## The Choice of a Cassegrain System for the 65-m Telescope

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## Summary

A discussion of several arguments leaves only one crucial reason for a secondary mirror: a large cluster of many feeds needed for a survey of the whole sky at shortest wavelength (which would take 25-900 years at the prime focus). A removable Cassegrain is suggested, to be used for $3.5 \mathrm{~mm} \leq \lambda \leq 3.8 \mathrm{~cm}$, while longer wavelengths must be observed at the prime focus (excessive horn length at secondary focus), using a spillover shield. The Cassegrain mirror should be mounted at three feed legs, on computer-controlled jacks, allowing $\pm 1$ inch movement in all directions with an accuracy of $\pm .002$ inch up and $\pm .010$ inch sideways.

For structural reasons, the secondary focus is located only 5 ft above the vertex. The cabin is $10 \times 10 \mathrm{ft}$ wide, and 12 ft long. The maximum feed cluster is $9 \times 9 \mathrm{ft}$. There should be two exchangeable cabins.

Two lower limits and four upper ones are derived for the diameter $d$ of the secondary mirror, resulting in $d=12 \mathrm{ft}$. The magnification, then, is 15.7 ; the longest wavelength at the Cassegrain focus is 3.8 cm ; the first side lobe is 22.8 dB , and the coma lobe is lower than 15 dB at the corner of the feed bluster for all wavelengths; the maximum number of feeds is 965 for $\lambda=6 \mathrm{~mm}$, and larger for smaller $\lambda$; the weight of the secondary mirror is about 900 lb , and its lowest dynamical frequency is 4.5 cps (sufficient for fast corrections of the pointing, bypassing the dynamical lag of telescope and towers).

## I. General Considerations

## 1. Reasons for Planning a Cassegrain System

Table 1 lists the six reasons mostly given in favour of a secondary mirror, plus one more connected with our special pointing system. They are listed in the order of increasing importance regarding the $65-\mathrm{m}$ telescope (zero means no difference, positive is in favour of a secondary, negative is against it).

Table 1. Reasons for a secondary mirror.

| Reason | Remarks for 65-m design | Estimated importance $(-5$ to +5$)$ |
| :---: | :---: | :---: |
| 1. Easy access | Service tower for prime focus is planned anyway. Access, then, is even easier there. | -2 |
| 2. Heavy equipment | Feed legs rest on most basic points of back-up structure. Additional weight at prime focus gives less surface deviation than at vertex (factor 3.4). | -2 |
| 3. Scanning ability | Telescope is fully steerable. | 0 |
| 4. Reduced spillover | A spillover shield at prime focus (Report 3, 1965) is just as good but cheaper; usable for $\lambda \leq 20 \mathrm{~cm}$ for $65-\mathrm{m}$. Needs to be tested at $140-\mathrm{ft}$. | 0 |
| 5. Multiple feed for alternative observations | Important for short wavelength and weather changes. But can also be done with rotating bottom of prime focus cabin, see Parkes telescope. | 1 |
| 6. Improvement of pointing accuracy | Bypassing the lowest dynamical frequency of the whole telescope ( 1.5 cps ) with a fast-correcting secondary mirror. Estimated $20-30 \%$ improvement of pointing accuracy; to be known better after $O$. Heine's platform experiment. | 3 |
| 7. Cluster of many feeds for simultaneous observations. | Survey of whole sky at short wavelength. Completely impossible at prime focus because of long duration. | 5 |

Regarding item 2, an experiment was run on the computer. It shows (1) that almost any load (up to 30 tons, say), if known in advance and not to be changed later, can be taken care of in our homology program by small changes of bar areas, such that no
additional rms surface deviation results. (2) Once the telescope is built for a given load, any additional load at the prime focus then gives additional rms surface deviations of $1.66 \times 10^{-4}$ inch/ton, and $5.67 \times 10^{-4} \mathrm{inch} /$ ton at the vertex (factor 3.41). This is tolerable up to about . 002 inch; which allows up to 12 tons additional load at the prime focus, but only up to 3.5 tons at the vertex.

There is only one really crucial reason for a secondary mirror: a cluster of many feeds needed for a survey at shortest wavelengths. For $\lambda=3.5 \mathrm{~mm}$, the beamwidth is only 13.3 arcsec. The observable $3 / 4$ of the whole sky then contains $2.89 \times 10^{9}$ beam areas. With an integration time of 10 sec per beam area, a complete survey then would take 983 years with a single feed, or still 25 years with 1 sec integration and four feeds at the prime focus (where a large number of feeds is prevented by coma lobes). The only way out seems to be a cluster of 100-1000 feeds at a secondary focus. According to $S$. Weinreb, receivers will get better in the future only down to some "natural limit" of noise temperature, and from then on they will get cheaper, and multiple feeds (and receivers) of several hundred may become feasible. For a wavelength of a few millimeter, a high number of bolometers in a common dewar may be considered.

Even if this reason were the only one, it still is important enough to decide on a secondary mirror. The system, thus, should be designed especially with regard to its use at shortest wavelengths. But for longer wavelengths, the prime focus must be accessible, too.

Whether to plan a Cassegrain (hyperbola below prime focus), or a Gregorian (ellipse above prime focus), seems not to be an important decision. Since a Gregorian needs a higher feed support sturcture, which is already rather high and slender in our design, and since Cassegrains are better understood, several discussions at NRAO resulted in planning a Cassegrain.

## 2. Mount of Cassegrain Mirror

The mirror should be mounted at three of the four feed legs. Removing and accurately remounting must be fast, with a design goal of 1 hour, say, and a maximum of $1 / 2$ day. It should be mounted on computer-controlled precision jacks, allowing the following movements:
(a) $\pm 1$ inch parallel up and down, z ; for homologous changes of focal length;
(b) $\pm 1$ inch parallel sideways, $x$; for homologous changes of axial direction;
(c) $\pm 1^{\circ}$ rotation about prime focus, $x$ and $y$; for alternating observations (1. 5 feet off-axis at vertex), and for on-off source observations; needs $\approx \pm 1$ inch movement sideways for Cassegrain mirror;
(d) $\pm 5$ arcsec fast tilt about its center, $x$ and $y$; for fast pointing corrections.

In summary, all three mounts must be able to move in all three directions by $\pm 1$ inch, with a precision of about $\pm .002$ inch up and down, and about $\pm .010$ inch sideways. Therefore, it is highly desirable to keep the secondary mirror as small and as light-weight as possible. If its weight goes with $\mathrm{d}^{2.5}$, its moment of inertia even goes with $d^{4.5}$.

## 3. Feed Movement Needed ?

(a) Secondary focus. A feed movement sideways is never needed because of the high flexibility demanded for the secondary mirror. But a feed movement along the telescope axis could be needed regarding the homologous change $d F$ of the focal length. If $F$ changes by dF , then the prime focus, the secondary mirror, and the secondary focus, all must move in parallel by dF. If the feed does not move, it then is axially displaced by $\Delta z=d F$. With $d F=1$ inch, and $\lambda=3.5 \mathrm{~mm}$, the feed is displaced by

$$
\begin{equation*}
\Delta z=7.26 \lambda \tag{1}
\end{equation*}
$$

According to equation (17) of Baars (1966), the relative gain loss, $L=1-\frac{G}{G_{0}}$, is for a parabolic illumination

$$
\begin{equation*}
\mathrm{L}=\frac{1}{18}\left[2 \pi \frac{\Delta \mathrm{z}}{\lambda}(1 \cos \varphi)\right]^{2} \quad \text { for } \quad \mathrm{L} \ll 1 \tag{2}
\end{equation*}
$$

which holds for a feed at the prime focus, and where $\varphi$ is the illumination angle, see Fig. 2. For a feed at the secondary focus, we must use $\varphi_{0}$ instead of $\varphi$, where (for $M \gg 1$ )

$$
\begin{equation*}
\cos \varphi_{0} \approx \frac{\mathrm{f}}{\sqrt{\mathrm{f}^{2}+\mathrm{D}^{2} / 4}} \approx 1-\frac{1}{2}(\mathrm{D} / 2 \mathrm{MF})^{2}=1-0.685 / \mathrm{M}^{2} \tag{3}
\end{equation*}
$$

which gives

$$
\begin{equation*}
\mathrm{L}=\frac{\pi^{2}}{18}(\mathrm{D} / 2 \mathrm{~F})^{4}(\Delta \mathrm{Z} / \lambda)^{2} \mathrm{M}^{-4}=1.03(\Delta \mathrm{Z} / \lambda)^{2} \mathrm{M}^{-4} \tag{4}
\end{equation*}
$$

and with (1)

$$
\begin{equation*}
\mathrm{L}=54.6 / \mathrm{M}^{4} \tag{5}
\end{equation*}
$$

If we demand, for example, $\mathrm{L} \leq 2 \%$, then

$$
\begin{equation*}
\mathrm{M} \geq 7.23 . \tag{6}
\end{equation*}
$$

Since we will see later that $M \approx 15$, we have only $L \approx 0.1 \%$. This means that a feed movement is not necessary at the secondary focus.
(b) Prime focus. From equation (2) and $\mathrm{dF}=1$ inch one can derive that an axial movement of the feed is needed for $\lambda \leq 14 \mathrm{~cm}$ if $\mathrm{L} \leq 2 \%$ is demanded, and for $\lambda \leq 9 \mathrm{~cm}$ if $\mathrm{L} \leq 5 \%$. This means that we certainly need an axial movement.

A lateral movement is not needed if $L=5 \%$ can be tolerated at $\lambda=2 \mathrm{~cm}$. But if wavelengths $\lambda \leq 2 \mathrm{~cm}$ are to be observed at the prime focus, too, then a lateral feed movement of 1 inch must be also provided (for homologous changes of axial direction).

## 4. Location of Secondary Focus, Cabin Size

Long feed horns can be avoided by placing the secondary focus higher above the vertex, closer to the secondary mirror. Fig. 1 shows the Goldstone antenna, with a vertex tower reaching 41 feet above the vertex, reducing the horn length by almost a factor of four as compared to a horn at the vertex.

This solution cannot be applied to our 65-m telescope because of the high surface precision needed for millimeter wavelengths. A long and heavy tower gives a large moment at its base when pointed at horizon, which is extremely difficult to counteract in the telescope structure. It was thus decided to mount the feed cabin right at the vertex, where it adds only a weight but no moment.

We need a feed cabin each at the prime focus and the secondary focus. For the latter, S. Weinreb suggested to have two exchangeable cabins, one to work with and one to work at. These cabins should be small enough for being removable; but they should be wide enough for housing two independent experiments, alternating with weather changes, and they should be long enough for the feed horns at the longest Cassegrain-wavelength, $\lambda_{m}$, As a compromise we agreed on

for both secondary and prime focus. Whenever this length is not needed, a higher second floor can be mounted inside.

Regarding the length of 12 feet, we then decided to have 5 feet above the surface and 7 feet below it. The distance $b$ between vertex and secondary focus in Fig. 2 then is

$$
\begin{equation*}
\mathrm{b}=5 \text { feet. } \tag{8}
\end{equation*}
$$

## 5. Longest Wavelength $\lambda_{m}$ at Secondary Focus

A large distance between primary and secondary focus, a finite cabin length, and the desire for a small secondary mirror, all tend to result in a small $\lambda_{m}$ (above which the prime focus must be used). Formulas for $\lambda_{m}$ will be given later. Here, we ask only: how small a $\lambda_{m}$ can be accepted from the future user's point of view?

First, there should be a certain overlap with existing or future large, high-precision telescopes which are shown in Table 2. This comparison results in a smallest value of about $\lambda_{\mathrm{m}}=3 \mathrm{~cm}$, because for $\lambda \geq 2 \mathrm{~cm}$ there are two telescopes larger than $65 \mathrm{~m}(=213 \mathrm{ft})$ available.

Table 2. Existing and future large telescopes.

| Telescope | D <br> feet | $\lambda_{\min }$ <br> cm |
| :--- | :---: | :---: |
| NEROC $\ldots \ldots \ldots \ldots$ | 410 | 2 |
| Bonn $\ldots \ldots \ldots$. | 330 | 2.5 |
| Goldstone $\ldots \ldots \ldots$ | 210 | 5 |
| Arecibo $\ldots \ldots \ldots$ | 1000 | 10 |

Second, we try to imagine how the scheduling might be done with $\lambda_{m}=3 \mathrm{~cm}$. This is shown in Table 3, assuming a good telescope site with 7 good months, say, and 3 bad ones like a rain season, and two intermediate ones in between. It is assumed that the scheduling always plans an alternate observation at longer wavelength to accompany any short-wavelength observations, changing from one to the other with changes of atmospheric conditions (and with day and night for the very shortest wavelengths). These changes, however, should not include a change between Cassegrain and prime focus.

Table 3. Example of scheduling, with $\lambda_{m}=3 \mathrm{~cm}$.

| Period/Year | $\lambda(\mathrm{cm})$ |  | Focus |
| :--- | :---: | :---: | :---: |
|  | shortest | alternate |  |
| 7 good months | 0.3 | $3=\lambda_{m}$ | Cassegrain $=7$ months <br> 2 intermediate months <br> 3 bad months |

Since $\lambda_{m}=3 \mathrm{~cm}$ seems reasonable with respect to both overlap and scheduling, whereas a longer cut-off still would be more convenient, we conclude

$$
\begin{equation*}
\lambda_{\mathrm{m}} \geq 3 \mathrm{~cm} \tag{9}
\end{equation*}
$$

## II. Magnification, Blocking, and Horn Size

## 1. Magnification

Definitions are given in Fig. 2. We call

$$
\begin{align*}
& m=D / d=\text { diameter ratio of mirrors }  \tag{10}\\
& M=f / F=\text { magnification }=\text { ratio of focal lengths. } \tag{11}
\end{align*}
$$

If we neglect the difference in curvature between primary, secondary, and equivalent mirror (valid for $m \gg 1$ ), we obtain from Fig. 2

$$
\begin{equation*}
\mathrm{m}=\mathrm{D} / \mathrm{d}=\mathrm{F} / \mathrm{a}=\mathrm{f} / \mathrm{g} \tag{12}
\end{equation*}
$$

and, with

$$
\begin{equation*}
g=F-b-a=F-b-F / m \tag{13}
\end{equation*}
$$

we derive in general

$$
\begin{equation*}
\mathrm{M}=\mathrm{m}(1-\mathrm{b} / \mathrm{F})-1 \tag{14}
\end{equation*}
$$

Using $\mathrm{F}=91 \mathrm{ft}$ and $\mathrm{b}=5 \mathrm{ft}$ for the $65-\mathrm{m}$ telescope, we have

$$
\begin{equation*}
M=0.945 \mathrm{~m}-1 \tag{15}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{m}=1.058(\mathrm{M}+1) . \tag{16}
\end{equation*}
$$

## 2. Blocking

The geometrical blocking is $(\mathrm{d} / \mathrm{D})^{2}=1 / \mathrm{m}^{2}$. Since the blocked area is illuminated, too, we get a factor of two for constant illumination, and another factor of about two for a tapered illumination where the center counts highest. In terms of signal/noise, the blocking then is

$$
\begin{equation*}
\mathrm{B}=4 / \mathrm{m}^{2} \tag{17}
\end{equation*}
$$

For example, we may demand

$$
\begin{equation*}
m \geq 14.1 \text { for } B \leq 2 \% \tag{18}
\end{equation*}
$$

## 3. Minimum Horn Length

Definitions for the feed horn are given in Fig. 3a. We use a paper of A. P. King (1950) about horn optimization. The shortest horn for a given gain is defined by

$$
l_{s}-l=0.3 \lambda
$$

which, for $\lambda \ll \ell$, gives a horn length of

$$
\begin{equation*}
\ell=0.417 \mathrm{~h}^{2} / \lambda \tag{19}
\end{equation*}
$$

The half-power beamwidth $\beta$ for this optimized horn is $1.221 \lambda / \mathrm{h}$ for the magnetic plane, and $1.047 \lambda / \mathrm{h}$ for the electric plane, with an average of

$$
\begin{equation*}
\beta=1.134 \lambda / \mathrm{h} . \tag{20}
\end{equation*}
$$

## 4. Tapered Illumination of Cassegrain Mirror

We demand that the illumination is 3 dB down at $1 / 2$ the Cassegrain radius:

$$
\begin{equation*}
\beta \mathrm{g}=\mathrm{d} / 2 . \tag{21}
\end{equation*}
$$

With equations (10) to (14), we find in general the horn width as

$$
\begin{equation*}
\mathrm{h}=2.27 \frac{\mathrm{~F}}{\mathrm{D}} \mathrm{M} \lambda \tag{22}
\end{equation*}
$$

Selecting the same $F / D=0.427$ as at all our NRAO telescopes,

$$
\begin{equation*}
\mathrm{h}=0.969 \mathrm{M} \lambda . \tag{23}
\end{equation*}
$$

Insertion of (22) into (19) yields the horn length in general as

$$
\begin{equation*}
\ell=2.15\left(\frac{\mathrm{~F}}{\mathrm{D}} \mathrm{M}\right)^{2} \lambda \tag{24}
\end{equation*}
$$

Or, with $\mathrm{F} / \mathrm{D}=.427$,

$$
\begin{equation*}
\ell=0.391 \mathrm{M}^{2} \lambda \tag{25}
\end{equation*}
$$

Demanding $\ell=12 \mathrm{ft}$ and $\lambda \geq 3 \mathrm{~cm}$, we find

$$
\begin{equation*}
M \leq 17.6 \tag{25a}
\end{equation*}
$$

## 5. Multiple Feed Cluster

We use a square feed cluster as shown in Fig. 3b. With respect to the cabin size of $10 \times 10$ feet, we adopt a maximum cluster size of, say,

$$
\begin{equation*}
c_{m}=9 \text { feet } \tag{26}
\end{equation*}
$$

In general, we have

$$
\begin{equation*}
c=h(\sqrt{n}-1) \tag{27}
\end{equation*}
$$

with $n=$ number of feeds, and using $h$ from (23) we find for $F / D=.427$

$$
\begin{equation*}
\mathrm{n}=(1+1.032 \mathrm{c} / \mathrm{M} \lambda)^{2} \tag{28}
\end{equation*}
$$

and with $c_{m}=9 \mathrm{ft}=275 \mathrm{~cm}$ finally

$$
\begin{equation*}
\mathrm{n}_{\mathrm{m}}=(1+283 \mathrm{~cm} / \mathrm{M} \lambda)^{2} \tag{29}
\end{equation*}
$$

Demanding, say, $\mathrm{n} \geq 1000$ for $\lambda=3.5 \mathrm{~mm}$, we obtain $\mathrm{M} \leq 26.5$, and with (15) we find a maximum diameter ratio of

$$
\mathrm{m} \leq 29.1
$$

## III. The Coma Lobe

The tolerable scan angle (off-axis feed displacement) of a Cassegrain system is limited by gain loss and/or by coma lobe increase. Both are caused by two effects. First, a one-sided spillover occurs beyond the rim of the primary mirror as shown in Fig. 4b. Second, the difference in path length between various rays yields phase errors. From the first effect, the gain loss is more harmful than the coma increase; in addition, any spillover toward the ground increases the noise temperature. From the first effect, the coma increase is more harmful than the gain loss. These distinctions are not clearly seen in many published papers.

## 1. Spillover and Critical Displacement

A critical scan angle $\Theta_{c r}$ is defined by White and DeSize (1962): for $\theta \geq \Theta_{c r}$ all incoming rays, after reflection at the primary, miss the secondary mirror, see Fig. 4a. The authors plot $\Theta_{\mathrm{cr}}$ for various $\mathrm{F} / \mathrm{D}$ ratios as a function of M . For $\mathrm{M} \gg 1$, the critical angle approaches $\theta_{c r}=0.72 / \mathrm{M}$, in radians, for $F / D=0.427$. The critical angle then can be transformed into a critical displacement, $r_{c r}$, using Baars' (1966) values of the beam deviation factor, BDF, for small M (using BDF as a function of $f / D$ instead of $\mathrm{F} / \mathrm{d}$ ):

$$
\begin{equation*}
\mathbf{r}_{\mathbf{c r}}=\frac{\mathrm{F} \theta_{\mathbf{c r}} \mathrm{M}}{\mathrm{BDF}} \tag{30}
\end{equation*}
$$

Of course, one cannot go until $r_{\text {cr }}$. From computer data about beam patterns published by Nihen and Kay (1963) it can be derived that a 3 dB gain loss is reached at about $0.4 \mathrm{r}_{\mathrm{cr}}$, while a 15 dB coma lobe is reached at about $0.3 \mathrm{r}_{\mathrm{cr}}$. But since the noise increase from spillover seems more serious, we suggest to regard

$$
\begin{equation*}
\mathrm{r}_{\mathrm{o}}=0.1 \mathrm{r}_{\mathrm{cr}} \tag{31}
\end{equation*}
$$

as a desirable limit. At this limit, the spillover is about $10 \%$ of the illuminated area, but the power spillover is considerably less, depending on the illumination taper. Fig. 5 then shows the maximum feed displacement $r_{0}$ for the $65-m$ telescope.

For $M \gg 1$, we find $r_{0}=6.5$ feet; this is just about the maximum feed cluster size adopted in (20) giving $r=9 / \sqrt{2}=6.37$ feet. This means that within a cluster size of $9 \times 9$ feet, the spillover creates a tolerable noise increase, and negligible gain loss and coma increase.

## 2. Coma Lobe from Phase Errors

Nihen and Kay (1963) have calculated beam patterns for 18 different combinations of $m, F / D, b, D / \lambda$, with 3 scan angles each. The results are shown graphically in detail, but are only poorly discussed: (a) the spillover effect is introduced in their description of the computer program, but is hardly ever mentioned regarding the results, although the vast majority of their cases is severely spillover-limited (mostly $\mathrm{D} / \lambda=200$ only), see the discussion of their Figs. 38 and 39. (b) They define the maximum scan angle by a 3 dB loss, although coma lobes up to 7 dB are much more severe for many applications. (c) They seem not to see that for a Cassegrain system it is mostly f/D = MF/D which matters and not F/D itself. But 4 of their graphs are essentially free of spillover effects and are used in the following.

We measure the scan angle $\Theta$ in terms of the half-power beamwidth $\beta$ and define

$$
\begin{equation*}
\mathrm{k}=\Theta / \beta \tag{32}
\end{equation*}
$$

We call $\gamma$ the level of the coma lobe (in dB , defined positive) at scan angle $\theta$, as compared to the main lobe at $\Theta=0$, because this is what matters for mapping or confusion problems. From Nihen and Kay's beam patterns we can derive approximately

$$
\begin{equation*}
\mathrm{k}=4.6 \mathrm{M} 10^{(15-\gamma) / 20} \quad \text { for } \quad \gamma \leq 20 \mathrm{~dB} \tag{33}
\end{equation*}
$$

or

$$
\begin{equation*}
\gamma=15-20 \log (\mathrm{k} / 4.6 \mathrm{M}) \quad \text { for } \quad \mathrm{k} \geq 2.6 \mathrm{M} \tag{34}
\end{equation*}
$$

For $\mathrm{k} \rightarrow 0, \gamma$ approaches quickly $\gamma_{0}$ of the first sidelobe, which is given by Baars (1964) as a function of the illumination taper and of the blocking by the secondary mirror. The limits given in (33) and (34) apply to $m \geq 7$; they are $\gamma \leq 22$ and $k \geq 2.1 \mathrm{M}$ for $\mathrm{m} \geq 15$.

Next, we use $\theta=r / f=c /(M F \sqrt{2})$ and $\beta=1.2 \lambda / D$ in (32) and obtain, in general and for $F / D=.427$,

$$
\begin{equation*}
\mathrm{k}=0.589 \frac{\mathrm{D}}{\mathrm{~F}} \frac{\mathrm{c}}{\mathrm{M} \lambda}=1.38 \frac{\mathrm{c}}{\mathrm{M} \lambda} . \tag{35}
\end{equation*}
$$

With (27) for c, and (22) for $h$, we then have in general

$$
\begin{equation*}
k=1.33(\sqrt{n}-1) . \tag{36}
\end{equation*}
$$

With (35) and (36) we thus can obtain the coma lobe from (34), for any given cluster size c , or given feed number n .

Demanding the full use of the cabin size at shortest wavelength, $\mathrm{c}=9 \mathrm{ft}$ and $\lambda=$ 3.5 mm , with a tolerable coma lobe of $\gamma \geq 15 \mathrm{~dB}$, we obtain

$$
\begin{equation*}
\mathrm{M} \geq 15.35 . \tag{37}
\end{equation*}
$$

## IV. Dynamical Properties

The total pointing error of the telescope consists of two parts. The first part is unknown, like thermal and wind deformations above the pointing platform, and reading errors. The remaining part is known but cannot be corrected fast enough because of the dynamical lag of the telescope structure. This second part is in proportion to $\nu^{-\alpha}$, where $\nu=$ lowest dynamical frequency of combined structure, and with $\alpha=1$ for slow perturbations, $\alpha=2$ for fast ones, and about $\alpha=1.5$ in the average. The combined structure (from ground to focal cabin) has $\nu=1.5 \mathrm{cps}$.

This second part of the pointing error can be reduced if the secondary mirror can be tilted with more than 1.5 cps . A considerable reduction by a factor of $\geq 5$, say, then demands a dynamical frequency of the secondary mirror of $\nu \geq 1.5 \times 5^{2 / 3}$, or

$$
\begin{equation*}
\nu \geq 4.4 \mathrm{cps} . \tag{38}
\end{equation*}
$$

The mirror itself will be very stiff because of its small size. What matters, then, is the bending stiffness of the long feed legs where the mirror is mounted. Scaled down from Report 22, for $D=213 \mathrm{ft}$, we have for the feed legs:

| Unguyed length | 610 inch |
| :---: | :---: |
| Prime focus - secondary mirror | 160 inch |
| Chord area (3 chords) | 5.88 inch $^{2}$ |
| Weight length | $3.28 \mathrm{lb} / \mathrm{inch}$ |
| Radial width | 46 inch |
| Radial moment of inertia | 2770 inch $^{4}$ |
| Tangential width | 21 inch |
| Tangential moment of inertia | 433 inch ${ }^{4}$ |

From these values, using formula (5) of Report 20, the dynamical frequencies from dead load only (neglecting the mirror weight) are
$\begin{array}{ll}\text { Radial } & \nu_{\text {rd }}=13.0 \mathrm{cps}, \\ \text { Tangential } & \nu_{\text {td }}=5.13 \mathrm{cps} .\end{array}$

The dynamical frequencies from the mirror weight W only (neglecting the dead load) are found from formula (6) of Report 20 as


Radial oscillation of the feed leg means a parallel translation of the mirror, where in (42) the full weight $W$ is applied to a single feed leg (worst case, with three-point mount). Tangential oscillation of the leg means a rotation of the mirror about the telescope axis, where for (43) only $\mathrm{W} / 3$ is applied to one leg.

Both radial oscillations give rather high frequencies which we neglect. The combined rotational frequency, $\nu_{t}$, is found from

$$
\begin{equation*}
\nu_{\mathrm{t}}^{-2}=\nu_{\mathrm{td}}^{-2}+\nu_{\mathrm{tw}}^{-2} \tag{44}
\end{equation*}
$$

as

$$
\begin{equation*}
v_{\mathrm{t}}=\frac{5.13 \mathrm{cps}}{\sqrt{1+0.303 \mathrm{~W} / \mathrm{kip}}} \tag{45}
\end{equation*}
$$

Demand (38) then leads to

$$
\begin{equation*}
\mathrm{W} \leq 1190 \mathrm{lb} \tag{46}
\end{equation*}
$$

For deriving a limit of the diameter $d$ of the secondary mirror, we use $\mathrm{W}=4100 \mathrm{lb}$ and $\mathrm{d}=22 \mathrm{ft}$ from the Goldstone antenna, and we assume that $\mathrm{W} \sim \mathrm{d}^{2.5}$, which gives

$$
\begin{equation*}
\mathrm{W}=571 \mathrm{lb}(\mathrm{~d} / 10 \mathrm{ft})^{2.5}, \tag{47}
\end{equation*}
$$

and (46) finally results in

$$
\begin{equation*}
\mathrm{d} \leq 13.4 \mathrm{ft} . \tag{48}
\end{equation*}
$$

## V. Size of Secondary Mirror

In the previous sections we have derived a total of 6 limits for $M, m$, or $d$. With (10) and (16), we convert the former limits into limits for $d$; this results in two lower limits and four upper ones. They are listed in Table 4.

Table 4. Limits for diameter d of secondary mirror.

| No. | Problem | Adopt | Demand | Equation | Limit |
| :---: | :--- | :--- | :--- | :--- | :---: |
| 1 | multiple feed, <br> cluster size | $\mathrm{c}=9 \mathrm{ft}$ <br> $\lambda=3.5 \mathrm{~mm}$ | $\mathrm{n} \geq 1000$ | $(29 \mathrm{a})$ | $\mathrm{d} \geq 7.3 \mathrm{ft}$ |
| 2 | limited cabin <br> length | $\mathrm{l}=12 \mathrm{ft}$ | $\lambda_{\mathrm{m}}^{\geq} 3 \mathrm{~cm}$ | $(25 \mathrm{a})$ | $\mathrm{d} \geq 10.8 \mathrm{ft}$ |
| 3 | full use of <br> cabin width | $\mathrm{c}=9 \mathrm{ft}$ <br> $\lambda=3.5 \mathrm{~mm}$ | $\gamma \geq 15 \mathrm{~dB}$ <br> coma | $(37)$ | $\mathrm{d} \leq 12.3 \mathrm{ft}$ |
| 4 | improved point- <br> ing accuracy | $\mathrm{W} \sim \mathrm{d}^{2} .5$ | $v \geq 4.4 \mathrm{cps}$ <br> dyn. freq. | $(48)$ | $\mathrm{d} \leq 13.4 \mathrm{ft}$ |
| 5 | blocking by sec- <br> ondary mirror | $\mathrm{B}=4 / \mathrm{m}^{2}$ | $\mathrm{B} \leq 2 \%$ <br> gain loss | $(18)$ | $\mathrm{d} \leq 15.1 \mathrm{ft}$ |
| 6 | homologous dF, <br> fixed feed | $\mathrm{dF}=1 \mathrm{inch}$ <br> $\lambda=3.5 \mathrm{~mm}$ | $\mathrm{L} \leq 2 \%$ <br> gain loss | (6) | $\mathrm{d} \leq 22.4 \mathrm{ft}$ |

Table 5. Secondary mirrors of various size.

| d | m | M | $\lambda_{m}$ | Side + Coma Lobe |  |  | Number $\mathrm{n}_{\mathrm{m}}$ of feeds, in $9 \times 9 \mathrm{ft}$ cluster |  | W | $\nu_{\mathrm{t}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | axis <br> $\gamma_{0}$ | $\begin{gathered} 6.37 \mathrm{ft} \text { off-axis, } \\ \gamma_{\mathrm{c}} \\ \hline \end{gathered}$ |  |  |  |  |  |
|  |  |  |  |  | $\begin{gathered} \lambda= \\ 6 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \lambda= \\ 3.5 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \lambda= \\ 6 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \lambda= \\ 3.5 \mathrm{~mm} \end{gathered}$ |  |  |
| ft |  |  | cm | dB | dB | dB |  |  | $1 b$ | cps |
| 8 | 26.6 | 24.1 | 1.6 | 23.4 | 23.1 | 21.4 | 424 | 1192 | 327 | 4.89 |
| 10 | 21.3 | 19.1 | 2.6 | 23.1 | 21.9 | 18.6 | 661 | 1875 | 571 | 4.74 |
| 12 | 17.7 | 15.7 | 3.8 | 22.8 | 19.8 | 15.3 | 965 | 2753 | 901 | -4. 55 |
| 14 | 15.2 | 13.4 | 5.2 | 22.5 | 17.3 | 12.6 | 1312 | 3758 | 1324 | 4.33 |
| 16 | 13.3 | 11.6 | 7.0 | 22.2 | 14.8 | 10.1 | 1738 | 4992 | 1849 | 4.11 |
| 18 | 11.8 | 10.1 | 9.2 | 21.9 | 12.4 | 7.7 | 2278 | 6561 | 2482 | 3.87 |



## Welcome to the <br> Mars Deep Space Station

The Mars Deep Space Station, ated at the Goldstone Deep Space Communication Complex near Barstow, Calfornia, is the site of the world's largest and most sensitive automatic tracking an tenna. The antenna is the newest addition to the Deep Space Network (DSN), which is operated by the Jet Propulsion Laboratory for the National Aero nautics and Space Administration. The DSN is responsible for tracking unmanned lunar and plan etary spacecraft missions and maintains and oper ates a network of tracking stations located at Goldstone, California; Cape Kennedy, Florida; As cension Island; Woomera and Canberra, Australia Johannesburg, South Africa; and Madrid, Spain.
JPL was responsible for establishing the performance and operational requirements of the antenna and directed the prime contractor, the Rohr Corporation of San Diego, California, and dozens of subcontractors and firms across the nation in the construction of the giant antenna. Construction at Goldstone was started in October 1963, and dedication ceremonies commemorating the antenna's 1966. Sin whe all the station on Aprii 29 bie. Sis hav, all percessfully achieved in and in actual tracking operations The Mars Stationg operato
a major step for ward in antenna design, reflector size, and instru-
mentationer deep space missions. The aluminum reflector, or "dish," is 210 feet in diameter, and the overall height of the structure, from the pedestal base to the apex, measures 234 feet, or the equivalent of a 21 -story building. The entire structure, including the massive pedestal, weighs 8,000 tons. Despite its great size and weight, the " 210 " can be maneuvered as easily and precisely as the smaller 85 -foot antennas used by the DSN.
The dish is carefully contoured to an accuracy of less than $1 / 4$ inch. It is designed to operate in winds up to 50 miles per hour and to withstand winds up to 120 miles per hour in a stowed position. The reflector and its supporting structure, weighing nearly 5 million pounds, rotate on a pressurized oil film about the thickness of a sheet of paper, by the use of three main hydrostatic thrust bearings. This great mass can be turned automatically or manually at desired tracking rates by drive motor The pedital capporting The pedestal supporting the antenna is 34 fee I!
The Mars Station 210 -foot antenna is the largest operated by the Jet Propulsion Laboratory's Deep Space Network for NASA. It it
one of six racking stations located at Goldstone, including the ioneer Echo, and Venus Stations of the DSN; the Manned Space Flight Network Apollo tracking station; and the Space
Tracking and Data Acquisition Network station.



Workmen are dwarfed by the Cassegrainian feed cone and surrounding aluminum tish surface of ere signals received from a spacecratt are reflected from the main dish to a subreflector (out of view) mounted on a truss-type
support (partially visible). The subreflector focuses the signal support (partially visible.). The subretlector focuses the signal
into the feed horn of the Cassegrainian cone, where it is ampli-
fied through a tied through a maser system.
walls are 42 -inch thick reinforced concrete, and the footings extend 11 feet underground. Within the pedestal are rooms for machinery and tracking equipment, electronic and data-handling facilities, offices, and maintenance shops. (A permanent display describing the function and role of the station is located on the second floor of the pedestal.)

Through the pedestal's center-but completely isolated from it-rises a 106 -foot tower which provides a stable, vibration-free platform for the sensitive angle-data master equatorial system. This unique system serves as the reference for antenna beam position and provides antenna pointing inforThe Mars Station is copab.
The Mars Station is capable of operation 24 hours a day, 365 days a year. Its transmitting and receiving capability provides six and a half times more with 85 foot-diameter the wh 85 the prent of two and a half times. This capability is a basic
requiremen oror the extension of the space explo ration effort. It allows spacecraft designers to in crease the information-gathering capability of tenna's capability, scientific experiments aboard the spacecraft can be designed to yield a greater amount of and more precise data.
It is anticipated that facilities with the capabilities the size of the Mars Station eventually will be constructed at other Deep Space Network stations around the Earth.

Antenna Dimensions

| Diameter |  |
| :--- | :--- |
| Focal length | 210 feet |
| Focal length/diameter ratio | 88.941 feet |
| Surface area | 0.4235 |
|  | 37,491 square feet |
|  | $(0.85$ acre $)$ |
| Depth of paraboloid | 31 feet |
| Pedestal wall thickness | 3.5 feet |
| Outside diameter of pedestal | 83 feet |
| Overall height of instrument tower** | 139 feet |
| Total concrete | 2500 cubic yards |

*Height of concrete section, 68 feet, including 33 feet below grade.
Antenna Weights, lb

|  |  |
| :--- | ---: |
| Overall | $16,000,000$ |
| $\quad$ On elevation bearings | $2,530,000$ |
| On azimuth bearings (including bearings) | $5,000,000$ |
| $\quad$ On soil | $16,000,000$ |
| Total rotating | $5,000,000$ |
| Total tipping | $2,500,000$ |
| Component | 4,100 |
| $\quad$ Hyperboloid | 62,000 |
| Feed cone and equipment | 39,000 |
| Quadripod | 58,000 |
| Primary reflector surface | $2,370,000$ |
| Reflector assembly (including reflector, | $2,200,000$ |
| $\quad$ wheels, and elevation counterweight) | 400,000 |
| Alidade and buildings | $10,000,000$ |
| Azimuth bearings | 96,000 |
| Pedestal and foundation | $1,151,000$ |
| Instrument tower (including wind shield) |  |
| $\quad$ Steel |  |
| Concrete |  |
|  |  |



Table 4 results in $10.8 \mathrm{ft} \leq \mathrm{d} \leq 12.3 \mathrm{ft}$. We suggest to regard

$$
\begin{equation*}
\mathrm{d}=12 \mathrm{ft} \tag{49}
\end{equation*}
$$

as the best choice, yielding for the longest wavelength at the Cassegrain focus

$$
\begin{equation*}
\lambda_{\mathrm{m}}=3.8 \mathrm{~cm} \tag{50}
\end{equation*}
$$

For other choices, Table 5 gives several quantities for comparison. The first side lobe level $\gamma_{0}$ (feed on axis) is taken from Baars (1964), and $\gamma_{c}$ is a combination of $\gamma_{0}$ and the coma lobe $\gamma$ from (34) and (35), at the corner of the feed cluster of $\mathrm{c}=9 \mathrm{ft}$. The lowest rotational mode of the secondary mirror, $\nu_{t}$, is found from (45).

The upper limit of $d$ is not so much given by $\nu_{t}$, which varies only very slowly; nor is it given in a strict way by the side lobe level $\gamma$, since the number of feeds actually used will probably be much less than $n_{m}$, which decreases the cluster size and thus the coma lobe. At it seems, the upper limit of $d$ is mostly given by the demand for having the secondary mirror just as small and as light-weight as possible, for easier handling and servo-control. With respect to all quantities, $\mathrm{d}=12 \mathrm{ft}$ seems to be a good choice. But this question is still open for discussion and will be decided later.

## References

J. W. Baars, 1964: Internal NRAO-Report.
J. W. Baars, 1966: Internal NRAO-Report.
A. P. King, 1950: Proc. IRE, p. 249.
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W. D. White and L. K. DeSize, 1962: IRE International Convention Record, Vol. X, 1, p. 44.


Fig. 2. Cassegrain system,
definitions and geometry,


Fig. 3. Geometry and definitions for:
a) single feed horn, side view;
b) multiple feed cluster, top view.


Fig. 4. Limit for scan angle $\theta$, from spillover.
a) Definition of the critical scan angle $\theta_{c r}$ : all incoming rays, after reflection at primary, miss secondary mirror.
b) One-sided spillover for $\theta \ll \theta_{c r}$. (The spillover is $100 \%$ for $\theta=\theta_{c r}$. )


Fig, 5. Limiting feed displacement, $r_{0}=0.10 r_{c r}$, as a function of the magnification $M$; for $F / D=0.427$.

Spring-Support for the 140-ft Panels<br><br>S. von Hoerner

## 1. Principle

When tilted from zenith to horizon, the $140-\mathrm{ft}$ structure deforms by about 10 mm , which increases the rms surface deviation from 1.2 mm to 4.5 mm . A new surface adjustment could decrease the zenith deviation from 1.2 mm to 0.7 mm , and it is at present discussed whether the gravitational sag under tilt should be removed by computercontrolled jacks at the adjustment studs of the surface panels. Fred Crews asked me whether there might be some easy way of making the deformations nearly homologous, by some automatical means. This seems indeed possible.

Most of the gravitational deviations arise from the difference between hard and soft surface points, and a good approximation to homology is an equal-softness structure, where each point sags by the same amount. Since one cannot make the soft points hard, one must make the hard points soft, which can be done by springs under the panels, using the panel weight as the active force.

There are several problems. First, the telescope gets softer for wind deformations, and it must be checked whether this can be tolerated. Second, the springs must give enough movement from the panel weight, but should not be over-stressed from survival loads. Third, demanding exact homology for each support point for zenith and horizon imposes two conditions per point, while a spring yields only one degree of freedom (its stiffness); thus, exact homology cannot be achieved in the general case. It can be achieved, if the deformation component perpendicular to the surface, of each support point, is in proportion to $\cos \theta$, where $\theta$ is the angle between the vertical and the surface normal at this point (because that is exactly what a spring-supported panel will do). This condition will not be fulfilled exactly, but it should hold in a good approximation, since the back-up structure, too, is something like a spring. Thus, the spring-supported panels should give a good improvement. It can be calculated from the available deformation data.

## 2. Method Suggested

The support should be flexible perpendicular to the surface, but very stiff against tangential movements and all rotations. It might be done with the stud moving up and down in some gliding bearings, but this would certainly give serious problems in horizon position, from friction and stiction. One should look for some frictionless mount, like a leaf-spring:

This arrangement has the following advantages:

1. flexible up and down;
2. very stiff otherwise;
3. no friction or stiction.


## 3. Wind Deformation

We use 20 mph of wind, giving $1.03 \mathrm{lb} / \mathrm{ft}^{2}$ pressure. This is to be compared with the panel weight of about $10 \mathrm{lb} / \mathrm{ft}^{2}$ which must yield $4.5_{\Lambda}^{\mathrm{mm}} \mathrm{rms}$ movement on the springs, when the telescope is tilted. Thus the rms wind deformation is $1 / 10$ of 4.5 mm . or 0.45 mm . This increases the (new adjusted) surface deviation from 0.70 to 0.83 mm , which can be tolerated.

The same calculation would apply to any other telescope. Call $w=$ weight/area of the panels, $p=$ wind pressure, $s=$ gravitational sag of the telescope to be corrected by the springs, and $\delta=$ desired rms surface accuracy. The demand then is
$s \mathrm{p} / \mathrm{w} \ll \delta$.
This is fulfilled for the $140-\mathrm{ft}$. But not for the $300-\mathrm{ft}$ ( $w$ too small), noc for our new $65-m$ design ( $\delta$ too small). In general, spring-supported panels will work for any telescope with large, heavy panels and for a moderate accuracy, yielding improvement factors of up to 10 but not much higher.

## 4. Plate Thickness from Survival Stress

We use the formulas for a circular flange, where the outer edge is fixed and supported, and the inner edge is fixed; with a uniform load along the inner edge.


The maximum stress at the extreme fiber, occurring at the inner edge, then is

$$
\begin{equation*}
S=\frac{3 \mathrm{P}}{2 \pi \mathrm{~h}^{2}} \mathrm{~K}_{1} \tag{2}
\end{equation*}
$$

with

$$
\begin{equation*}
K_{1}=1-\frac{2 \ln \alpha}{\alpha^{2}-1} \quad \text { and } \quad \alpha=D / d \tag{3}
\end{equation*}
$$

| $\alpha$ | $K_{1}$ | $K_{2}$ |
| :---: | :---: | :---: |
| 10 | .956 | .778 |
| 15 | .976 | .861 |
| 20 | .985 | .907 |

Some values of the correction term $K$, are given in the table. We see $\alpha$ does not make much diffference as long as $\alpha \gg 1$. For the following estimates we adopt

$$
\begin{equation*}
\alpha=15 \tag{4}
\end{equation*}
$$

The stress then is

$$
\begin{equation*}
S=.466 \mathrm{P} / \mathrm{h}^{2} . \tag{5}
\end{equation*}
$$

For the survival load we use $20 \mathrm{lb} / \mathrm{ft}^{2}$, and for a panel size of $12 \mathrm{x} 20=240 \mathrm{ft}^{2}$, the load is 4800 lb . We add the panel weight of 2500 lb , and we divide by 2 since the load goes on two plates, yielding a survival load $P_{s}$ on each plate of

$$
\begin{equation*}
P_{s}=3650 \mathrm{lb} . \tag{6}
\end{equation*}
$$

The thickness $h$ then is given by

$$
\begin{equation*}
\mathrm{h} \geq .683 \sqrt{\mathrm{P}_{\mathrm{s}} / \mathrm{S}_{\mathrm{m}}} \tag{7}
\end{equation*}
$$

where $S_{m}$ is the maximum allowed stress of the steel used. Since it turns out that the diameter $D$ would get inconveniently large for normal steel, we choose high-stress
steel with, say, $S_{m}=75000 \mathrm{lb} /$ inch $^{2}$. The minimum plate thickness, for the softest springs, then is

$$
\begin{equation*}
h=0.150 \text { inch } \tag{8}
\end{equation*}
$$

## 5. Cylinder Diameter from Deformation

The central deformation of the circular flange is
with

$$
\begin{align*}
z & =\frac{3}{16 \pi}\left(1-v^{2}\right) \frac{P D^{2}}{E h^{3}} K_{2}  \tag{9}\\
K_{2} & =1-\alpha^{-2}-\frac{4(\ln \alpha)^{2}}{\alpha^{2}-1} \quad \alpha=D / d \tag{10}
\end{align*}
$$

and with $v=.25=$ Poisson's ratio. Using again $\alpha=15$, and $E=3 \times 10^{7} \mathrm{Ib} /$ inch $^{2}$, we get

$$
\begin{equation*}
z=0.0500 \frac{P D^{2}}{E h^{3}} \tag{11}
\end{equation*}
$$

In order to have a deformation $z$, the cylinder diameter then must be

$$
\begin{equation*}
D=4.47 \sqrt{E z h^{3} / \mathrm{P}} \tag{12}
\end{equation*}
$$

The load $P$ on each plate is $P=12501 b=1 / 2$ the panel weight, and the softest spring must move by $z=10 \mathrm{~mm}=.394$ inch. With h from (8) we obtain

$$
\begin{equation*}
D=25.3 \text { inch }=2.11 \mathrm{ft} . \tag{13}
\end{equation*}
$$

In general, if the loads are given, $D \sim z^{1 / 2} S_{m}^{-3 / 4}$. The diameter would decrease if still better steel would be used for the plates, which is possible. Also, we have used for 2 the maximum deformation, which would be decreased by using the maximum deviation from a proper best-fit paraboloid. But even a diameter of two feet is not excessively large, and this method thus looks possible.

## 6. Various Stiffness

For the softest spring, $h$ is given by (8) and D by (13). Any higher stiffness can be achieved by either increasing $h$, or decreasing $D$, or both; whatever is more convenient for the manufacturing.

If there is an actual interest in this method, I would volunteer to derive the formulas, for obtaining the needed spring stiffness at each panel support from the measured or calculated deformation data.

