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Price and Performance for Various Diameters

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Summary

Based on the proposed 300-ft design, a scaling method is derived which leaves the survival stability constant for all diameters D. All items of price P, surface deformation Δz, and pointing error $\Delta\theta$ are investigated in order to find with which power of D they increase.

The combined results are shown in Table 4 and plotted in Fig. 1. For $D \geqslant 300$ ft, a best-fit yields $P \sim D^{2.34}$, $\lambda = 16\ \Delta z \sim D^{1.17}$, and $\Delta\theta$ = const.

Since the price/area increases only very slowly, as $D^{0.34}$, the present 300 ft design (if actually built) may be considered as a model of a future larger telescope, to be scaled from the same design.

For example, a telescope with diameter D = 500 ft would cost P = 23.6 M$; it would have a shortest wavelength λ = 2.60 cm on calm sunny days, and λ = 1.80 cm for all other time (including winds up to 15 mph).

I. Introduction

Design and performance of the proposed 300-ft homologous telescope are described in Report 25 (March 12, 1969). The shortest wavelength, defined as 16 times the rms surface deviation from a best-fit paraboloid of revolution, was found as λ = 1.50 cm for sunny calm days and λ = .98 cm for all other time including winds up to 15 mph (2/3 of all time); both values for a zenith distance of 60° with the telescope being adjusted at zenith.

The pointing error depends on the instrumental error for which we used 5 arcsec for the present system and 3 arcsec for an improved one. In the following calculations we shall adopt 4 arcsec for the instrumental error; the combined pointing error for 300 feet diameter then is 7.38 arcsec for sunny calm days, and 5.50 arcsec for all other time (winds up to 15 mph).

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The price estimate was worked out by O. Heine (Feb. 18, revised March 14, 1969). This estimate includes (A) surface, dish and azimuth towers; complete drive system with motors and gears; optical pointing system with seven light beacons, console and computer; tracks and foundations; (B) service tower, building, and site development. An uncertainty was left open, regarding the price/weight of steel structure for telescope and towers, including erection. O. Heine used \$1.15/lb, while a cost estimate of LTV would lead to 0.90 \$/lb. In the following calculations we shall use an average of 1.03 \$/lb; with this value, items (A) yield 5.98 M\$ for 300 feet diameter, and items (B) yield 0.40 M\$, with a total of 6.38 M\$. Adding a contingency of 10% finally yields a total of 7.02 M\$.

The aim of the present investigation is to develop a fail-safe scaling method for price and performance, and to apply it to telescopes of various diameters (but all of the same design). This scaling method should give reliable results if the diameter is scaled up or down within a factor of 1.5. We have chosen D = 210, 250, 300, 350, 410 feet; the lowest one for comparison with existing large steerable telescopes, the highest one for comparison with the proposed NEROC telescope which has 440 ft diameter, reduced by the shadow of the radome to an effective diameter of 410 ft.

II. Method of Scaling

Instead of scaling both D and λ independently and then finding the most economical $\lambda(D)$, we scale only D, but in the most predictable way. This will yield a $\lambda(D)$ which still will be close to the most economical one.

1. Telescope Structure

Many single bars of the surface panels and of the members of layer 1 (beneath the surface) have survival stresses close to the maximum allowed stress. We call

Λ = ℓ/r = slenderness ratio of single pipe;

S_Λ = maximum allowed stress;

S_s = stress from survival loads;

S_g = stress from dead loads;

k = S_g/S_s; (1)

$$Q = (S_s + S_g) / S_\Lambda = (1 + k)\, S_s/S_\Lambda = \text{stress ratio;} \tag{2}$$

$$d = D/300 \text{ ft} = \text{scaling factor.}$$

All bars of the 300-ft design have $Q < 1$ for stability. We now decide to scale all bar areas according to

$$A \sim D^\alpha \tag{3}$$

and we want to find α such that $\underline{Q = \text{const}}$. This means we scale with constant survival stability. We always want to use standard steel pipes, where $r \sim A^{2/3}$ yields a good fit to the tables of the Steel Manual; thus

$$\Lambda \sim D/A^{2/3} \sim D^{1-2\alpha/3} \tag{4}$$

An inspection of the 300-ft design shows that all bars with $Q \gtrsim .70$ have $\Lambda \geq 100$, and then $S_\Lambda \sim \Lambda^{-2}$; thus

$$S_\Lambda \sim D^{(4\alpha/3)-2} . \tag{5}$$

Furthermore, we have

$$S_s \sim D^2/A \sim D^{2-\alpha} \tag{6}$$

and

$$S_g \sim W/A \sim D. \tag{7}$$

The demand of Q = constant then yields, with (1), (2), (5), (6), and (7),

$$(1 + k\, d^{\alpha-1})\, d^{4-7\alpha/3} = 1 + k, \tag{8}$$

and for $d \approx 1$ finally

$$\alpha = \frac{12 + 9k}{7 + 4k} . \tag{9}$$

Inspection of the 300-ft design shows that all bars with $Q \geqslant .70$ have $0.20 \leq k \leq 0.35$, yielding $1.769 \leq \alpha \leq 1.803$. We thus adopt

$$\alpha \;=\; 1.80 \text{ for all telescope bars.} \qquad (10)$$

2. Tower Structure

The highest stresses in the tower members are due to lateral survival loads (85 mph wind), where $F \sim D^2$. The main chords are very heavy, $\lambda \leq 40$, where S_λ increases only slowly with decreasing λ. For simplicity we use S_λ = const. The demand Q = const then leads to

$$A \sim D^2 \text{ for all tower bars.} \qquad (11)$$

3. Other Items

We use the same type of surface plates for all D, but vary the length ℓ of the plates as $\ell \sim D$, which means we have a constant number of plates.

Drive motors, gears and bearings are scaled in proportion to the weight above them, and the price of foundations and tracks, service tower and cables is scaled in proportion to D.

As being independent of D are regarded: console system, optical pointing system, computer, feed mount, building and site development.

III. Scaling to Various Diameters

1. The Price

All items of O. Heine's price estimate are listed in Table 1, together with the 300-ft price, and with the power γ of D as used for the scaling.

Table 1. Scaling the Price
(γ defined by $P \sim D^{\gamma}$)

Item	300-ft price M$	Subtotals M$	γ
1. Aluminum surface, studs	.70		2.0
2. Dish structure	2.32		2.8
3. Tower structure	1.39		3.0
4. Truck assemblies	.30		2.9
5. Foundations + tracks	.12		1.0
6. Elevation bearings	.10		2.8
7. Pintle bearing	.06		2.0
8. Elevation gear	.03		2.8
9. Azimuth gear	.16		2.9
10. Feed mount	.05		0
		5.23	
11. Optical pointing (7 beacons, platform, encoders, servo)	.35		0
12. Drive system (amplifiers, console)	.30		0
13. Computer	.10		0
		.75	
14. Cabling + catwalks	.03		1.0
15. Service tower	.11		1.0
		.14	
16. Building (3000 ft^2)	.12		0
17. Power + transformer	.08		0
18. Water, sewer, road	.03		0
19. Site preparation	.03		0
		.26	
	Total	6.38	
	add 10% contingency	7.02 M$	

Table 2 contains the different groups of γ, with the 300-ft price. The resulting prices for various D are given in Table 4 together with wavelength and pointing error. All results are also plotted in Fig. 1. A contingency of 10% is always included.

The prices should be considered realistic or even conservative, since we now are starting a final optimization of the dish structure which should reduce the total price by about 0.20 to 0.30 M$.

Table 2. Price Groups Regarding γ, from Table 1.

γ	300-ft price M$
3.0	1.39
2.9	.46
2.8	2.45
2.0	.76
1.0	.26
0	1.06
Total	6.38

2. The Shortest Wavelength

All gravitational deformations go with D^2, and all thermal deformations with D, both independent of the bar areas A. If we vary the size of the surface plates as $\ell \sim D$, then the first three items of Table 3 vary in proportion with D. If the adjustment is done with constant angular accuracy, then the linear accuracy is also in proportion with D.

The wind force goes with D^2, and the wind deformation thus with D^3/A. Since $A \sim D^{1.8}$ for the dish, we have for the wind deformation:

$$\Delta z \sim D^{1.2} \tag{12}$$

Table 3. Scaling the rms surface deviation from the best-fit paraboloid. (γ defined by rms(Δz) $\sim$ D^{γ})

	300-ft rms (Δz) mm	γ
1. Parabola/flat plate		
2. Dev. from flatness		
3. Shorter bulges	.447	1.0
4. Adjustment accuracy		
5. Adjuster level	.127	0
6. Dev. from homology, ΔH		
7. Use of standard pipes		
8. Sag of plates	.219	2.0
9. Sag of ribs		
10. Sag of panels		
11. Ext. load, panels		
12. Wind on plate + ribs		
13. Wind on panels	.241	1.2
14. Wind on telescope structure		
15. Thermal def ΔT = 5.0 °C	.788	1.0
15. Thermal def ΔT = 1.5 °C	.236	1.0

Table 3 shows all items considered in Report 25. For all gravitational deformations we have assumed that the telescope is adjusted at zenith and then tilted down by 60°. For the wind deformations we have used v = 15 mph; the wind is below this value for 2/3 of all time at Green Bank. The results of the scaling are shown in Table 4 and Fig. 1.

3. The Pointing Error

The thermal pointing error is independent of D and A. The instrumental pointing error (including optical reading, servo and drive systems) amounts to 5 arcsec for the present design worked out by O. Heine; a possible improvement may bring it down to 3 arcsec. For the present calculations we shall adopt 4 arcsec.

The wind-induced pointing error depends on τ, the duration of the longest (one-sided) wind deformation which is not omitted by the optical pointing system. For D = 300 feet, we have $\tau = 1.5$ sec. In general, with stiffness $K \sim A/D$ and weight $W \sim DA$, we have

$$\tau \sim \sqrt{W/K} \sim D \tag{13}$$

or

$$\tau = 1.5\ d. \tag{14}$$

The pointing error then is treated according to equations (4) and (6) of Report 23 (March 1969), using v = 15 mph, and scaling all wind deformations with $D^{1.2}$ as in (12). The resulting wind-induced pointing error ranges from 1.10 arcsec for D = 210 ft up to 2.94 arcsec for D = 410 ft, and it is always smaller than the thermal pointing error due to $\Delta T = 1.5\ ^{\circ}C$. Since both will not occur simultaneously, we take the thermal one only and disregard the wind-induced error.

Thermal and instrumental pointing error then add up to

$$\Delta\theta = \begin{cases} 7.38 \text{ arcsec, sunny calm days;} \\ 5.50 \text{ arcsec, all other time.} \end{cases} \tag{15}$$

All results are shown in Table 4 and Fig. 1. For $D \geqslant 300$, we read from Fig. 1

$$P \sim D^{2.34}, \text{ for } D \geqslant 300 \text{ ft,} \tag{16}$$

and

$$\lambda \sim D^{1.17}, \text{ for } D \gtrsim 300 \text{ ft.} \tag{17}$$

Finally, since the total price per telescope area increases only slowly,

$$P/\text{area} \sim D^{0.34}, \text{ for } D \geqslant 300 \text{ ft,} \tag{18}$$

we may also consider larger telescopes designed by the same principles. The results are shown in Table 4. Maybe the proposed 300-ft telescope, although extremely useful by itself, could be considered as a model of a future larger telescope.

Table 4. Price and Performance for Various Diameters D.
λ = 16 x rms(Δz) = shortest wavelength;
β = half-power beamwidth;
$\Delta\theta$ = pointing error.

D	P(+10%)	P/Area	Sunny calm days			All other time		
			λ	β	$\Delta\theta$	λ	β	$\Delta\theta$
Ft	M$	$/ft^2	cm	arcsec	arcsec	cm	arcsec	arcsec
210	3.48	100.3	1.05	44.5	7.38	.68	25.7	5.50
250	4.76	96.9	1.25	42.8	"	.80	25.4	"
300	7.02	99.3	1.50	42.3	"	.98	25.9	"
350	10.03	104.4	1.77	41.9	"	1.17	26.5	"
410	14.80	112.2	2.10	41.7	"	1.42	27.4	"
extrapolated:								
500	23.6	120.3	2.60	41.6	"	1.80	28.7	"
600	36.1	127.5	3.15	41.7	"	2.22	30.0	"

26

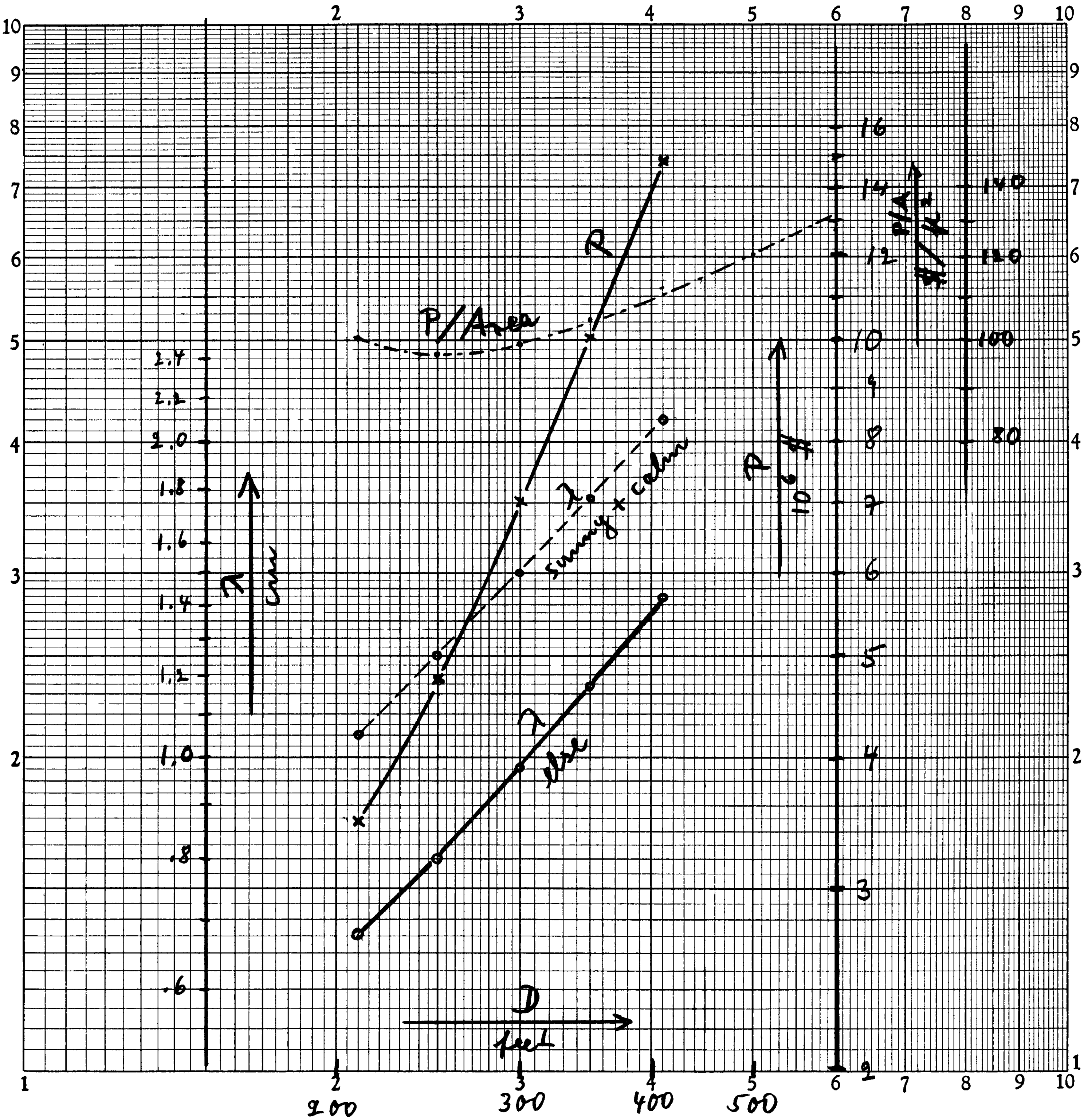

Fig. 1. Wavelength and Price as functions of diameter.

D = telescope diameter
λ = shortest wavelength (16 rms Δz)
P = price (complete, with drives, servos, computer, foundations building and service tower)
P/A = price per telescope area.