes the method so easy is the fact that the tensor $\partial K/\partial A$ does not depend on the bar areas A; its elements are very simple geometrical terms. (It is not necessary to store the tensor, its elements are calculated whenever needed). The combined task of (a) achieving homology and (b) selecting that solution which is most similar to the first guess is solved by the method of "Lagrangean multipliers." Since a simultaneous solution of all n variables is wanted, a set of n linear equations must be solved in each iteration step.

The method is described in detail in Appendix 2. It was first published as Report 4 (Nov. 1965). It was programmed in 1966 by R. Jennings and M. Biswas in the Department of Civil Engineering at the University of Virginia, Charlottesville, Va. The application to simple telescope structures (N = 9 to 13) gave mostly a fast convergence (ΔH decreasing by a factor of 3 or more for each iteration step). The final accuracy depends only on the calculating accuracy of the computer and is $\Delta H = 10^{-4}$ to 10^{-5} inch for 300 ft diameter telescopes. The method was reprogrammed in 1967 by W. Y. Wong of NRAO, minimizing time and memory requirements. Furthermore, it now converges only to such solutions where each bar has at least a minimum bar area demanded for survival conditions. If no such solution exists, the program yields enough information for finding out which geometrical changes might be necessary for obtaining convergence, or which bars should be omitted. The new method actually calculates only one quadrant of an x,y-symetrical structure.

The new method allows the treatment of more complicated structures, up to N = 57 equally-spaced homologous points on the total surface, with $p \leq 60$ structural joints and $m \leq 190$ members in one telescope quadrant. On our IBM 360-50, the complete analysis of the original structure then takes 20 minutes, and each iteration step takes 80 minutes. Auxiliary programs check the clearance between any two members, and the separating angle between two members at a common joint. Whereas simple structures mostly converge on first try, we found that more complicated ones depend much more on the geometry and thus need many initial tries until convergence occurs.

For survival conditions, we have chosen 20 lb/ft^2 of snow or ice, or a wind of 85 mph (one chance in 100 years) in stow position, with a safety factor of 1.92 below the yield point. For the highest wind during observation, we have taken 18 mph (third quartile), where the program calculates the average surface deformation from design (no best-fit used). The homology iterations are stopped when $\Delta H \leq .005$ inch is reached.

Our present structure was iterated down to $\Delta H = .003$ inch. Replacing all calculated bar areas by commercially available pipes from the Steel Manual increased it to $\Delta H = .015$ inch. The axial and lateral displacements of the focus are about one inch.

2) The Optical Pointing System

In most radio telescopes the pointing is measured at the axes or drive rings (too far away from the telescope surface), and with respect to some structural elements or rails (stressed by heavy loads). The most logical way seems to be: measuring the pointing where it matters (right at the apex), and with respect to something unstressed and unmovable (fixed points on the ground). This can be done by optical means.

Some satellites, rockets and balloon telescopes use already optical pointing devices "locked-in" to the bright rim of earth or sun, or to some brighter stars. With the help of J. Findlay we have started an investigation into the availability, accuracy, and cost of such devices. The basic idea of their application is described in Figure 2. In principle, we need three beacons, and only two if the direction of gravity is measured independently by some pendulum. Actually we should have at least twice as many, because the light paths will occasionally be blocked by structural parts. This method does not work in heavy fog or cloudburst, but then we cannot observe at short wavelengths anyway; and since no high accuracy is needed for long wavelengths, the telescope might have an additional pointing system of conventional type for those cases.

The method has two major advantages. First, it keeps the pointing accuracy completely independent of the accuracy of elevation rings and azimuth rails. As far as pointing is concerned, one could as well drive the telescope on a dirt road. Actually, we use standard railroad equipment for the azimuth ring, (see Appendix 3), with 100,000 dollars per mile for material and erection, 400 dollars per mile and year for maintenance, and an accuracy of 1/4 to 1/2 inch vertical and lateral. The maximum lateral load is 5 per cent, and the maximum longitudinal load 10 per cent, of the downward load.

Second, with respect to thermal deformations, constant wind loads and all gusts slower than the servo loops, we omit completely all deformations occurring between apex and ground (in telescope suspension, bearings, elevation ring, towers, rails and foundations). The pointing errors from wind deformations thereby are cut down by a factor of 12, and those from thermal deformations by a factor of 4 (both values from our present design of dish and towers). At the same time, the price of foundations and rails is cut down by a factor of 7 (from a cost estimate by Sidney Smith of NRAO).

O. Heine has studied various possibilities of its realization. He finally suggested a scheme where the beacons in Fig. 2 are autocollimated lasers, and where the platform does not carry telescopes but a six-sided mirror, with six beacons on the ground which gives enough redundancy with regard to blocking by structural members. This system will yield a pointing accuracy of 5 arcsec at least; maybe it would be improved to 3 arcsec without much extra cost.

The most elegant method would be to use a gyro compass and a tilt sensor, without any optical connection to the ground. O. Heine found that even this seems possible with available parts, but it will need some further study regarding its behaviour on a moving telescope shaking in the wind. Thus, in the present study we have adopted an optical system with 5 arcsec accuracy.

3) Automated Manufacturing of Members and Panels

The proposed telescope design looks somewhat more complicated than a conventional one. In order to facilitate a very accurate and fast structural analysis as used in each iteration step of the homology program, we have designed the telescope from long, built-up members, see Fig. 3, which are not connected or braced to each other; their properties are analyzed separately, and in the homology program they are represented by solid rods of "equivalent bar area" and "equivalent density" (to be defined in the following section). The members are built from steel pipe. The telescope has 646 built-up members, each member has 219 single pieces of pipe, with 6-8 pipes meeting at each joint at various angles. All connections should be welded. The 88 surface panels are triangles of 43 feet average length, with 1253 single pieces each. In each telescope quadrant, all 22 panels are slightly different from each other.

For the manufacturing, all detailed information should be provided by NRAO in form of punched cards or tape, and the manufacturer must have (or find) automatic machinery working with punched cards. Each single piece of pipe of a member or panel is represented by a punched card, numbered in order of decreasing diameter. All pieces are cut, and welded together, in the order of their numbers. An automatic saddle cutter provides both ends of each pipe with saddles fitting the pipes already present at both joints. Each saddle is described by three parameters punched on the card (diameter of other pipe, angle between the two, angle on periphery). In the average, each pipe end needs 1.5 saddles. In both members and panels, the thick pipes (which are assembled first) provide a complete outer framework, into which the smaller pipes then can be welded with flexible jigs. The total length of a member or panel should be accurate within ± 1/4 inch, but no special accuracy is demanded for the location of all other joints.

A similar procedure is suggested for cutting the triangular surface plates from aluminum sheets. If fully automated cutters are available, NRAO provides all information on punched cards. If not, we provide templets from a Calcom plotter, see Section II, 3.

II. Description of the Design

1) The Built-up Members

In order to facilitate a fast and accurate structural analysis of the telescope, the built-up members must be stable in themselves without any bracing with each other. In Appendix 4 (Report 10, June 1966) we found that rectangular shapes, for same stiffness, pick up 80% more wind force than pipes; we thus decided to use standard steel pipes for the single pieces. Second, we found that the members should have 3 main chords (equilateral triangular cross section), which gives smaller ℓ/r ratios, and also reduces the wind force to $\sqrt{3/4}$, as compared to the usually used 4 main chords. Third, we decided to use double bracing (two diagonals for each rectangle) in order to let the bracing contribute to the axial stiffness. Fourth, for decreasing the ℓ/r ratio of the diagonals, we introduced small triangles as secondary bracing. Fifth, a detailed study showed that a parabolic shape of the whole member, as seen from the side, is the best compromise for opposing demands like maximum axial stiffness/weight, minimum lateral sag, minimum Euler-buckling, etc. The result is shown in Fig. 3.

Next, we adopted the same bar area for all chords, battens, diagonals of one member, but left the ratios, batten/chord and so on, as free parameters. A method was developed for optimizing these ratios such that, for increasing axial load, chords, battens and diagonals become unstable at the same time (no waste of steel for any of them). These calculations were done by A. Rahim of NRAO. Finally, all computed areas can be replaced by those from the Steel Manual which come closest.

For each member, we calculate an "equivalent bar area"

$$A_{eq} = \frac{F\ L}{E\ \Delta x} \tag{3}$$

with F = axial force, Δx = resulting change of length L, and E = modulus of elasticity. With W = total weight of the member, we define an "equivalent density" as

$$\rho_{eq} = \frac{W}{L\ A_{eq}} . \tag{4}$$

In whatever structure these members are used, the analysis of that structure then regards these members as solid rods of bar area A_{eq} (for stiffness) and material density ρ_{eq} (for dead loads).

Simple approximation formulas have been developed for wind force, stress under various loads, and buckling criteria; they have been checked by Simpson, Gumpertz and Heger with good agreement ($\pm$ 3%). All these results will be published in a separate report.

The over-all procedure then is the following. First, we develop with the homology program a structure with $\Delta H \leq .005$ inch, where every member is stable against survival according to our approximation formulas. Second, all single calculated bar areas are replaced from the Steel Manual. Fourth, the member program calculates A_{eq} and ρ_{eq} for every built-up member. Fifth, with these changed values, the homology program calculates the final ΔH. Sixth, a final check for stability in survival conditions is done by Simpson, Gumpertz and Heger.

Several members which need a special design are discussed in Appendix 5 (Report 22, Feb. 1969): feed legs and focal cabin, suspension of the telescope, and the center bar with optical pointing system.

O. Heine mentioned wind-induced vibrations in long slender pipes, provided the basic formulas, and suggested high-strength steel tubing as a possible solution. This problem is treated in Appendix 6 (Report 24, March 1969). The result is that standard-weight pipes can be used for all telescope members (no tubing needed); 80% can be of normal A36 steel, 20% of all members need high-strength steel with a yield point of $S_y = 60$ ksi. Under these conditions, 33 members never vibrate in resonance. The pipes of the remaining 107 members will vibrate in resonance at the proper critical wind velocity (depending on length and diameter); but the resulting stress in the extreme fiber, together with that from external axial loads, is always below (1/3) S_y, adopting a safety factor of 1.5 and an endurance factor of 2. A similar investigation of the surface panels might be necessary, but since they are more shielded against the wind they should be less critical than the built-up members.

2) Dish, Towers and Foundations

The dish structure is shown in Fig. 4 and is described in detail in Appendix 7 (Report 21, Jan. 1969). The fundamental structure is an octahedron; it has a basic square, above is the focus, below is the "antifocus" or elevation-holding point. The basic square defines an octagon, and all eight corners are connected to the antifocus by the "cone members." From the octagon and its center point, the members of "Layer 2" hold 40 of the 45 points of the "Subsurface". From the subsurface, the members of "Layer 1" hold the 57 points of the telescope surface.

The basic square, the center point and the antifocus are held with 8 "suspension members" from the two elevation bearings which are mounted on top of two azimuth towers.

The two towers are tetrahedrons, see Fig. 5. Two legs of each tower go down to the azimuth ring (points 2 and 3), the third leg goes to a central pintle bearing (point 1). This third leg needs a slight bend at about its middle (point 5), giving enough clearance for the dish structure in horizon or serving position. Point 5 is braced against the bottom points (2 and 3) of the other two legs. In the center of the tower system, above the pintle, a platform is mounted for the elevation drive (point 6). Its strong stiffness is provided by members 6-7 and 1-6. There is no member 6-8, for clearance in horizon position. The dish can rotate in elevation from zenith to horizon in one direction, and by 45° in the opposite direction. It rotates in azimuth by $\pm$ 360°. A detailed description of the towers, with load conditions and constraints, is given in Appendix 8 (Report 19, Oct. 1968) and Appendix 9 (Report 20, Dec. 1968).

The azimuth ring is plain normal railway, double track, with usual gravel, ties and rails. This yields all the accuracy we could ask for ($\pm$ 1/2 inch) and needs only a cheap and fast maintenance once in 1 or 2 years, according to settlements and movements of the ground underneath. See Appendix 3.

O. Heine has designed a truck assembly using standard railroad wheels, axles and other parts, to be mounted under each tower leg. We have calculated all loads under various conditions, and they are all within the allowed range.

The permitted 1/2 inch deformation of the rails introduces some additional small stress in the tower members which has been added when their stability was checked.

Each truck assembly has four drive units (16 total) for azimuth rotation, using friction on the rails. Our calculation shows that the amount of friction actually prevailing under various conditions of load, wind and rail deformation is high enough to provide the needed forces, for driving the telescope in any orientation in winds up to 50 mph. Above that, the dish should be in stow position (zenith) where no high torques will occur.

Two of the eight cone members are split up in such a way that their outer chords provide a circle for the elevation drive. Since we do not want any accuracy of this circle, and since in strong winds or sunshine it will move up and down by $\pm$ 1.0 inch anyway, O. Heine has designed a flexible mount for the elevation drive which follows the inaccuracy or movements of the elevation wheel, and which introduces no torque nor vertical loads on point 6 of the tower.

All strong horizontal wind loads are taken up by the central pintle bearing, leaving none for the rails. This results in high stresses in all horizontal tower members. For this reason, point 8 is introduced which closes the circle of basic tower points, see Fig. 5. Point 8 carries no vertical loads, but this configuration yields the maximum stiffness/weight in survival winds (85 mph), as shown for a similar case in Appendix 10 (Report 9, May 1966). The maximum loads at the foot of one tower leg, on top of the truck assembly, are:

dead load (tower + dish)	527 kip	
rail deformation (max.)	177	
counterweight	230	
total without wind	934 kip;	(5)
wind of 85 mph on dish and towers	780	
total in survival	1714 kip.	(6)

The maximum loads on the central pintle are

down	2458 kip;	(7)
horizontal	1592 kip.	

3) The Surface

The dish surface provides 57 homologous structural surface points. They form a pattern of 88 triangles, in the average about equilateral with 43 feet side length, see Fig. 4. Each of these triangles is a large surface panel with a design as shown in Fig. 6 for 1/6 of a panel. Each panel consists of 1253 single pieces of pipe. The design of Fig. 6 shows a "standard" (or typical) panel, and each actual panel can be derived from this standard by a simple geometrical transformation formula derived for this purpose. The holding point is point 51 in Fig. 6 (homologous point of dish structure).

In order to keep the dead-load sag low, the panel must have a certain vertical depth. We have chosen 10 ft., which actually could be less (7-8 ft). There are axial forces between any two holding points, up to 10 kip, which would give large vertical surface deformations, if the depth of the panel would extend only below the holding points. Thus, in the design of Fig. 6, the holding point is placed at half the depth. The remaining vertical deformation from axial loads was calculated and is small enough.

At their upper surface, the panels provide a triangular pattern onto which the adjustment studs are welded which hold the triangular surface plates, see Fig. 7 and Fig. 8.

The triangular plates are of aluminum sheet (1/8 inch; alloy 6061, T6) and have riveted channels under their sides. Plates and ribs are designed for holding a man of 200 lb on any point, without permanent deformation. They are much stronger than needed for survival, and their gravitational sag is only .008 inch maximum. The triangular plates have a side-length of only 3.12 ft for two reasons: (a) with this size they can be flat instead of curved; (b) the deviations form flatness, if cut from sheets as they come from the mill, are small enough. Thus, using standard available sheets gives already the accuracy wanted without any costly curving, molding or adjusting. The total surface needs 18,000 such plates, held by 9,000 adjustment studs (our present NRAO 300-ft has 15,000 adjustment studs).

The design of the adjustment stud is shown in Fig. 8. It fulfills five demands. (1) It never needs a man below the surface; putting on plates or taking them off, and adjusting their height, can all be done from above, and with the combination tool as shown in Fig. 8a, the adjustment is done fast and in a most easy way. (2) After adjustment, everything is turned tight; no loose or rattling pieces. (3) The leaf-spring connections to the plate corners allow thermal expansion and contraction of the plates, but are extremely stiff for all other movements or rotations. They hold the weight of a man. (4) After adjustment, a lid is screwed on and provides a closed surface. (5) The studs are made from stainless steel, and their final design should make use of as many available standard pieces as possible.

The gaps between two plates, and between plates and lid, are about 1 mm, allowing for manufacturing tolerances and thermal expansion. The sum of these gaps then is 1/400 of the total surface.

Studs and plates must fit each other exactly ($\pm$ 1/2 mm, say). Where should we demand this accuracy, in the positions of the studs on the panel structure, or in the size of the plates? After many detailed considerations, we have chosen the second way. This means the studs are welded on the proper places of the panel structure but without any special demand on accurate locations. Then the distance between any two neighboring studs is measured exactly. The resulting slightly different sizes of the 18,000 plates then are given on punched cards to the manufacturer, assuming there is a cutting machine on the market working with punched cards. If not, we rent a large-size Calcom plotter and produce 18,000 templets (triangles drawn on paper) to be sent to the manufacturer. The only accuracy needed is for the plate itself, for the length and straightness of each side; the position of the ribs is not important. A price estimate is under way with two manufacturing firms and should be completed soon.

The total weight of all aluminum plates and ribs is 91 (US) tons, the weight of all surface panels is 213 tons (which could be decreased by further optimization). The weight and stiffness of the panels is represented in the homology program by surface bars with (result of STRUDL analysis)

$$\begin{aligned} A_{eq} &= 3.89 \text{ inch}^2; \\ \rho_{eq} &= 1.47 \text{ lb/inch}^3. \end{aligned} \tag{7}$$

III. Performance

1) Survival Conditions

Wind data and their statistical treatment are given in Appendix 11 (Report 16, Dec. 1966), where the upper quartile was found as 17 mph to be used for winds during observation, and where the demand of 1% chance in 30 years yielded a survival wind of 89.6 mph. Meanwhile, some more recent wind data were added, without any significant change of the results; the upper quartile now is:

$$\text{wind below } v_{obs} = 18 \text{ mph, for } 3/4 \text{ of all time.} \tag{8}$$

The survival demand was lowered to one chance in 200 years, which gives 79 and 75 mph for the two best-fitting extrapolation curves. In order to be on the safe side, we adopt

$$v_{sv} = 85 \text{ mph.} \tag{9}$$

For the highest possible snow load we have adopted for the whole surface

$$P_{sv} = 20 \text{ lb/ft}^2 \text{ of snow or ice} \tag{10}$$

which is a solid layer of ice 4 inch thick, or about 2 feet of snow. Since we must be able to dump snow, we have demanded the full load of (10) in any elevation angle (which is grossly overdone). Since our homology program must be fast, while actual wind calculations for every member are time-consuming, and since (9) gives a higher over-all load than (10), the homology program uses only (10) but demands it for any elevation angle; it iterates only to those homology solutions where every single bar is stable against this survival condition. After a solution is obtained, we exchange x and y directions and apply (10) again, as an approximation of heavy storm in any horizontal direction.

Our present structure thus is stable against (10) in any horizontal or vertical direction; a final check with the actual wind force (9) on each member is in preparation with Simpson, Gumpertz and Heger.

As to the highest wind for driving the telescope in azimuth, we have adopted

$$50 \text{ mph for azumuth drive.} \tag{11}$$

2) Lowest Dynamical Mode

Before the servo system can be designed, one must know the lowest dynamical mode, or natural frequency, of the combined telescope structure, regarding all items like single long members, dish structure, suspension, drive and gear, towers, trucks, rails and ground.

In Appendix 9 (Report 20, Dec. 1968) I have developed and used a simple approximate method for calculating the dynamical frequencies of various modes, using our computer output for mass, stiffness, moments of inertia, and sag of different parts of the structure. This method is supposed to give good estimates somewhat on the safe side, which was checked on occasion. The application to 5 different modes of translations and rotations yielded a lowest mode of f = 1.23 cps for rotation in elevation angle with the telescope looking at horizon. This result included members, dish, suspension and towers; but not drive and gear, trucks, rails and ground. When the stiffness of these items became available, trucks, rails and ground reduced the result to f = 1.15 cps, and drive and gear to f = 1.11 cps. Finally, I suggested using for the servo design

$$f = 1.10 \text{ cps.} \tag{12}$$

Meanwhile, O. Heine has made a rigorous dynamical computer analysis of the combined system, with Philco-Ford Corp. The resulting lowest mode is only f = .88 cps. It seems, however, that this low value results (1) from changes in the tower design, and (2) from using too low a stiffness for dish and suspension (factor 4). For the following performance calculations I will assume a compromise of f = 1.0 cps, and a servo bandwidth of

$$b = (1/3)\ f = .333 \text{ cps.} \tag{13}$$

3) The rms Surface Deformation

Surface deformations and pointing errors from wind and thermal effects are treated in Appendix 13 (Report 23, March 1969), using measurements and experiments on thermal behaviour from Appendix 12 (Report 17, Jan. 1967), and some recent high-resolution wind statistics to be published later.

We adopt white protective paint as used on the 140-ft telescope, which gives a difference between sunshine and shadow of $\Delta T \leq 5^{\circ}$ C on sunny calm days ($v < 5$ mph). The time lag of heavy members during fast changes of ambient air temperature gives $\Delta T \leq 6.1^{\circ}$ C per inch of wall thickness for the fastest daily change on 3/4 of all calm days. In Appendix 13 we concluded that the six heavy cone members of the telescope should be made from open shapes (not pipes) which reduces the time-lag by a factor 2.

Table 1 shows the various contributions to the rms surface deviation from the best-fit paraboloid of revolution. In detail, we have:

1. Difference between <u>flat plate and paraboloid</u>. If the paraboloid goes through the three corners of the plate, the plate center is too high by .0709 in the average over the dish surface. For finding the rms deviation from the best-fit paraboloid, this item must be combined with the next one.

2. <u>Deviation from flatness</u>. We have cut (sheared) two triangular plates, 1/8 inch aluminum sheet as it comes from the mill, in our NRAO workshop. After clamping the three sides onto aluminum channels 3/2 inch deep, the maximum deviation at the plate center, from a plane defined by the three corners, was about $\Delta z_m = .025$ inch for both plates. Then one plate was welded to the ribs and gave a completely useless deformation after cooling. The other plate was riveted to the ribs and gave a maximum center deviation of .032 inch (the ribs were straight within .005 inch). This accuracy is good enough for our purpose, and it was achieved without any selection before, or trimming after, the cutting and riveting. Originally, then, we decided to specify for the manufacturer a maximum deviation of $\Delta z_m = \pm .050$ inch of the plate center from the corner-plane. Adding up this item with all other deformations then resulted in a $\lambda = 2$ cm telescope. A comparison of all items showed that item 1 and 2 were the largest contributions under most observing conditions.

   We thus decided to go one step further, specifying

$$0 \leq \Delta z_m \leq .120 \text{ inch} \tag{14}$$

   which means that 1/2 of all plates are turned around before the exact cutting and the riveting, in case of Calcom templets (Section II, 3). In case of a punched-card cutter, 1/2 of all plates must be bent down after riveting in some way. The average $\overline{\Delta z_m}$ then will be about .050 inch for all plates, and we will adopt the limits $.030 \leq \overline{\Delta z_m} \leq .075$. Under this assumption, and adding .005 inch for the rms difference between the levels of the six leaf springs in Fig. 8, as well as adding .010 inch for the rms amplitude of shorter waves in the plate, we find that the rms deviation of the riveted plates, from a best-fit paraboloid of revolution, is

$$\mathrm{rms}(\Delta z_p) = .014 \text{ inch} = .356 \text{ mm} \tag{15}$$

for the combined effect of using flat plates, deviations from flatness, and manufacturing tolerance of the adjuster.

3. Sag of plate and ribs. The center sag of the plate, of side length ℓ and thickness h, was calculated for dead load $q(1.70 \text{ lb/ft}^2)$ as

$$\Delta z_m = \frac{q\ \ell^4}{162\ E\ h^3} = .00722 \text{ inch} = .184 \text{ mm} \tag{16}$$

and the center sag of the rib was found as

$$\Delta z_m = \frac{W\ \ell^3}{E\ I} = .00161 \text{ inch} \tag{17}$$

Since (16) and (17) go in parallel, they add up linearly to $\Delta z_m = .00883$ inch. This sag will be about the same for all plates, which means the best-fit paraboloid goes down by $\overline{\Delta z}$.

In general we derived by an integration, for triangular objects with a constant radius of curvature, that

$$\mathrm{rms}(\Delta z - \overline{\Delta z}) = .193\ \Delta z_m. \tag{18}$$

The dead load sag then yields for the riveted plate

$$\mathrm{rms}(\Delta z_p) = .193 \times .00883 = .00171 \text{ inch} = .043 \text{ mm}. \tag{19}$$

I would like to mention that the stiffness for distributed load of the riveted plate was checked experimentally and was found a few per cent higher than the theoretical one.

4. Wind on plates and ribs. With 18 mph the pressure is .83 lb/ft^2. A face-on wind then gives a central deformation of

$$\Delta z_m = .00409 \text{ inch} = .104 \text{ mm} \tag{20}$$

or, with (18)

$$\mathrm{rms}(\Delta z_p) = .00079 \text{ inch} = .020 \text{ mm.} \tag{21}$$

Finally, we add (21) and (19) quadratically, since gravity and wind are perpendicular to each other, and obtain for wind and dead load in the worst combination

$$\mathrm{rms}(\Delta z_p) = .00189 \text{ inch} = .048 \text{ mm.} \tag{22}$$

5. Sag of panels. The panel of Fig. 6 was investigated with the STRUDL program, including welded joints. The maximum sag, occurring at point 6, is

$$\Delta z_m = .028 \text{ inch} = .711 \text{ mm,} \tag{23}$$

the average sag is $\overline{\Delta z}$ = .0214 inch, and the rms deviation from the average is

$$\mathrm{rms}(\Delta z_p) = .00624 \text{ inch} = .159 \text{ mm.} \tag{24}$$

This deviation from the average is also the deviation from the best-fit paraboloid, if all panels sag by the same amount, which can easily be obtained by making the smaller central panels less thick than the larger outer panels.

6. Wind on panels. From the STRUDL analysis, we find for 18 mph face-on a maximum deformation, at point 6, of

$$\Delta z_m = .0070 \text{ inch} = .178 \text{ mm} \tag{25}$$

and the deviation from the average is

$$\mathrm{rms}(\Delta z_p) = .00156 \text{ inch} = .040 \text{ mm.} \tag{26}$$

7. External loads on panels. Since the panel stiffness contributes to the dish stiffness, the panels take up loads which vary with elevation angle. From the stress analysis of the homology program we find loads up to 15 kip with an rms of 7.8 kip. If two holding points are pressed toward each other

with 7.8 kip, this load is distributed into two panels. And if the panel of Fig. 6 is pressed with 3.9 kip at point 51, we find from a STRUDL analysis a maximum deformation (at points 14 and 19) of

$$\Delta z_m = .00350 \text{ inch} = .089 \text{ mm}. \tag{27}$$

Since the panels deform different from each other depending on their external loads, we cannot subtract an average deformation and use the rms of the deformation itself (from design) which from the analysis is

$$\text{rms}(\Delta z) = .00149 \text{ inch} = .038 \text{ mm}. \tag{28}$$

8. Standard pipes. The present structure was iterated with the homology program down to ΔH = .003 inch. After replacing all calculated bar areas by those from the Steel Manual which come closest, the rms deviation from homology went up to

$$\Delta H = .0158 \text{ inch} = .402 \text{ mm}, \tag{29}$$

for a tilt from zenith to horizon. A further improvement seems possible but has not been done.

9. Surface adjustment. Unless a better method is found, we assume that a number of surface points are declared as "key points" and are measured from the apex with theodolite and tape. The remaining adjustment points then are measured by their deviation from straight lines between key points.

   Some theodolites on the market claim an accuracy of $\pm$ 1 arcsec. For the present estimate, we adopt an rms error of $\pm$ 1.5 arcsec. At the rim of the telescope, this gives Δz = .0131 inch = .333 mm, and for the average over the dish we adopt 213 of that, or Δz = .0088 inch = .222 mm for the rms error of the key points. We further assume that the rms measuring error, when going from the key points to the normal points, is .20 mm = .0079 inch. This adds up for the normal points to an rms error of

$$\text{rms}(\Delta z) = .0118 \text{ inch} = .299 \text{ mm}. \tag{30}$$

10. Wind on whole telescope. In Appendix 13 (Report 23, March 1969) we divide the surface into three equal parts 100 ft apart, with different gusts acting on them. From our high-resolution wind statistics we find that the pressure difference, responsible for the non-rigid deformation of a 300 ft telescope, is $.415/\sqrt{2}$ = .294 of the total pressure, for winds of 18 mph. From the analysis part of the homology program we know the stiffness of the structure.

Without subtracting a best-fit paraboloid, we then obtain

$$\mathrm{rms}(\Delta z) \;=\; .0135 \text{ inch} \;=\; .343 \text{ mm} \tag{31}$$

11. Thermal deformations are also treated in Appendix 13. The deformation of the feed legs gives a gain decrease which can be neglected ($\leq$ 1%). The aluminum plates have a floating mount and do not contribute. The only strong contribution comes from the cone members and is (ΔT in $^{\circ}$C):

$$\mathrm{rms}(\Delta z) \;=\; .0062 \;\Delta T \text{ (inch)} \tag{32}$$

For sunny calm days ($v < 5$ mph) we use $\Delta T = 5^{\circ}$ C, and for all other conditions $\Delta T = 1.5^{\circ}$ C (wind, ambient air temperature change, nights). Thus

$$\mathrm{rms}(\Delta z) \;=\; \begin{cases} .0310 \text{ inch} \;=\; .788 \text{ mm, sunny and calm;} \\ .0093 \text{ inch} \;=\; .236 \text{ mm, else.} \end{cases} \tag{33}$$

Table 2 lists 15 different items. They are combined in four groups. First, all items independent of observing conditions. Second, all items depending on elevation tilt. Third, wind deformations. Fourth, thermal deformations. All four groups give about the same values which shows that we have a nicely optimized design.

The final summary for the shortest wavelength,

$$\lambda \;=\; 16 \times \mathrm{rms}(\Delta z), \tag{34}$$

is given in Fig. 9 as a function of elevation tilt for winds below 10 mph, and in Fig. 10 as a function of wind velocity for $\zeta = 60^{\circ}$ zenith distance. From these results we conclude that the shortest wavelength of observation is

$$\lambda \;=\; \begin{cases} 1.5 \text{ cm, on calm sunny days;} \\ 1.0 \text{ cm, for all other time.} \end{cases} \tag{35}$$

Table 2. The rms surface deviation.

No.	Item	Δz_m mm	rms(Δz) mm	combined rms(Δz) mm	
1.	Parabola/flat plate	1.801	.356	.465	Telescope, at zenith, no wind, $\Delta T = 0$.
2.	Dev. from flatness	1.270			
3.	Shorter bulges	.254			
4.	Adjuster level	.127			
5.	Adjustment accuracy	.533	.299		
6.	Dev. from homology, ΔH	.186	.076	.437	for tilt of 90^o; otherwise $\sim (1 - \cos \zeta)$.
7.	Use of standard pipes, ΔH	.912	.402		
8.	Sag of plate	.184	.043		
9.	Sag of ribs	.041			
10.	Sag of panels	.711	.159		
11.	Ext. load, panels	.089	.038		
12.	Wind on plate + ribs	.104	.020	.347	for 18 mph; otherwise $\sim v^2$
13.	Wind on panels	.178	.040		
14.	Wind on telescope		.343		
15.	Thermal def. $\Delta T = 5.0^o$ C		.788	.788	sunny, calm $\sim \Delta T$
	Thermal def. $\Delta T = 1.5^o$ C		.236	.236	else $\sim \Delta T$

4) The Pointing Error

The pointing error is treated in detail in Appendix 13 (Report 23, March 1969). The optical pointing system eliminates all thermal deformations, and all wind deformations slower than the servo system, between the center of the dish structure and the ground. Furthermore, it eliminates the need for any accuracy of the azimuth ring.

The servo bandwidth adopted in (13) means that deformations slower than 3 sec are eliminated while faster ones still give pointing errors. For our estimate we assume a sharp cut-off at 3 sec (this might be optimistic, and for that reason we left a safety factor of $\sqrt{2}$ when we applied the statistics of pressure differences in Appendix 13). Three seconds then is a full wavelength of the longest remaining deformation, thus the longest one-sided deformation is of duration

$$\tau = 1.5 \text{ sec.} \tag{36}$$

From our wind measurements we find that the rms pressure difference, between adjacent time averages of duration τ, is the fraction

$$P(\tau) = .375 \tag{37}$$

of the average pressure. Furthermore, gusts of 1.5 sec duration have a size of only 1.5 sec x 18 mph = 40 ft, and a telescope of 300 ft diameter then is hit simultaneously by 57 gusts which are uncorrelated if all longer gusts are eliminated. From our computer analysis we know the stiffness of dish and towers, and the resulting pointing error is entered into Table 3.

The contribution of the feed support legs to the thermal pointing error would be rather high (12.6 arcsec for sunshine). We thus suggested in Appendix 12 to blow ambient air at 20 mph through the chords of the feed legs, (4 inch diameter pipes) which reduces the temperature difference from 5° C to 2° C. The resulting pointing errors are shown in Table 3. As to the combined errors, we should mention that the effects of feed legs and of back-up structure cannot add up for any telescope orientation, thus the maximum was taken.

Table 3. Pointing errors.

Source	Item	rms($\Delta\zeta$) arcsec	Combined arcsec
Wind (18 mph)	towers	1.29	2.83
	dish	1.37	
	feed legs + cabin	1.02	
Thermal (sunshine)	back-up structure	6.20	6.20
	feed legs	5.02	
Thermal (else)	back-up structure	1.86	3.77
	feed legs	3.77	
Instrumental	present design	5.0	
	possible	3.0	

The instrumental pointing error (optical readout, servo system, drive units) was originally specified as 5 arcsec. This demand is met by the design of O. Heine. It even seems possible to reduce it to 3 arcsec without much extra cost, which will be tried in a further investigation.

Table 4. Combined pointing error.

Instrumental arcsec	rms($\Delta\zeta$) sunny calm day arcsec	rms($\Delta\zeta$) else arcsec
5.0	7.9	6.2
3.0	6.9	4.9

The combined pointing error is summarized in Table 4 for various conditions. Finally, we compare the pointing error, $\Delta\zeta$, with the half-power beamwidth, $\beta = 1.17\ \lambda/D$. Table 5 shows that the ratio between pointing error and beamwidth (.200 ···· .235), for the present pointing system, is about the same as it is for the NRAO 140-ft telescope at 2 cm wavelength ($\Delta\zeta = .22\ \beta$); which is not too good but still tolerable.

Table 5. Wavelength λ, beamwidth β, and pointing error $\Delta\zeta$.

	λ cm	β arcsec	$\Delta\zeta$ for 5 arcsec instrum.	$\Delta\zeta$ 3 arcsec instrum.
sunny calm day	1.5	39.6	7.9 arcsec = .200 β	6.9 arcsec =.174 β
else	1.0	26.4	6.2 arcsec = .235 β	4.9 arcsec =.186 β

It seems that the pointing error during sunny calm days cannot be easily improved. But during all other time, the highest contribution comes from the thermal deformation of the feed legs, where we have used $\Delta T = 1.5^{\circ}$ C. The actual ΔT may well be smaller if we use the blowers all the time. And the wind deformation as given in Table 3 refers mostly to pointing errors of only short duration, of 1.5 seconds. Actually, much longer integration times are used for almost all observations, and the short-time contributions then will average out to a high degree. It thus is quite probable that the actual pointing error is considerably smaller than that of Table 4.

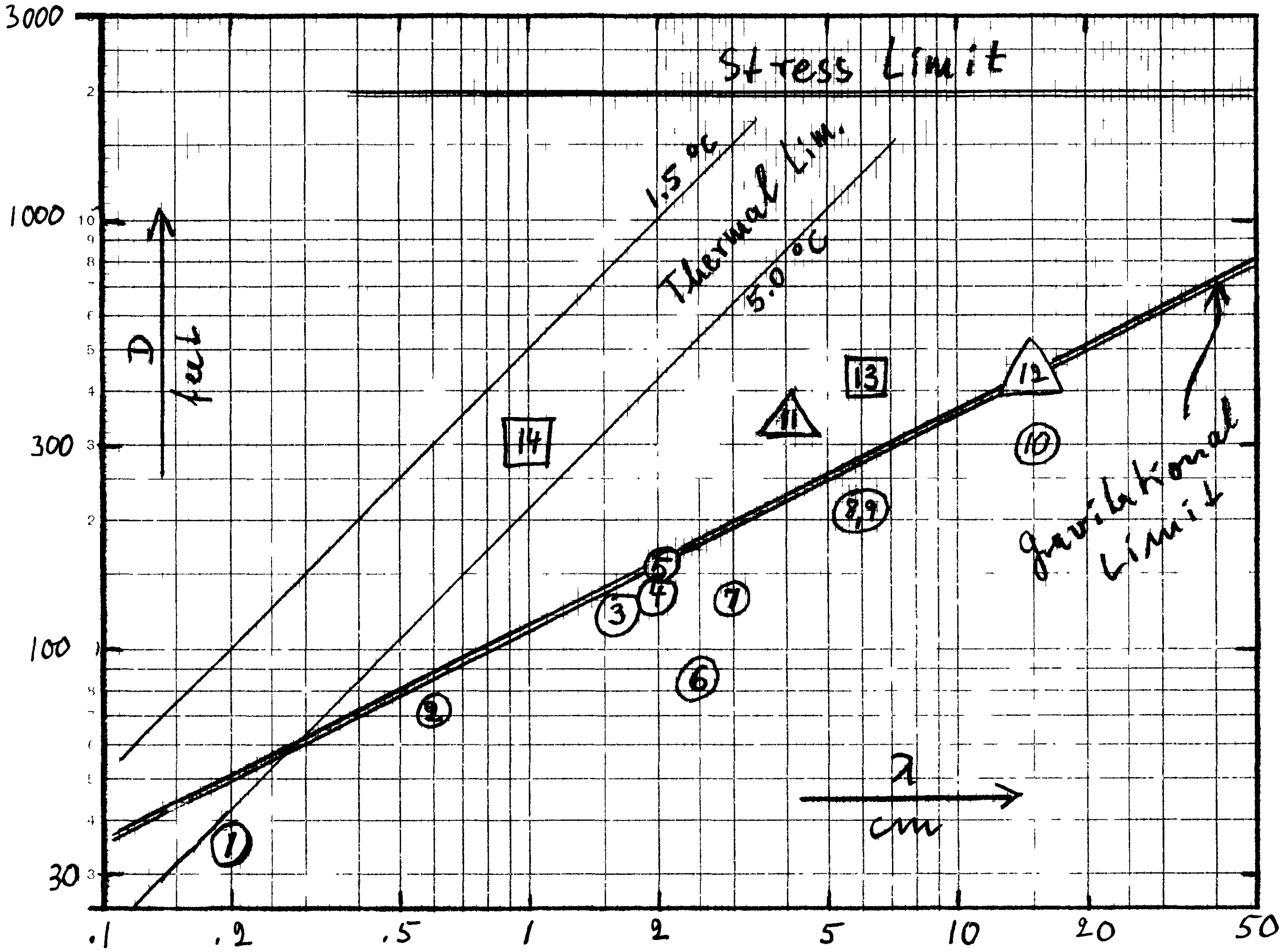


Fig. 1.

Diameter D and shortest wavelength λ. Three natural limits for tiltable, conventional telescopes.

Existing ○

1. 36 ft, NRAO Kitt Peak
2. 22 m, Lebedev, Serpukhof
3. 120 ft, MIT, Haystack
4. 140 ft, NRAO, Green Bank
5. 150 ft, ARO, Canada
6. various 85 ft telescopes
7. 130 ft, Owens Valley
8. 210 ft, Parkes, Australia
9. 210 ft, JPL, Goldstone
10. 300 ft, NRAO, Green Bank

Within 1-2 Years △

11. 100 m, Bonn, Germany
12. 450 ft, Jodrell Bank, GB

In Preparation □

13. 440 ft, CAMROC
14. 300 ft, Homologous Design

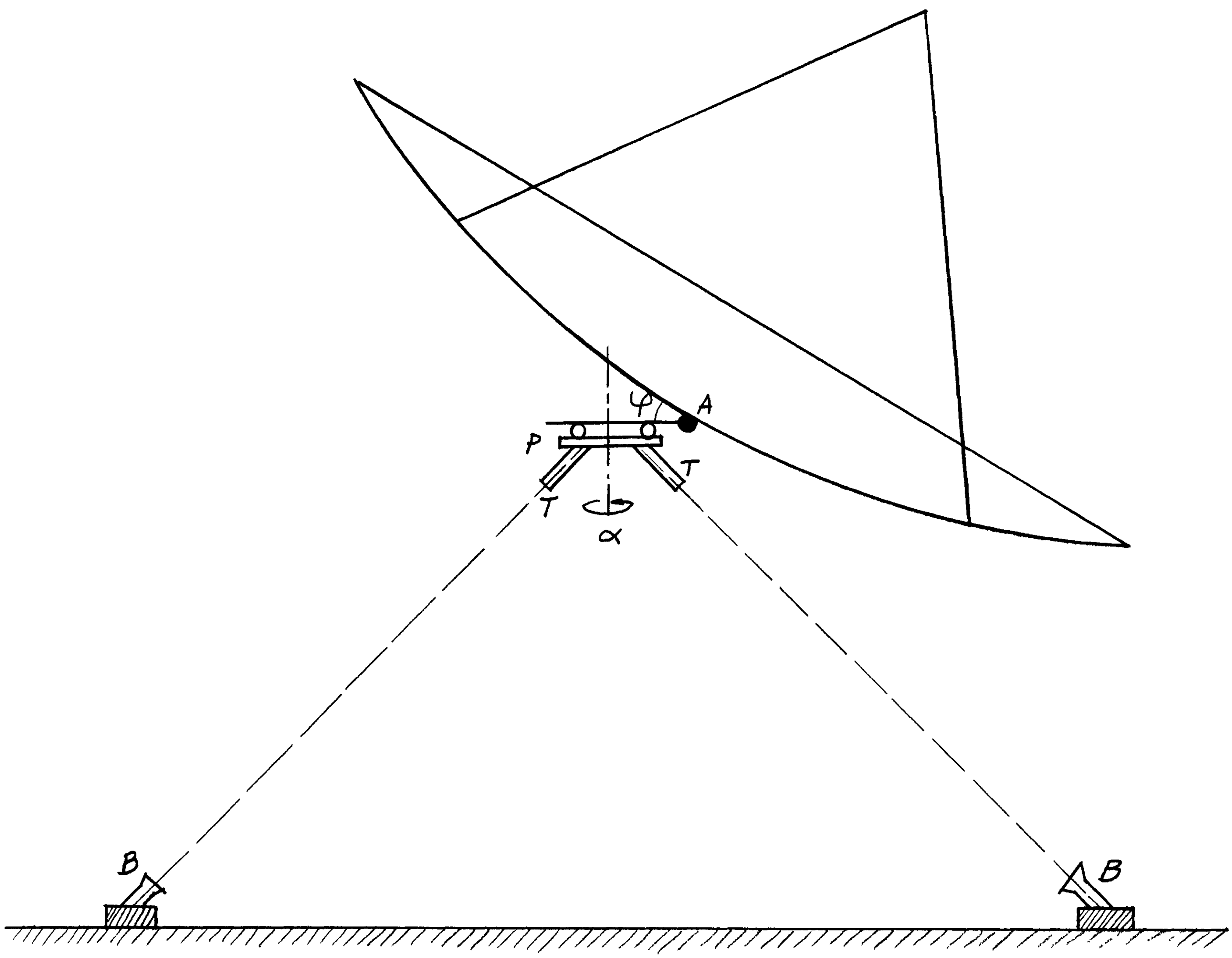


Figure 2. Position measurements by optical means.

A small tiltable and rotatable platform P is mounted behind the apex A and looks with about six theodolites T to as many optical beacons B fixed at the ground. Three servo motors keep the platform "locked-in" to the beacons; elevation φ and azimuth α then are measured between structure and platform. In this way, the position is measured where it matters and with respect to something unstressed and unmovable. No high accuracy is required for foundations, azimuth rails and elevation ring; also, all slow deformations between apex and ground are omitted.

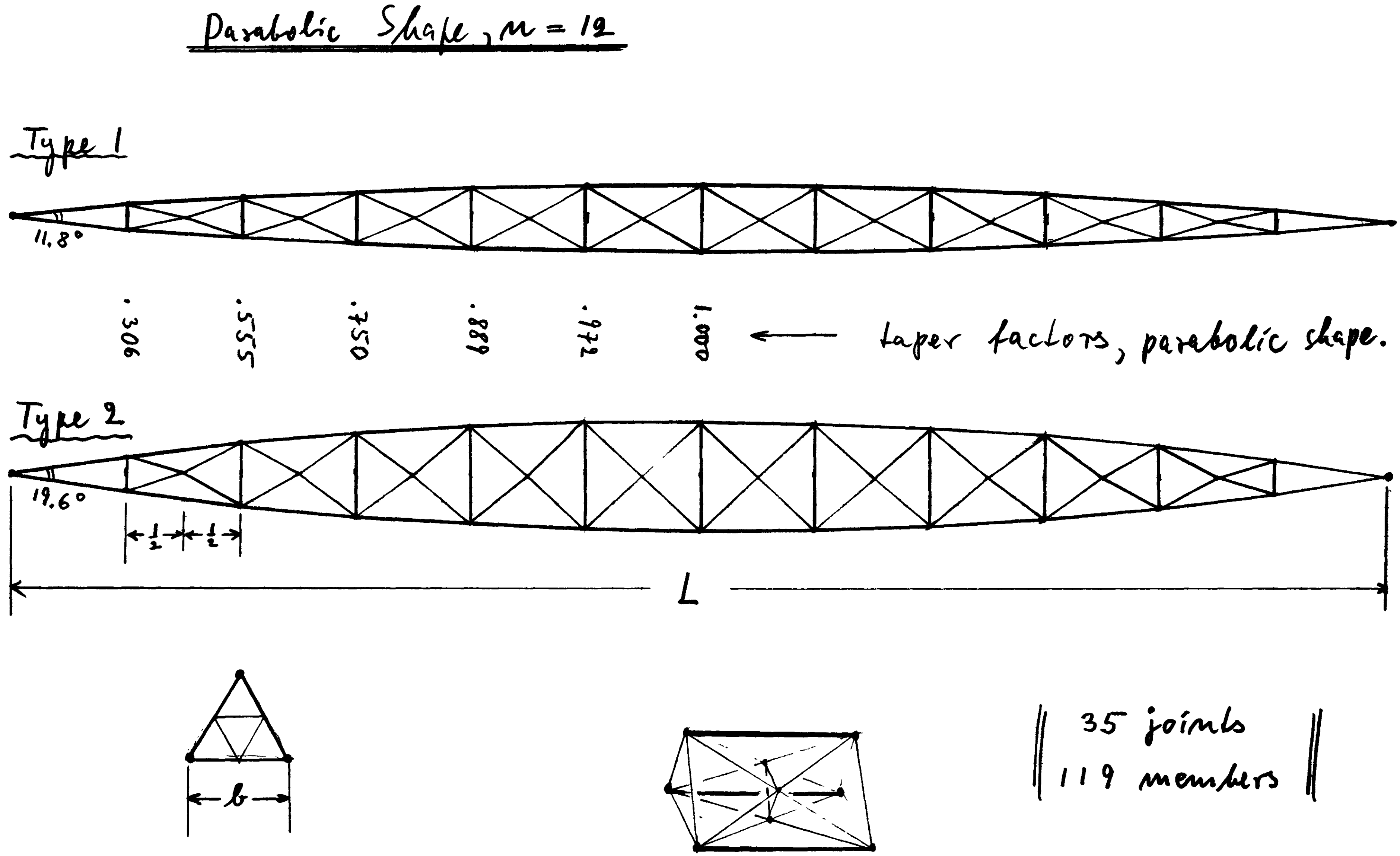
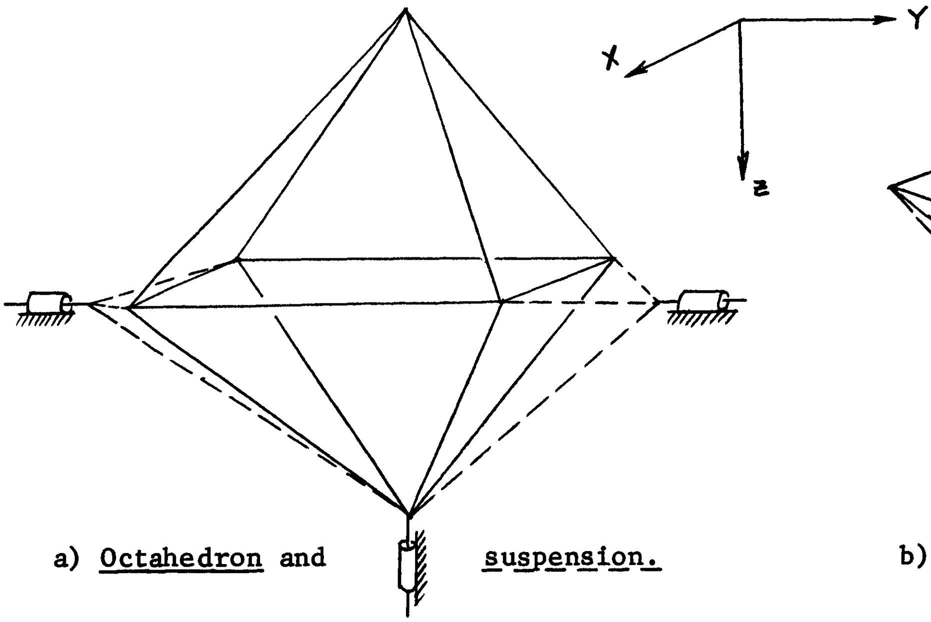
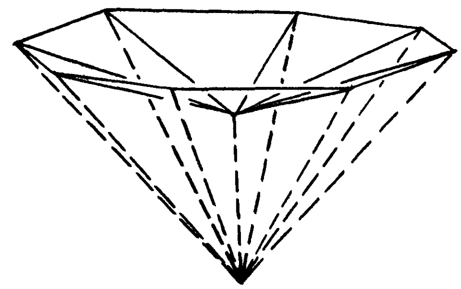


Fig. 3: Geometry of built-up members.

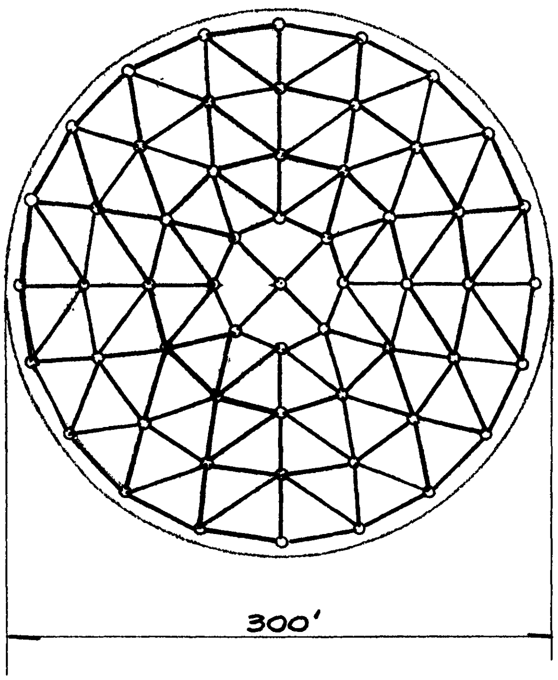
Type 1 is used for all telescope members and heavy tower members;
Type 2 is used for long, light-weight tower members.



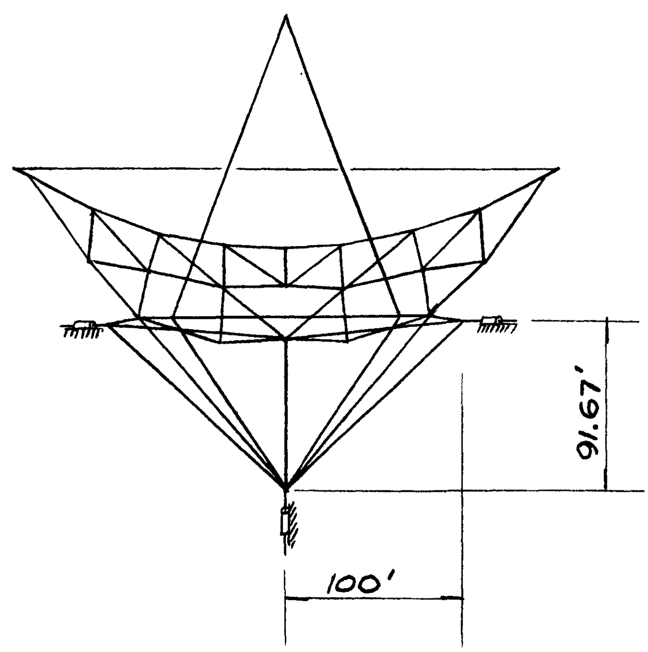
a) Octahedron and suspension.



b) Octagon and cone.



c) Surface.
Circle = rim of surface panels.



d) Side view of telescope, octahedron and suspension.

Figure 4. Geometry of Structure. The basic structure is an octahedron, held by a suspension from two elevation bearings mounted on top of two towers. This structure has 57 homologous surface points, a total of 149 joints and 646 members.

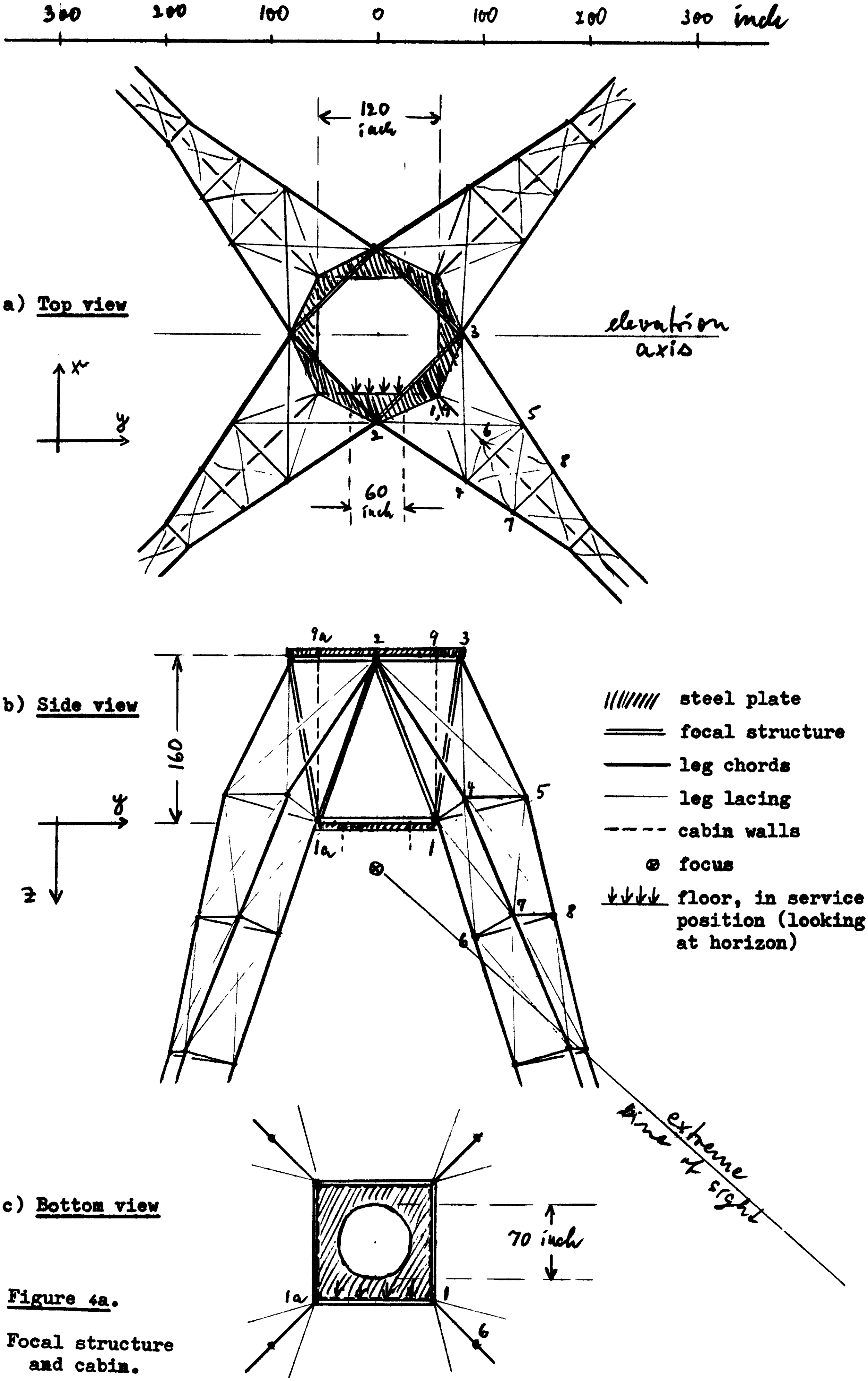


Figure 4a.

Focal structure and cabin.

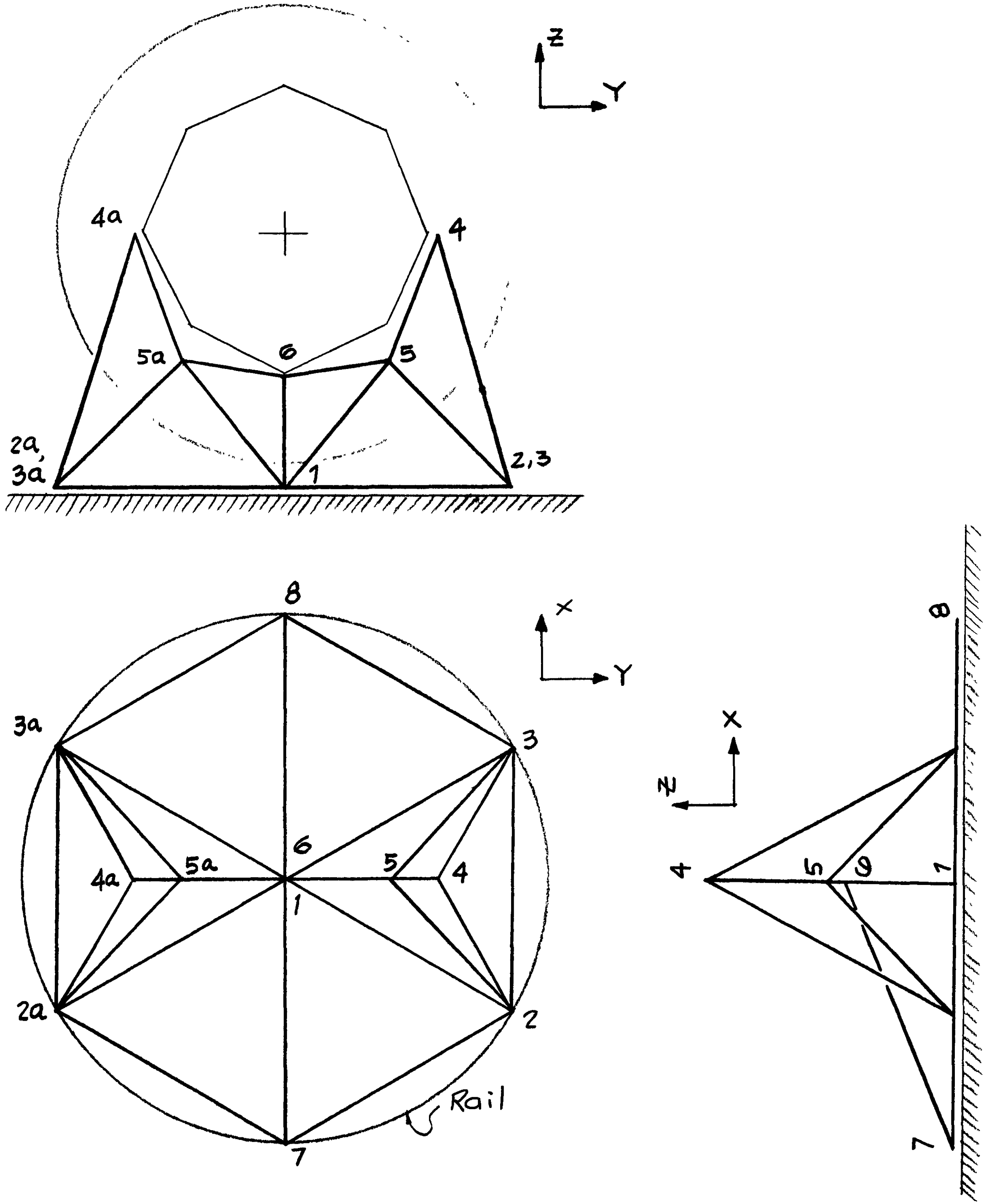


Figure 5. Azimuth Towers.

Point 1 is a strong pintle bearing, taking up 1/3 of the total weight, and all horizontal wind forces. The elevation drive is at point 6, the elevation bearings at 4 and 4a. Points 2, 2a, 3, 3a, 7 and 8 drive on track assemblies on a circular railway track; 2, 2a, 3 and 3a have drive units with friction wheels.

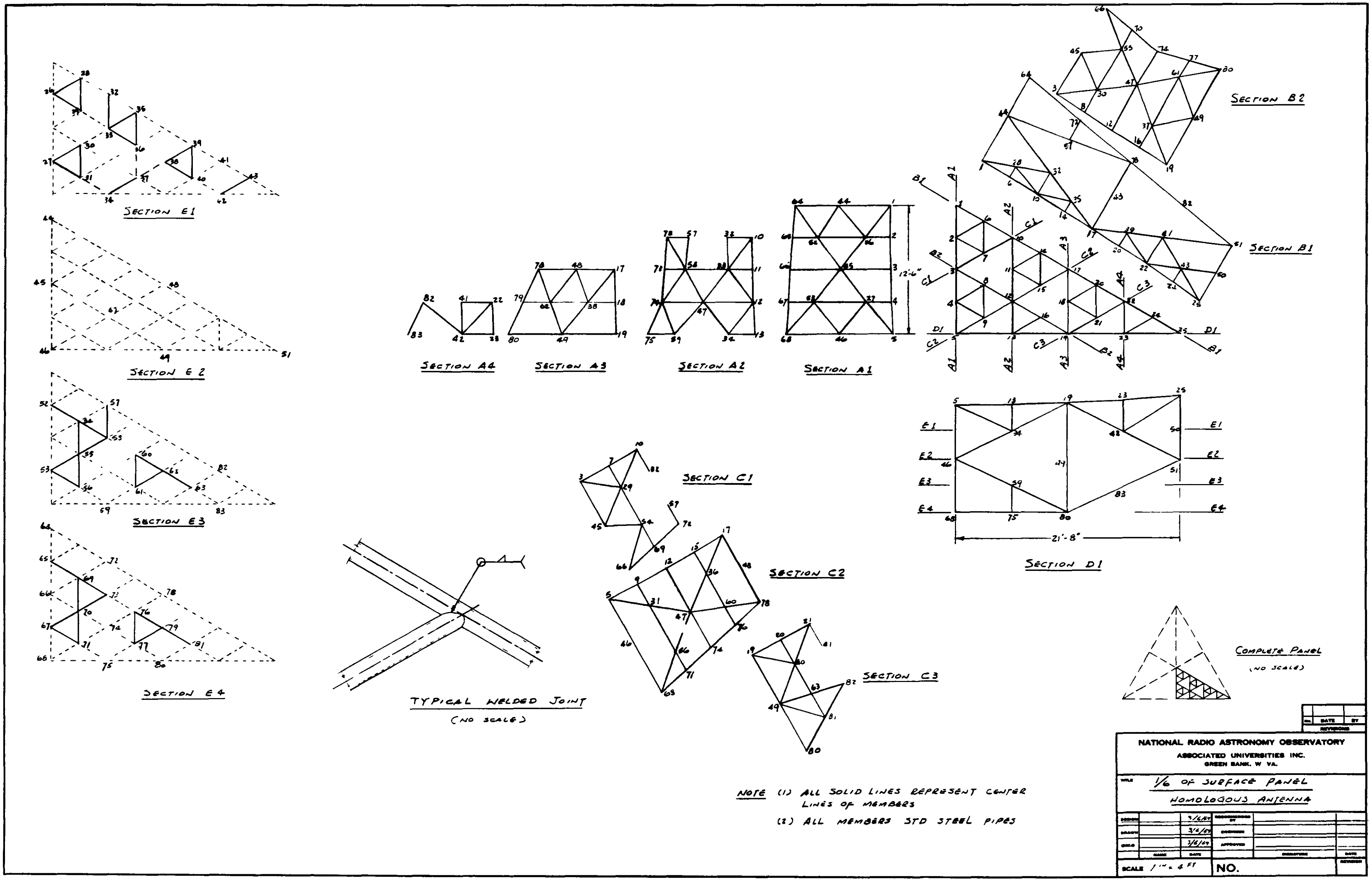


Figure 6. Design of 1/6 of a surface panel

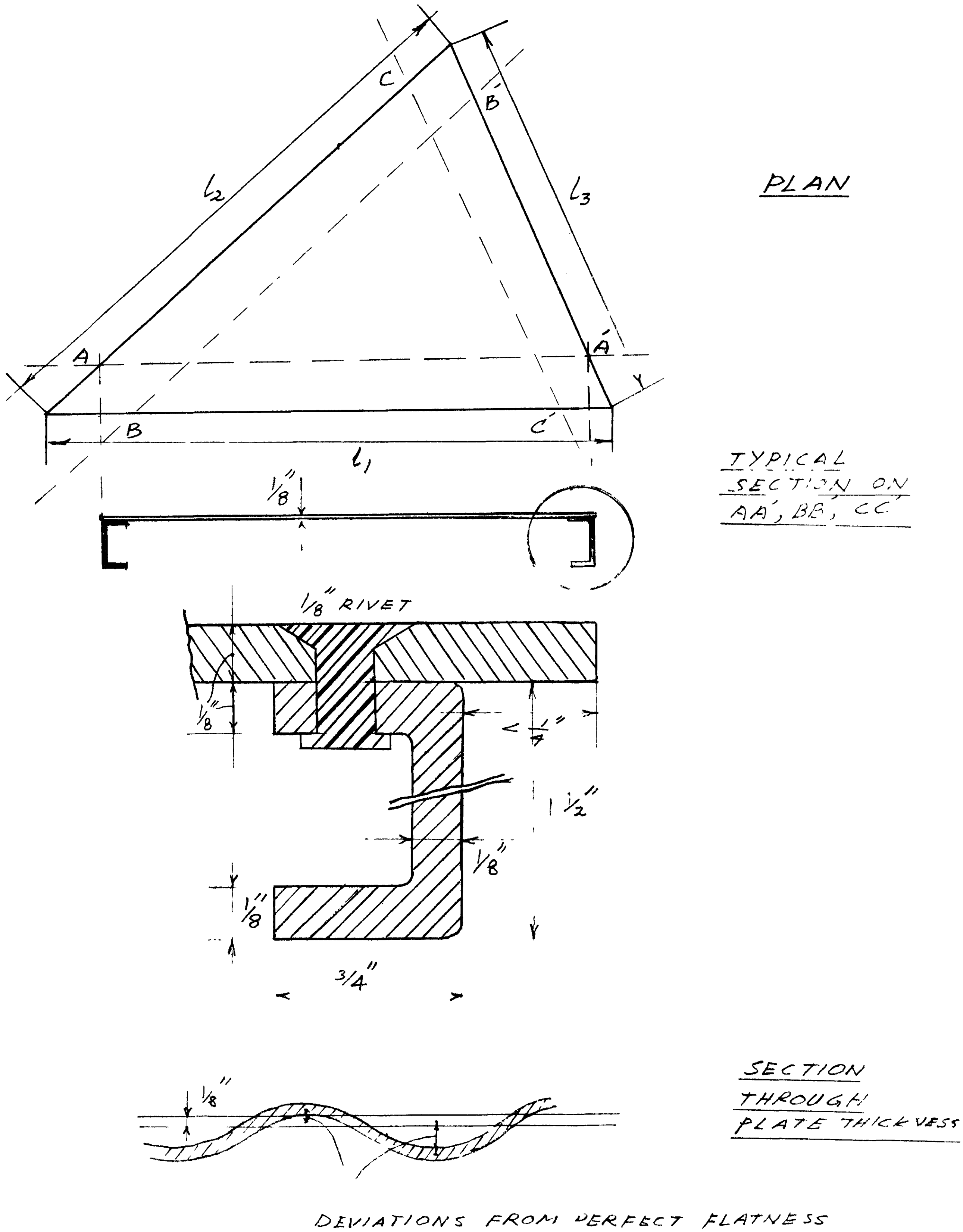


Fig.7. Surface plate with ribs.

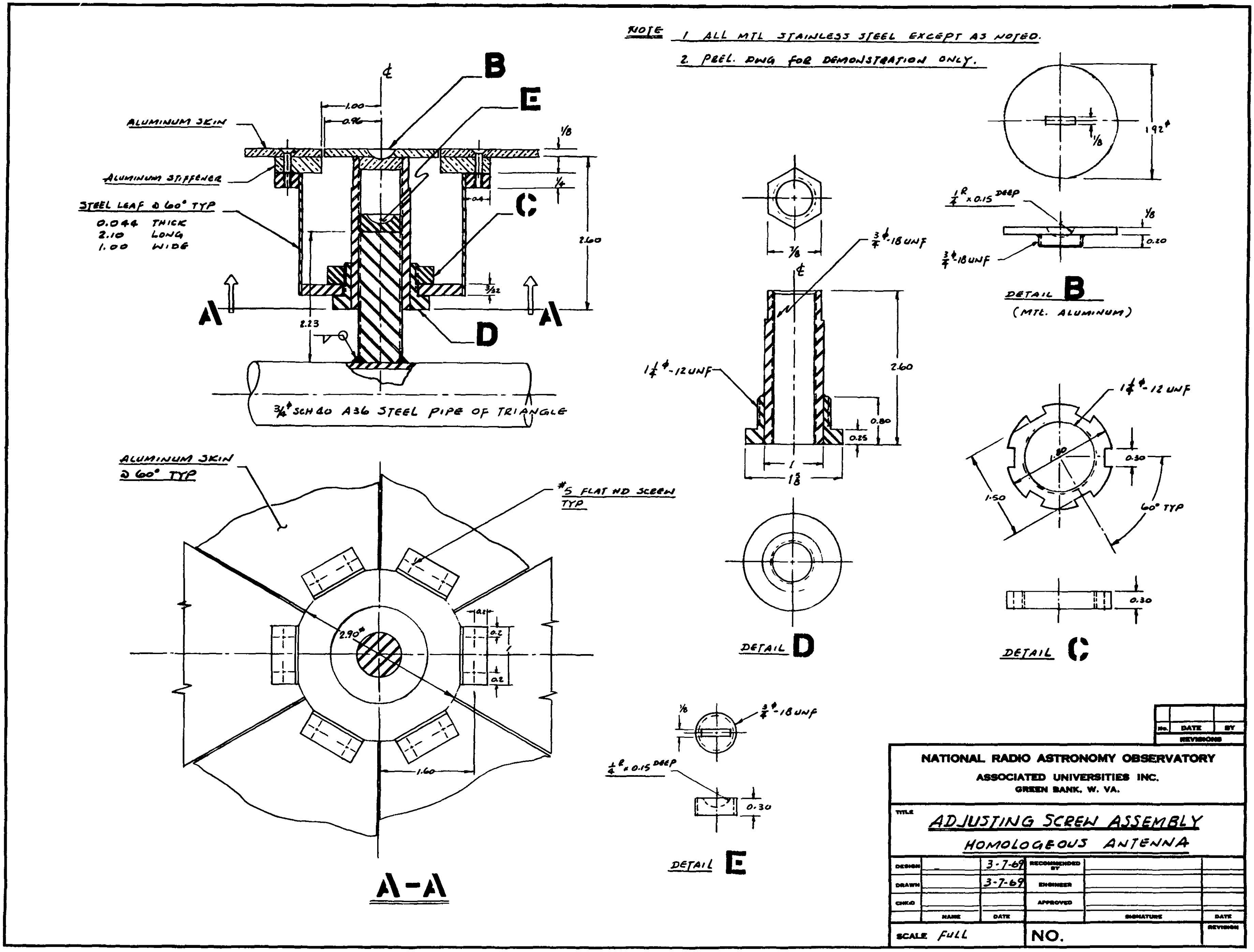


Figure 8. Adjustment stud for surface plates

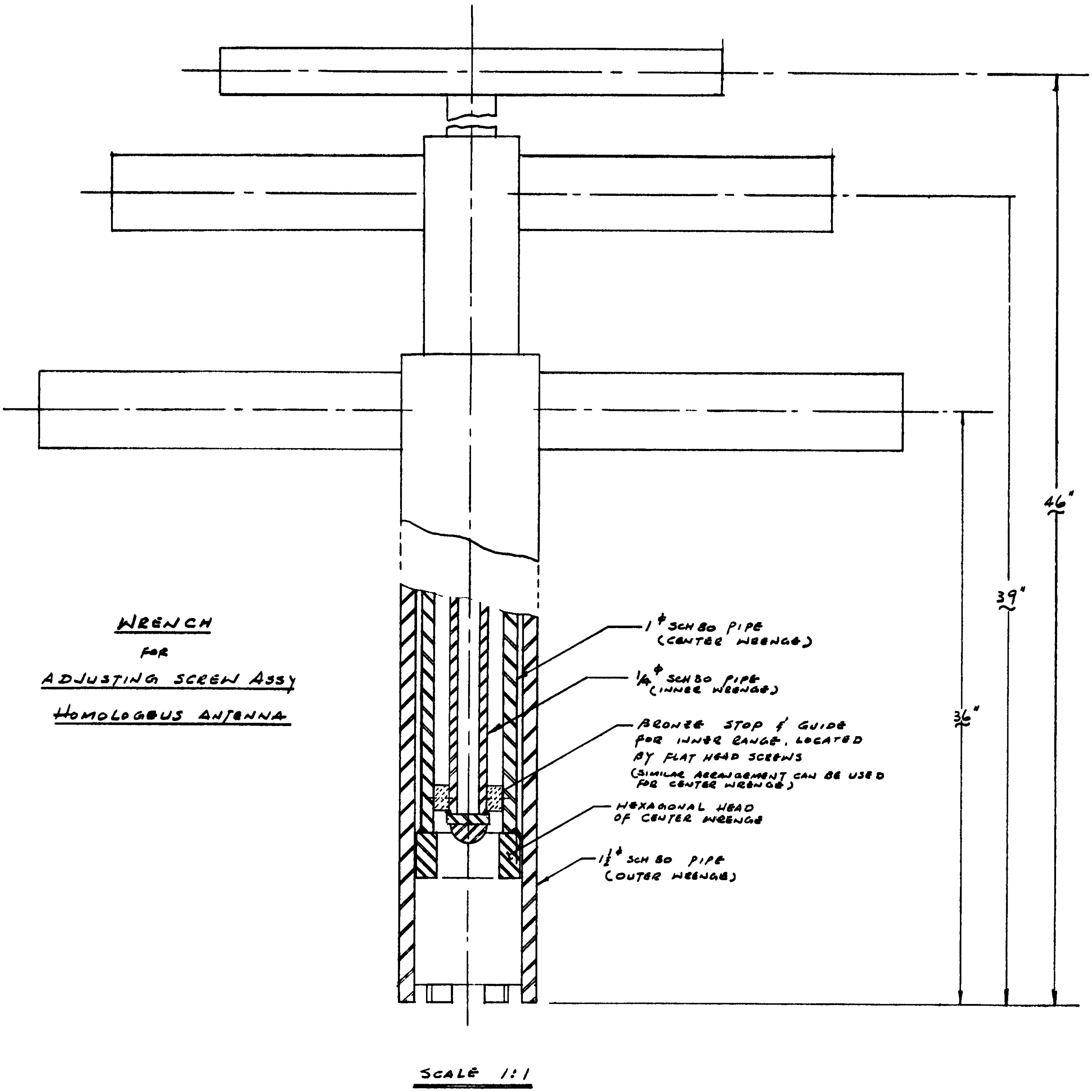
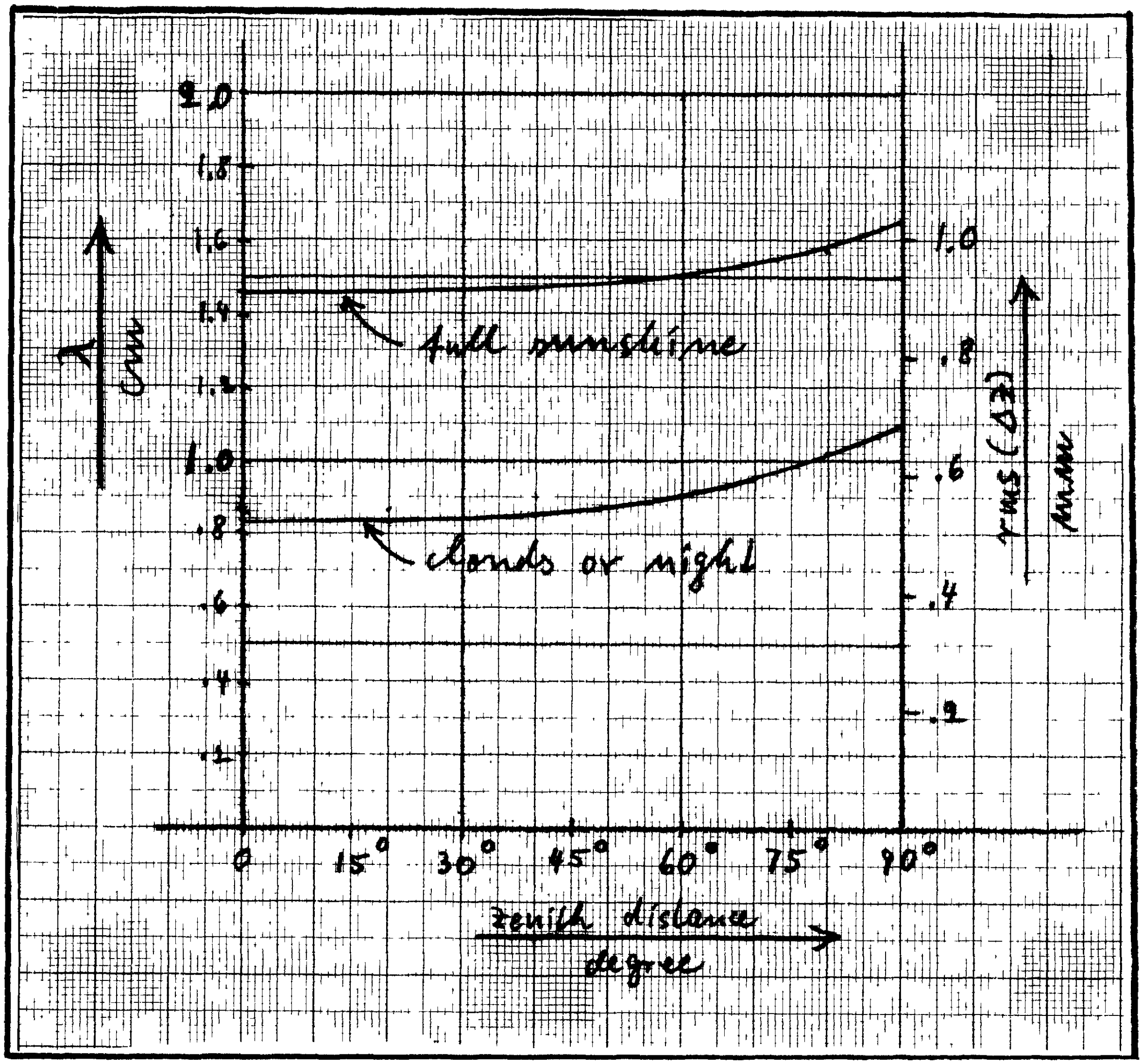


Figure 8a. Combination tool for adjusting



<u>Fig. 9.</u> The shortest observational wavelength as a function of elevation tilt, for low winds ($\leq$ 10 mph).

rms(Δz) = deviation from best-fit paraboloid
λ = 16 x rms(Δz) = shortest wavelength

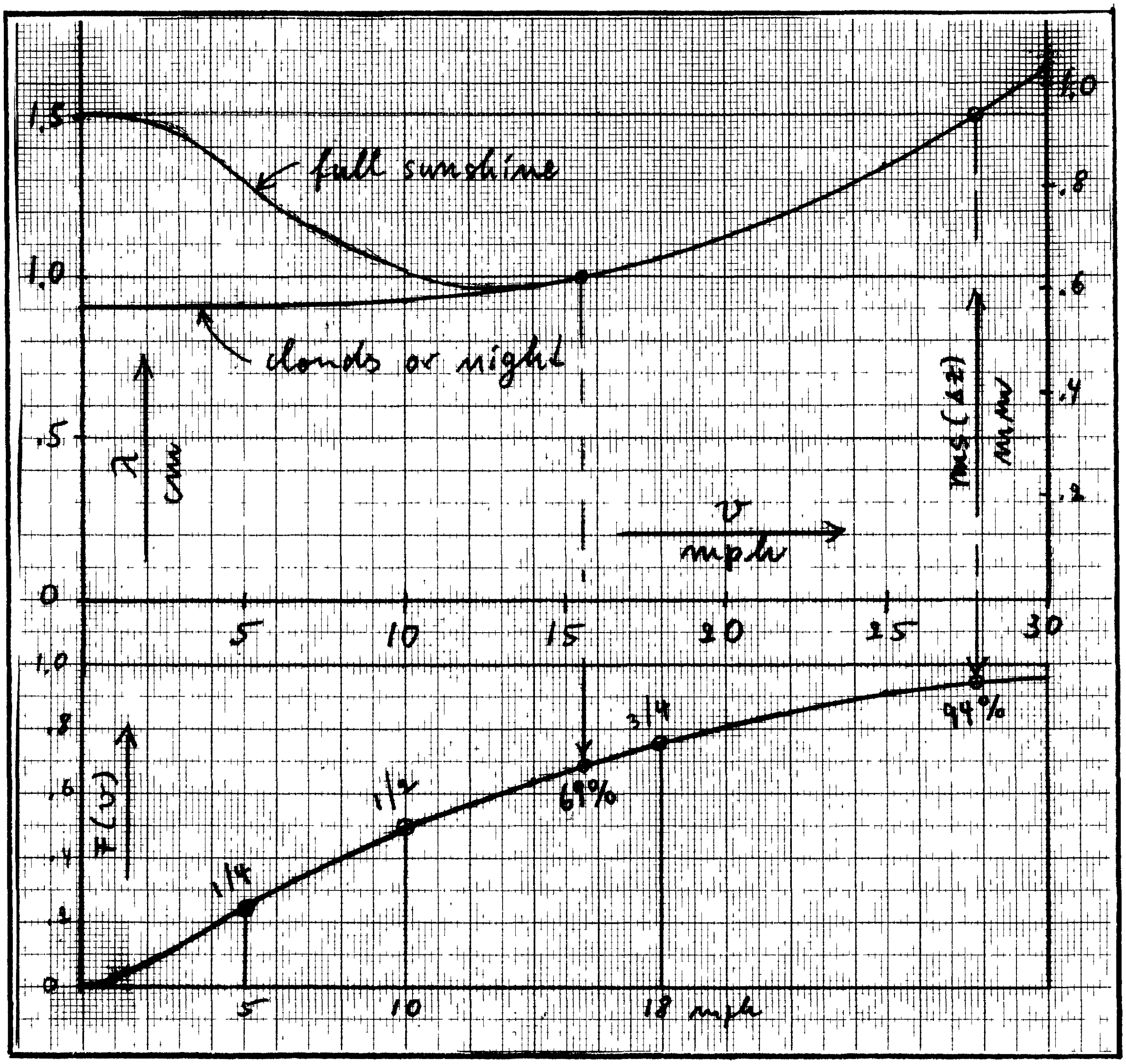


Fig. 10. Shortest wavelength λ as a function of wind velocity v, for a zenith distance of $\zeta = 60^o$.

$$F(v) = \int_o^v f(v)dv = \text{cumulative wind distribution.}$$

for 94% of all time, $\lambda \leq 1.5$ cm;
for 69% of all nights, $\lambda \leq 1.0$ cm.