Report 32
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# Tower Data, and Dynamics of Combined System

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## Summary

Coordinates and bar areas for the azimuth towers of the 65-m telescope are given. All bars are single pipes. The weight of the towers is 318 tons, the equipment has 270 tons (trucks, bearings, drives), with a total to 588 tons. From 6 load conditions a computer analysis derives stiffnesses, maximum stresses and reactions.

Dynamical data of the dish structure are given, and the combined stiffness of tower, dish and equipment is calculated. The lowest dynamical frequencies of the combined system are calculated for translations and rotations. The lowest value is $\nu$ = 1.87 cps for elevation rotation.

Movements of the reference platform are given. Large translations (defining the size of the mirrors) result from gravitation, wind and temperature; they are all $\leq \pm$ 1.10 inch. Angular perturbations at the platform (to be corrected by the servo system) are 22 arc sec for longer periods from average wind; the gusts give largest perturbations of 6 arc sec for durations of 8 sec, with 18 mph average wind. For 30 mph the maximum of 20 arc sec occurs at durations of 4 sec. The horizontal accelerations at the platform were estimated; the maximum occurs at .3 sec duration and ranges between 0.1 and 1.0 cm/sec$^2$ for v between 18 and 30 mph.

## 1) Tower Design Data

The tower for the 65-m telescope is shown in Figures 1, 2 and 3. Member 1-6 points tangentially at the elevation wheel (for maximum dynamical stiffness), and point 6 is 210 inch away from the point of contact (size of gear box and link). The telescope then still can look about 35° beyond zenith. The tower coordinates are given in Table 1. Points 1, 2, 3, 7, 8 are 96 inch above ground (height of trucks and rails).

The bar areas are shown in Table 2; all bars are single pipes welded from sheets. The bars going to points 4, 5, 6, 9 and 10 are defined by dynamical stiffness and are over-designed regarding survival stress and even wind deformations. The bars parallel to the ground are defined by buckling criteria and lateral vibrations. The long bars 2-3 and 3-8 need a diameter of 130 inch. The rim of the dish has 7 inch clearance from bar 3-8, and 18 inch clearance above ground, if going to horizon or service position (the height of trucks and rails is 96 inch).

Two things are left open at present. First, how to balance the weight of the elevation drive (46 tons) with a spring or a counterweight. Second, how to brace point 11, needed for the cable wind-up. (Cryogenic equipment should rotate with the tower).

Table 1. Tower 4; Coordinates (inch)

Center angle = 74°

| point | x | y | z |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | -888 | 1179 | 0 |
| 3 | 888 | 1179 | 0 |
| 4 | 0 | 800 | 1200 |
| 5 | 0 | 580 | 620 |
| 6 | -460 | 0 | 515 |
| 7 | -888 | -1179 | 0 |
| 8 | 888 | -1179 | 0 |
| 9 | 0 | - 800 | 1200 |
| 10 | 0 | - 580 | 620 |
| (11 | 0 | 0 | 350) |

Table 2. Tower 4; Bar Areas ($inch^2$)

center angle 74°

| bar | points | A |
|---|---|---|
| 1 | 1-2 | 40 |
| 2 | 1-3 | 40 |
| 3 | 1-5 | 95 |
| 4 | 1-6 | 156 |
| 5 | 1-7 | 40 |
| 6 | 1-8 | 40 |
| 7 | 1-10 | 95 |
| 8 | 2-3 | 40 |
| 9 | 2-4 | 127 |
| 10 | 2-5 | 35 |
| 11 | 2-7 | 102 |
| 12 | 3-4 | 127 |
| 13 | 3-5 | 35 |
| 14 | 3-8 | 102 |
| 15 | 4-5 | 95 |
| 16 | 5-6 | 20 |
| 17 | 6-10 | 20 |
| 18 | 7-8 | 40 |
| 19 | 7-9 | 127 |
| 20 | 7-10 | 35 |
| 21 | 8-9 | 127 |
| 22 | 8-10 | 35 |
| 23 | 9-10 | 95 |

The weight is given in Table 3. All equipment at present is taken unchanged from the old 300-ft. design and is thus rather overdesigned (by maybe 90 tons). The weight of the towers could be reduced by about 80 tons if a lower dynamical stiffness were chosen (over-all frequency 1.4 - 1.5 cps).

Table 3. Weight of Towers and Equipment

| Items | | US tons | |
|---|---|---|---|
| 2 | towers and azumuth structure | 318 | |
| 4 | trucks + azimuth drives (pts. 2,3,7,8) | 180 | |
| 2 | elevation bearings (points 4,9) | 40 | 270 |
| 1 | elevation drive + link (point 6) | 46 | |
| 1 | pintle bearing (point 1) | 4 | |
| | Total weight | 588 | |

Some of the tower weight should be added to the weight of the dish for the dynamical estimates. This is 1/2 of all bars meeting in points 4 and 9 (64 tons), the elevation bearings (40 tons), and the elevation drive (46 tons), totaling 150 tons. This holds regarding parallel translations. As for rotations, we mostly have only two of the three items actually contributing, and thus use 100 tons:

$$\text{add to dish} \begin{cases} \text{translations} & 150 \text{ tons} \\ \text{rotations} & 100 \text{ tons} \end{cases} \tag{1}$$

2) Tower Analysis

a) Loads and Constraints

The towers were analyzed with the STRUDL program, adopting pin joints, and calculating deformations, reactions and stresses.

We use two different constraint conditions. In $C_1$ the azimuth trucks are fixed along the rails (blocked drive) but free perpendicular to the rails (wheel tilt), to be used for calculating the dynamical stiffness. In $C_2$ the trucks are free in both directions, to be used for calculating the maximum stress; see Table 4.

Table 4. Constraint Conditions

| | Point 1 | | | Points 2, 3, 7, 8 | | | used for |
|---|---|---|---|---|---|---|---|
| | x | y | z | along rail | perpend. rail | z | |
| $C_1$ | con. | con. | con. | con. | free | con. | stiffness |
| $C_2$ | con. | con. | con. | free | free | con. | stress |

Table 5 lists six different load conditions. The first three are used for calculating the stiffness of the tower tops in all three directions, with an arbitrary load. Load conditions $L_4$ and $L_5$ combine force F from a wind of 85 mph with the dead load W of dish and towers. The full side area of the complete dish is A = 15,000 ft$^2$, if we consider all open spaces between members as being closed (projected area of telescope envelope). The pressure is 18.8 lb/ft$^2$ for 85 mph. The wind force on the dish (in stow position) then is F = 183 tons, to which we add 1/3 for representing the wind force on the towers, which certainly is overestimated, resulting in

$$F = 250 \text{ tons.} \tag{2}$$

The weight of the dish we use as 650 tons; adding 40 tons for the bearings gives

$$W = 690 \text{ tons + dead load tower.} \tag{3}$$

The last one, $L_6$, combines (3) with an ice or snow load on the dish, of 20 lb/ft$^2$ (4 inch of solid ice, or 3 ft. dry snow), resulting in a load of

$$I = 356 \text{ tons.} \tag{4}$$

Table 5. Load Conditions

| | Points 4 and 9 | | | Constraints | Used for |
|---|---|---|---|---|---|
| | x | y | z | | |
| $L_1$ | F | | | $C_1$ ($C_2$) | stiffness |
| $L_2$ | | F | | | |
| $L_3$ | | | F | | |
| $L_4$ | F | | W | $C_2$ | stress |
| $L_5$ | | F | W | | |
| $L_6$ | | | W,I | | |

The stiffness of point 6 at the elevation drive is calculated directly from bar area A and length L as

$$K_6 = \frac{EA}{L} . \tag{5}$$

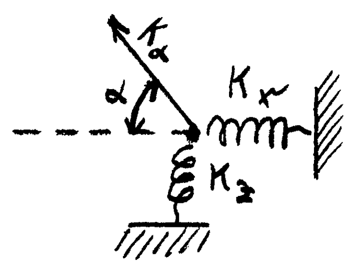
b) Stiffness

The resulting stiffnesses are given in Table 7, first for the tower only as described up to now. In addition, the flexibility of the equipment must be added; the data given by O. Heine (Memo of Feb. 7, 1970) are shown in Table 6. "Combined" means that the stiffnesses of truck, drive and ground are added reciprocally, and the result then is multiplied by 4 for the four points contributing. The numbers in Table 6 thus refer to the total structure, not to the single points.

Table 6. Stiffness of Equipment

| Points | K($10^6$ lb/inch) x | y | z | Remarks | |
|---|---|---|---|---|---|
| 1 | 20 | 20 | 10 | 1 | pintle bearing + ground |
| 4,9 | 20 | 20 | 20 | 2 | elevation bearings, combined |
| 2,3,7,8 | along rail | 4 | 40 | 4 | trucks (drives, ground) combined |
| 6 | axial | 5 | | 1 | elevation drive (box, link) |

Next, we consider that various equipment contributions add up at several points. First, $K_x$ and $K_z$ of the pintle bearing result in an axial stiffness along direction 1-6 of




$$K_\alpha = \frac{K_x}{1 + (\frac{K_x}{K_z} - 1) \sin^2\alpha}$$

$$= 12.85 \times 10^6 \text{ lb/inch.} \tag{6}$$

which adds up reciprocally with the elevation drive stiffness $K_e = 5 \times 10^6$ lb/inch from Table 6 to

$$K_{e6} = (K_\alpha^{-1} + K_e^{-1})^{-1} = 3.60 \times 10^6 \text{ lb/inch.} \tag{7}$$

Second, the tower tops are influenced from trucks and pintle, which can be estimated from the geometry and Table 6. Since the results are not critical, we omit the derivation and just enter the results in Table 7. The last column of Table 7 finally gives the combined stiffness of tower and equipment.

Table 7. Stiffness of Towers and Equipment ($10^6$ lb./inch)

| | | Towers only | | equipment | towers + equipment combined; $C_1$ |
|---|---|---|---|---|---|
| | | $C_1$ | $C_2$ | | |
| Pts. 4,9 combined | $K_x$ | 2.46 | .784 | 5.35 | 1.685 |
| | $K_y$ | .864 | .852 | 20 | .828 |
| | $K_z$ | 6.22 | 5.34 | 30 | 5.15 |
| Pt. 6, axial | $K_6$ | 6.55 | 6.55 | 3.60 | 2.32 |

c) Maximum Stress

The stresses $S_{42}$ and $S_{52}$ from load conditions $L_4$, $L_5$ and $L_6$ with constraints $C_2$, are given in Table 8.

Table 8. Stresses (ksi). $S_m$ is the maximum stress

| bar | points | $S_{42}$ | $S_{52}$ | $S_{62}$ | $S_r$ | $S_m$ |
|---|---|---|---|---|---|---|
| 1 | 1-2 | 7.36 | 2.35 | 3.08 | .15 | 7.51 |
| 2 | 1-3 | 3.33 | 2.05 | 2.78 | .15 | 3.48 |
| 3 | 1-5 | 3.83 | .16 | 5.48 | 2.82 | 8.30 |
| 4 | 1-6 | .14 | .14 | .14 | 0 | .14 |
| 5 | 1-7 | 7.36 | 1.98 | 3.08 | .15 | 7.51 |
| 6 | 1-8 | 3.33 | 1.68 | 2.78 | .15 | 3.48 |
| 7 | 1-10 | 3.83 | 7.50 | 5.48 | 2.82 | 10.32 |
| 8 | 2-3 | 4.62 | 5.95 | 6.71 | 1.00 | 7.71 |
| 9 | 2-4 | 3.79 | 3.90 | 3.06 | 1.40 | 5.30 |
| 10 | 2-5 | 3.37 | .37 | 4.72 | 2.30 | 7.02 |
| 11 | 2-7 | .59 | .52 | .76 | 0 | .76 |
| 12 | 3-4 | .37 | 3.90 | 3.06 | 1.40 | 5.30 |
| 13 | 3.5 | 3.08 | .08 | 4.43 | 2.30 | 6.73 |
| 14 | 3.8 | 1.67 | .56 | .80 | 0 | 1.67 |
| 15 | 4-5 | 3.86 | .19 | 5.68 | 3.11 | 8.79 |
| 16 | 5-6 | .59 | .59 | .59 | 0 | .59 |
| 17 | 6-10 | .59 | .59 | .59 | 0 | .59 |
| 18 | 7-8 | 4.62 | 3.29 | 6.71 | 1.00 | 7.71 |
| 19 | 7-9 | 3.79 | .26 | 3.06 | 1.40 | 5.19 |
| 20 | 7-10 | 3.37 | 6.37 | 4.72 | 2.30 | 8.67 |
| 21 | 8-9 | .37 | .26 | 3.06 | 1.40 | 4.46 |
| 22 | 8-10 | 3.08 | 6.08 | 4.43 | 2.30 | 8.38 |
| 23 | 9-10 | 3.86 | 7.91 | 4.68 | 3.11 | 11.02 |

Next, we ask for the stress resulting from rail deviations $\Delta z_r$ (settlement of ground), for which we use:

$$\Delta z_r = \pm 0.5 \text{ inch max.} \tag{8}$$

If all tower legs (pts. 2,3,7,8) go down by $\Delta z_r$, points 4 and 9 move inward by

$$\Delta y_4 = \frac{z_4}{y_2} \Delta z_r = 1.018 \, \Delta z_r = .509 \text{ inch} \tag{9}$$

The stiffness $K_y$ at <u>one</u> tower top, combined with equipment, is for $C_2$

$$K_{y2} = .377 \times 10^6 \text{ lb/inch.} \tag{10}$$

If the dish were infinitely stiff, movement (9) with stiffness (10) results in a force

$$F_r = \Delta y_4 \, K_{y2} = 192 \text{ kip.} \tag{11}$$

We call $S_{22}$ the stress from $L_2$ with $C_2$ from the STRUDL output, obtained from $F_y$ = 250 kip at <u>one</u> tower top, and we obtain the stress from rail deformation, $S_r$, as

$$S_r = \frac{192}{250} S_{22} = .768 \, S_{22}, \tag{12}$$

which is shown in Table 8. Finally we call

$$S_m = S_\gamma + \text{Max } (S_{42}, S_{52}, S_{62}) \tag{13}$$

the maximum stress in each member, which also is given in Table 8. This includes: weight of dish, dead loads of tower, wind of 85 mph in stow position, and 0.5 inch rail deviation. All values of $S_m$ are rather moderate, meaning that the towers are overdesigned regarding survival.

d) <u>Reactions at Supports</u>

The STRUDL output also gives the reactions, meaning the forces acting on bearings and trucks. Table 9 gives the z reactions at all supports from dead load $L_3$, and dead load plus snow $L_6$.

Table 9. Vertical Reactions at all Tower Supports, from Dead Loads, $L_3$, and Dead Loads plus 20 $lb/ft^2$ of snow or ice $L_6$.

| | Reaction (kip) | |
|---|---|---|
| point | $L_3$ | $L_6$ |
| 1 | 619 | 848 |
| 2 | 352 | 473 |
| 3 | 347 | 467 |
| 7 | 352 | 473 |
| 8 | 347 | 467 |

3) Dish Data

The present dish structure is "cleaned up" geometrically: no two bars cross or touch each other, no two bars meet at a common joint with less than 15° in between, space and stiffness is provided for a vertex cabin of 10 x 10 x 12 feet and its surrounding structure, and the elevation wheel is as large as possible. Furthermore, all members are single pipes only (no built-up members); except for the feed legs and the suspension bars which need special designs. The external geometry is final: location of bearings, wheel, surface, and prime focus. Also, bar areas of suspension and back-up cone will not be changed. The weight of this present structure (EE 401, i = 2) is 610 tons; in addition we have 56 tons for surface plates, 14 tons for equipment and cabins at prime and Cassegrain focus together, and a counterweight of 45 tons; adding up to 725 tons total.

The next step is optimizing the weight while keeping the stiffness constant. From some tries and estimates it seems that the total weight may be reduced to a goal of about 650 tons, and certainly to 680 tons or smaller. Similar estimates were made regarding inertia and stiffness: for every number, a design goal is established which might be reached, and a limit is estimated which certainly can be reached. Only the latter values are used in the following. For the total dish (except bearings and elevation drive) we use:

$$W_o = 680 \text{ tons} = 1.36 \times 10^6 \text{ lb.} \tag{14}$$

a) Total Weight

For dynamics, we use according to (1), with tower tops, bearings and drive:

$$\text{Translations} \quad W_t = 830 \text{ tons} = 1.66 \times 10^6 \text{ lb.} \tag{15}$$

$$\text{Rotations} \quad W_r = 780 \text{ tons} = 1.56 \times 10^6 \text{ lb.} \tag{16}$$

b) Dimensions, Inertia

| | | | | |
|---|---|---|---|---|
| 1/2 Distance between bearings | | $r_b$ | = 800 inch | (17) |
| Radius of elevation wheel | | $r_w$ | = 781 inch | (18) |
| Radius of gyration | x - axis | $r_x$ | = 660 inch | (19) |
| | y - axis | $r_y$ | = 615 inch | (20) |
| | z - axis | $r_z$ | = 665 inch | (21) |

c) Stiffness

| | | |
|---|---|---|
| at both bearings, combined | $K_x = 3.02 \times 10^6$ lb/inch | (22) |
| | $K_y = 2.67$ " " | (23) |
| | $K_z = 2.52$ " " | (24) |
| at softest point of elevation wheel, tangentially, | $K_6 = .909$ " " | (25) |

The stiffness at the bearings is derived as follows. Let the whole telescope sag under dead loads if held with gliding cylinder bearings, and call s the average sag of all surface points. Then $K = W_o/s$. The stiffness of the wheel was investigated by a STRUDL analysis, the smallest value $K_o$ is obtained when held at pt. 43 (upper end of wheel), which means a telescope looking 41° beyond zenith. With respect to rotation, we get $K_w = (r_m/r_w)^2 K_o = .489 K_o$ as the effective stiffness of the wheel, where $r_m = 546$ inch is the closest distance of line 43-58 (the wheel chord) from the y - axis. Similarly, the stiffness $K_c$ of the four cone members 45-58 regarding rotation is calculated, and both are combined as

$$K_{wc} = [K_w^{-1} + (K_c + K_w)^{-1}]^{-1}. \tag{26}$$

Finally, the contribution of the structural layers between the octagon (upper end of cone) and the surface was obtained from dead-load deformations, resulting in

$$K_6 = .80\ K_{wc}. \tag{27}$$

By this procedure, $K_o = 3.67 \times 10^6$ lb/inch is reduced to $K_6 = .909 \times 10^6$ lb/inch.

4) Dynamics of Combined System

The following is an estimate of the lowest dynamical frequencies of the combined system of dish, towers and equipment. The results are supposed to be correct within 6%. A computer analysis is being prepared by O. Heine using a proper combination of dish, tower and equipment, but still using the estimated dish values of this report. Later on, a rigorous computer analysis of the dish dynamics is to be made by Simpson, Gumpertz and Heger, and O. Heine's calculations will be repeated using these dish values, and including lateral vibrations of single members.

a) Parallel Translations

We consider the elevation drive as being disconnected such that translations in x and z do not couple with elevation rotation. We add reciprocally the tower and equipment stiffness from Table 7, and the dish stiffness from (22) - (24), for obtaining the combined stiffness K. For a one-mass one-spring system, we have

$$\nu = 3.127\sqrt{K/W} \quad \begin{cases} \nu \text{ in cps} \\ K \text{ in lb/inch} \\ W \text{ in lb.} \end{cases} \tag{28}$$

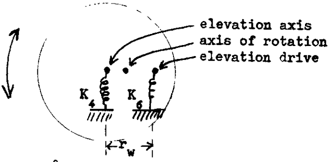
and with $W_t = 1.66 \times 10^6$ lb. from (15) we obtain the values given in Table 10. Going from zenith to horizon, the tower stiffnesses do not change, but $K_x$ and

$K_z$ of the dish are exchanged. Directions x, y, z of Table 10 always refer to the tower.

b) Rotations

The elevation drive now is connected (which matters for y rotation but not for x and z). For rotation about the x-axis, we use $K_z$ of the tower; and $K_z$ of the dish if looking at the zenith, but $K_x$ if looking at horizon; similar combinations hold for rotation about the z-axis. We calculate $\nu$ from (28) with $W_r = 1.56 \times 10^6$ lb. from (16), but multiply $\nu$ with $r_b/r_x$ for x-rotation and with $r_b/r_z$ for z-rotation. The results are found in Table 10.

Rotation about the y-axis (elevation axis) cannot be treated independently, since it couples with translations in x and z direction. The dish actually rotates about a free axis parallel to the y-axis, located between y-axis and elevation drive. We use a one-mass, two spring system as sketched. We call




$$\beta = \left(\frac{r_w}{r_y}\right)^2; \quad \beta_o = \frac{\beta}{1+\beta} \tag{29}$$

$$\gamma = K_4/K_6 \tag{30}$$

$$C = \frac{4\beta\gamma}{(1 + \beta + \gamma)^2} \tag{31}$$

and a combined stiffness

$$K = [(\beta K_6)^{-1} + (\beta_o K_4)^{-1}]^{-1} \tag{32}$$

The dynamical frequency then can best be written as

$$\nu = \underbrace{3.127 \sqrt{K/W}}_{\nu_o} \Gamma \tag{33}$$

with an approximation $\nu_o$, and a correction $\Gamma$ given by

$$\Gamma = \sqrt{\frac{1 - \sqrt{1-C}}{C/2}} \tag{34}$$

In this way of writing, $\beta K_6$ in (32) is the "rotational stiffness" regarding rotation about point 4, while rotation about point 6 is described by $\beta_o K_4$. Equation (32) considers both as being in series. The quantity C has its maximum

$$C_{max} = \beta_o \leq 1, \quad \text{for} \quad \gamma = 1+\beta. \tag{35}$$

The correction $\Gamma$ is always $\Gamma \geq 1$ with

$$\Gamma_{max} = \begin{cases} 1.000 & \text{for} \quad \beta \rightarrow 0 \\ 1.082 & '' \quad \beta = 1 \\ \sqrt{2} & '' \quad \beta \rightarrow \infty \end{cases} \tag{36}$$

Table 10. Lowest dynamical frequencies in cycles per second. Including dish, towers, bearings, and drives (but not the vibrations of single members).

| Mode | | Zenith | Horizon | 41° beyond zenith |
|---|---|---|---|---|
| Translations<br>Elev. drive disconnected | x | 2.53 | 2.44 | |
| | y | 1.93 | 1.93 | |
| | z | 3.16 | 3.35 | |
| Rotations<br>Elev. drive connected | x | 3.95 | 4.16 | |
| | y | | | 1.87 |
| | z | 3.14 | 3.05 | |

The results are summarized in Table 10. We see that

$$\text{all} \quad \nu \geq 1.87 \text{ cps.} \tag{37}$$

This result should be somewhat improved after finishing the optimization of the dish structure. On the other side, the vibrations of single members are not yet included, which will result in a somewhat lower frequency.

5) Movements of the Reference Platform

The reference platform of the optical pointing system is located below point 56, such that the mirror block is at the crossing of both telescope axes. The movements of point 56 matter for three reasons: (a) large parallel translations define the size needed for the mirrors; (b) wind gusts give the amount of pointing deviation the servo system must correct for; and (c) the accelerations from wind gusts tell how "shockproof" the whole platform system must work.

a) Large Translations

The single contributions for point 56 are (max.):

| | | |
|---|---|---|
| gravitational sag of dish, | zenith | .63 inch |
| | horizon | .29 inch |
| wind deformation (18 mph) of | dish, x, y | $\pm$.02 " |
| | towers, x | $\pm$.06 " |
| | " , y | $\pm$.04 " |
| thermal def. of tower height (-25 to +40°C) | | $\pm$.47 " |
| rail and ground (after some years) | | $\pm$.50 " |

Added up properly, the maximum movement would be 1.39 inch horizontally if the platform axis is mounted exactly at the actual elevation axis after erection; if mounted at the design elevation axis, the maximum movement is 1.48 inch vertically. The best mount of the platform axis is .30 inch below the actual elevation axis or .33 inch above the design axis (the difference of .63 inch being the sag in zenith position). For this case the maximum movements of the platform are given in Table 11.

Table 11. Maximum Movements (inch) of Platform

| | Zenith | Horizon |
|---|---|---|
| Δx | $\pm$ .76 | +1.09<br>- .42 |
| Δy | $\pm$ .57 | $\pm$ .57 |
| Δz | + .55<br>-1.15 | + .52<br>-1.10 |

b) Angular Perturbations

First, we estimate the rotation in elevation angle caused by a steady wind of $v_o$ = 18 mph, blowing face-on into the dish in horizon position (worst case). With a shape factor $C_s$ = 1.56 the pressure is $p_o$ = 1.29 lb/ft$^2$, and the force is

$$F = 46 \text{ kip}. \tag{38}$$

The rotation then is $\Delta\phi = F/(Kr_w)$. For a truely steady and symmetrical force, we should use the translational stiffness $K_x$, but regarding gusts and movements, we should use the stiffness for elevation rotation, which is 40% smaller and is found from $\nu$ = 1.87 of Table 10, with $\nu = 3.127 \sqrt{K/W}$ and $W = 1.56 \times 10^6$ lb from (16) as

$$K = .558 \times 10^6 \text{ lb/inch}, \tag{39}$$

yielding a rotation of

$$\Delta\phi = 21.5 \, (v/v_o)^2 \text{ arc sec}. \tag{40}$$

Second, we consider turbulent gusts. We call $\tau$ the one-sided duration of a perturbation (1/2 wavelength), and $P(\tau)$ the rms pressure difference between adjacent time averages of duration $\tau$, divided by the average pressure, as given in Report 23 (March 1, 1969) from high-resolution wind measurements at Green Bank. If $p_o$ is the average pressure, the amplitude of the fluctuations then is (1/2) $p_o P(\tau)$.

A gust of duration $\tau$ and speed $v$ has a size $s = v\tau$, and the number of such gusts hitting the dish of diameter D simultaneously is $n = (D/s)^2$, or, with $v_o$ = 18 mph,

$$\sqrt{n} = D/s = \frac{D}{v\tau} = \frac{v_o}{v} \, \frac{8.07 \text{ sec}}{\tau} \, . \tag{41}$$

The amplitude of the pressure difference on the whole dish surface, for dish movements of duration $\tau$, then is

$$a_p = (v/v_o)^2 \, p_o \, Q(\tau) \tag{42}$$

with a pressure fraction $Q(\tau)$ given by

$$Q(\tau) = \begin{cases} 1/2\ P(\tau)/\sqrt{n} = \dfrac{v}{v_o}\ \dfrac{\tau\ P(\tau)}{16.14\ \text{sec}} & \text{for } \tau \leq \tau_g \\ 1/2\ P(\tau) & \text{"}\ \ \tau \geq \tau_g \end{cases} \tag{43}$$

and a limiting duration $\tau_g$ where the gust size equals the dish size

$$\tau_g = (v_o/v)\ 8.07\ \text{sec.} \tag{44}$$

The amplitude of the angular perturbations of duration $\tau$ is with (40)

$$\Delta\phi(\tau) = 21.5\ (v/v_o)^2\ Q(\tau)\ \text{arc sec} \tag{45}$$

and the frequency is $\nu = 1/(2\tau)$. Results are given in Table 12 and Fig. 4.

If the dynamical frequency is 1.8 cps, the servo-bandwidth is about 1/3 of that or 0.6 cps, corresponding to a one-sided duration of

$$\tau_s = .83\ \text{sec.} \tag{46}$$

Table 12. The rms amplitude $\Delta\phi(\tau)$ of angular perturbations of duration $\tau$ at the platform.

Frequency $\nu = 1/(2\tau)$; pressure fraction $Q(\tau)$ defined in (41). For wind speed $v_o$ = 18 mph.

| $\tau$ sec | $\nu$ cyc/min | $Q(\tau)$ | $\Delta\phi(\tau)$ arc sec | Remarks |
|---|---|---|---|---|
| .50 | 60 | .0071 | .15 | |
| .83 | 36 | .0161 | .35 | limit of servo |
| 2.50 | 12 | .0678 | 1.46 | |
| 8.07 | 3.7 | .271 | 5.81 | gust = dish size |
| 30 | 1.0 | .239 | 5.13 | |
| 100 | .3 | .160 | 3.43 | |

c) Accelerations from Wind Gusts

For slow gusts, the amplitude is limited by the stiffness, rms $x = F/K$, and the rms acceleration then is, using $\Delta\phi = F/(Kr_w)$ already calculated,

$$\text{rms } \ddot{x} = (2\pi\nu)^2 F/K = (\pi/\tau)^2 \Delta\phi(\tau)\, r_w, \quad \text{for } \tau \geq \tau_d \tag{47}$$

with $\tau_d$ given by the lowest dynamical frequency $\nu_d$ = 1.8 cps as

$$\tau_d = 1/(2\nu_d) = .28 \text{ sec.} \tag{48}$$

For fast gusts the stiffness does not matter, but the acceleration is limited by the inertia M

$$\text{rms } \ddot{x} = F/M = (\pi/\tau_d)^2 \Delta\phi(\tau)\, r_w, \quad \text{for } \tau \leq \tau_d \tag{49}$$

In numbers

$$\text{rms } \ddot{x} = \begin{cases} \dfrac{.095 \text{ cm}}{\tau^2} \dfrac{\Delta\phi(\tau)}{\text{arc sec}} & \text{for } \tau \geq .28 \text{ sec.} \\ 1.213 \dfrac{\text{cm}}{\text{sec}^2} \dfrac{\Delta\phi(\tau)}{\text{arc sec}} & \text{" } \tau \leq .28 \text{ sec} \end{cases} \tag{50}$$

Results are given in Fig. 5; they represent only a rough order-of-magnitude estimate.

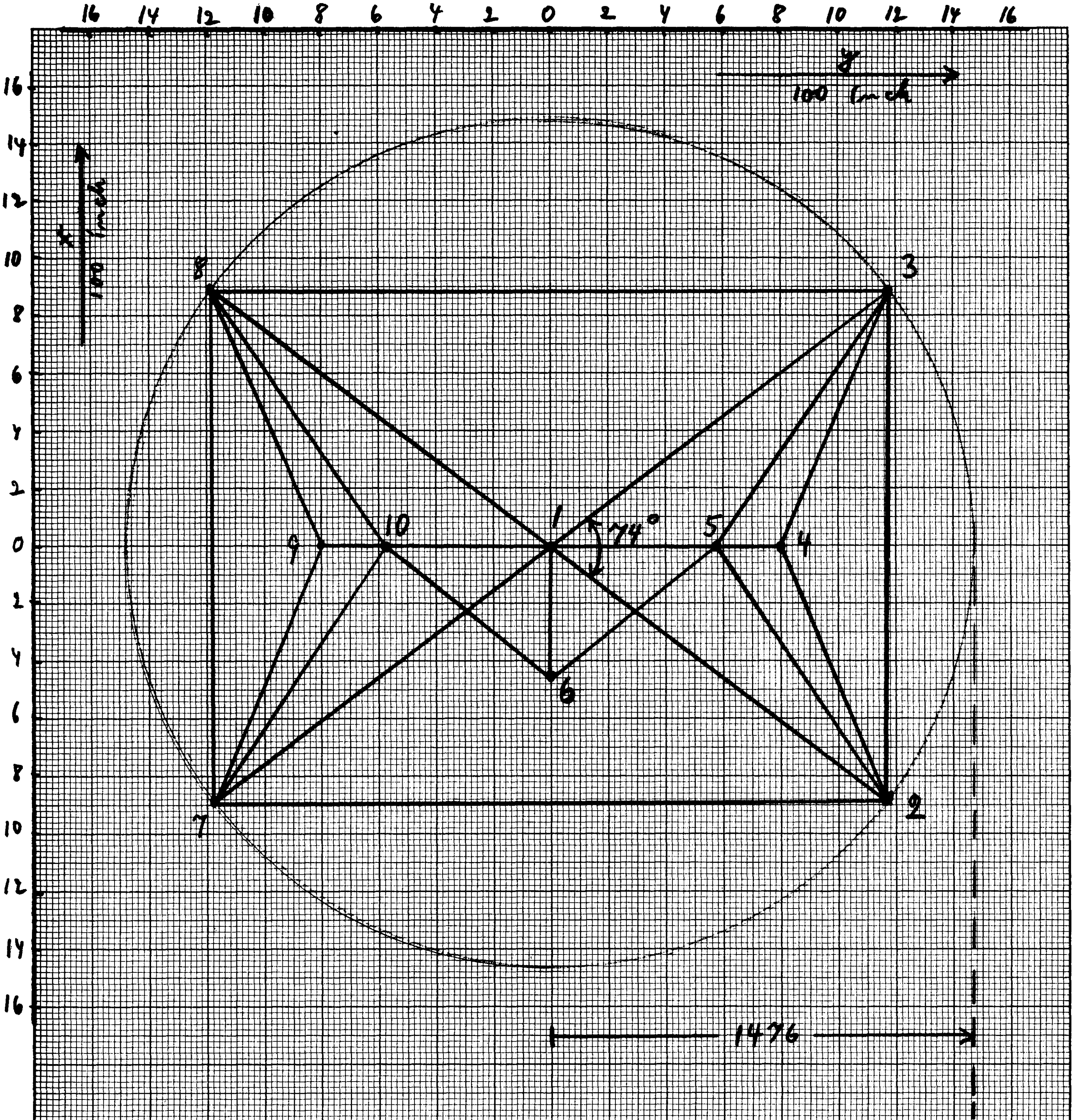



Fig. [illegible]. Tower, plan view

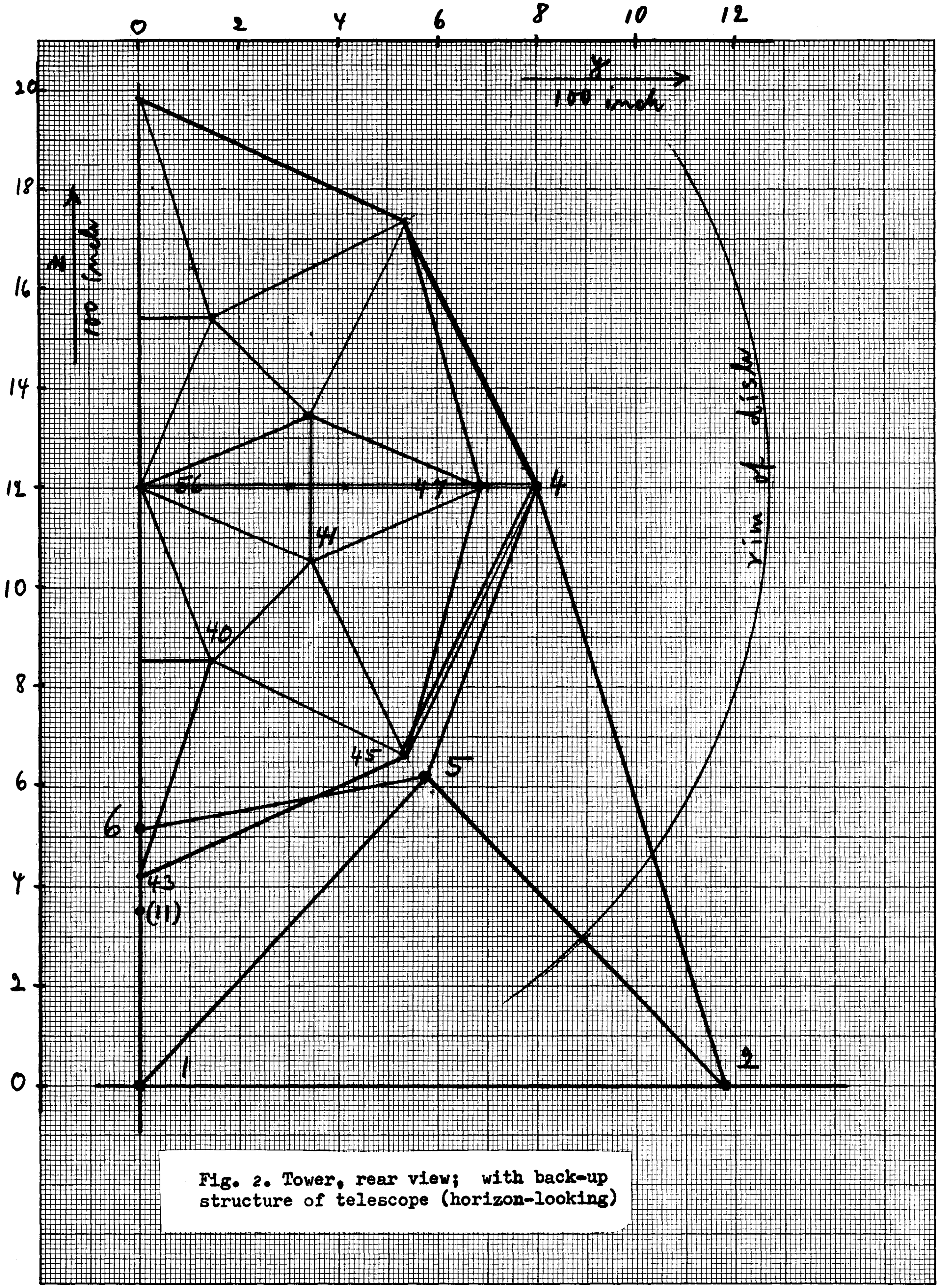



Fig. 2. Tower, rear view; with back-up structure of telescope (horizon-looking)

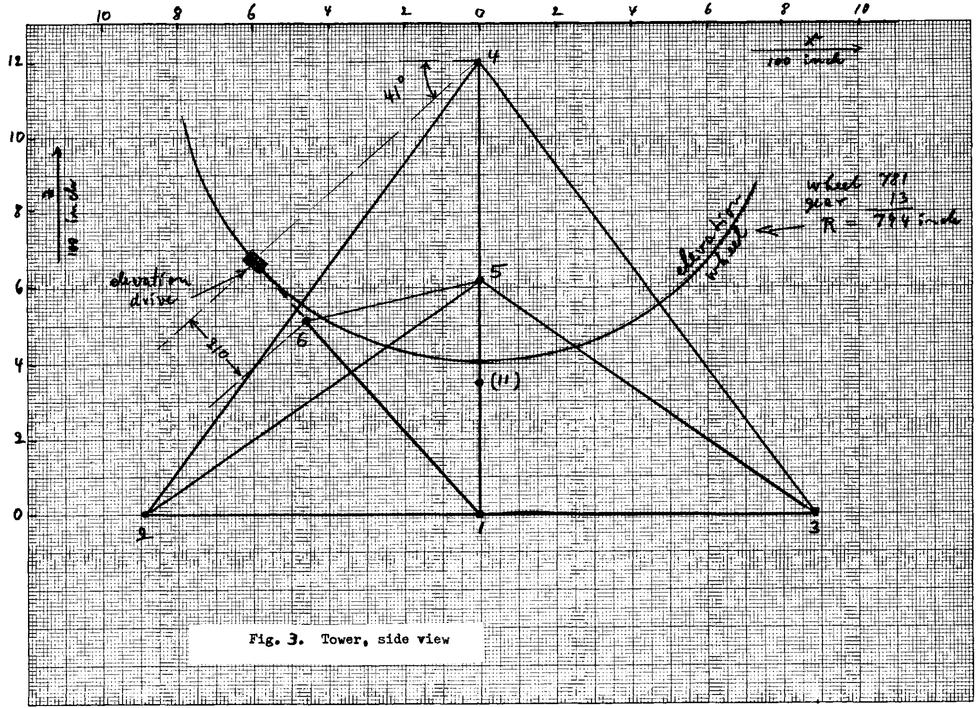



Fig. 3. Tower, side view

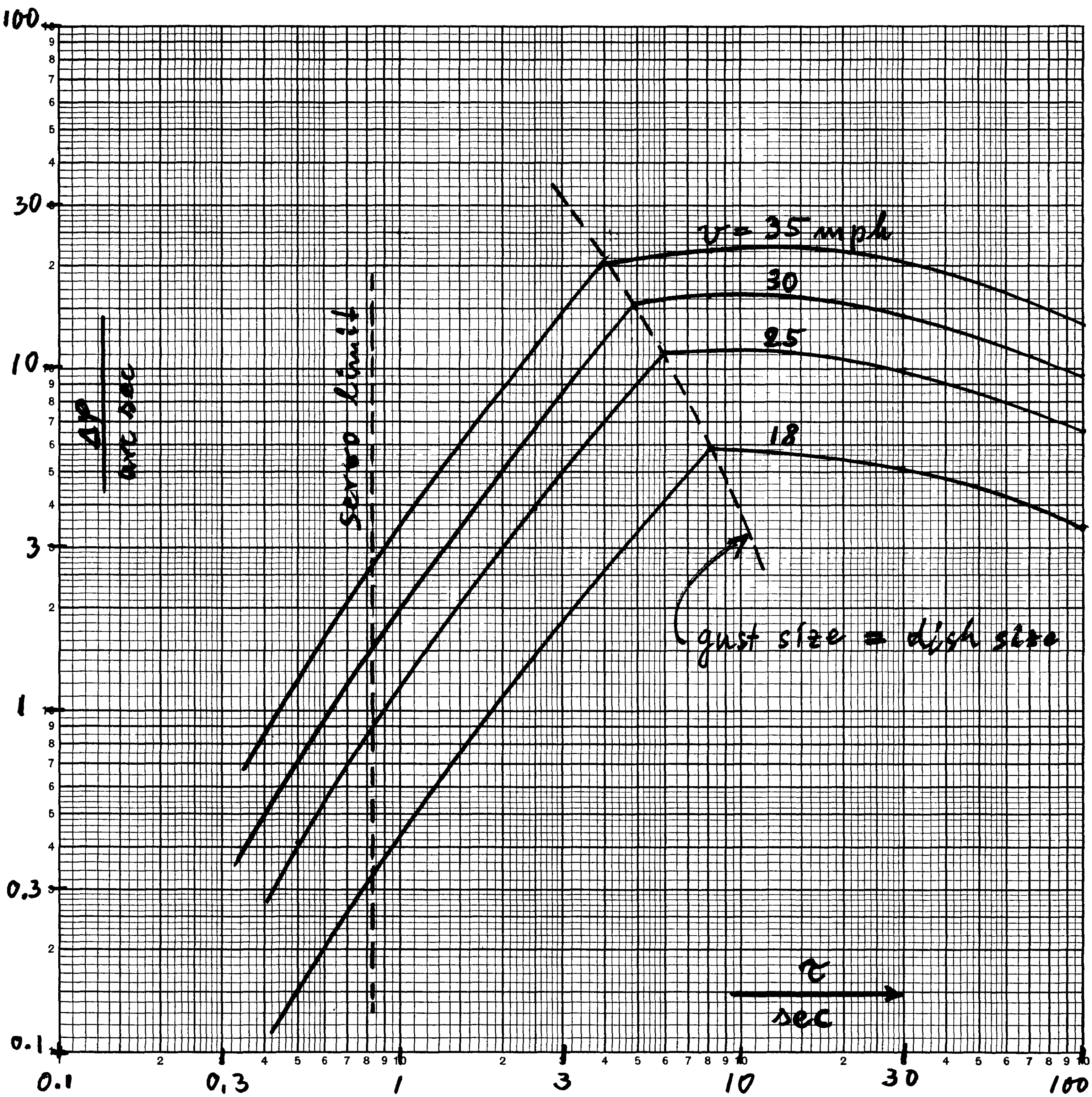



Fig. 4. The rms amplitude Δφ, of angular perturbations at the platform; for perturbations of duration τ, from wind of velocity v.

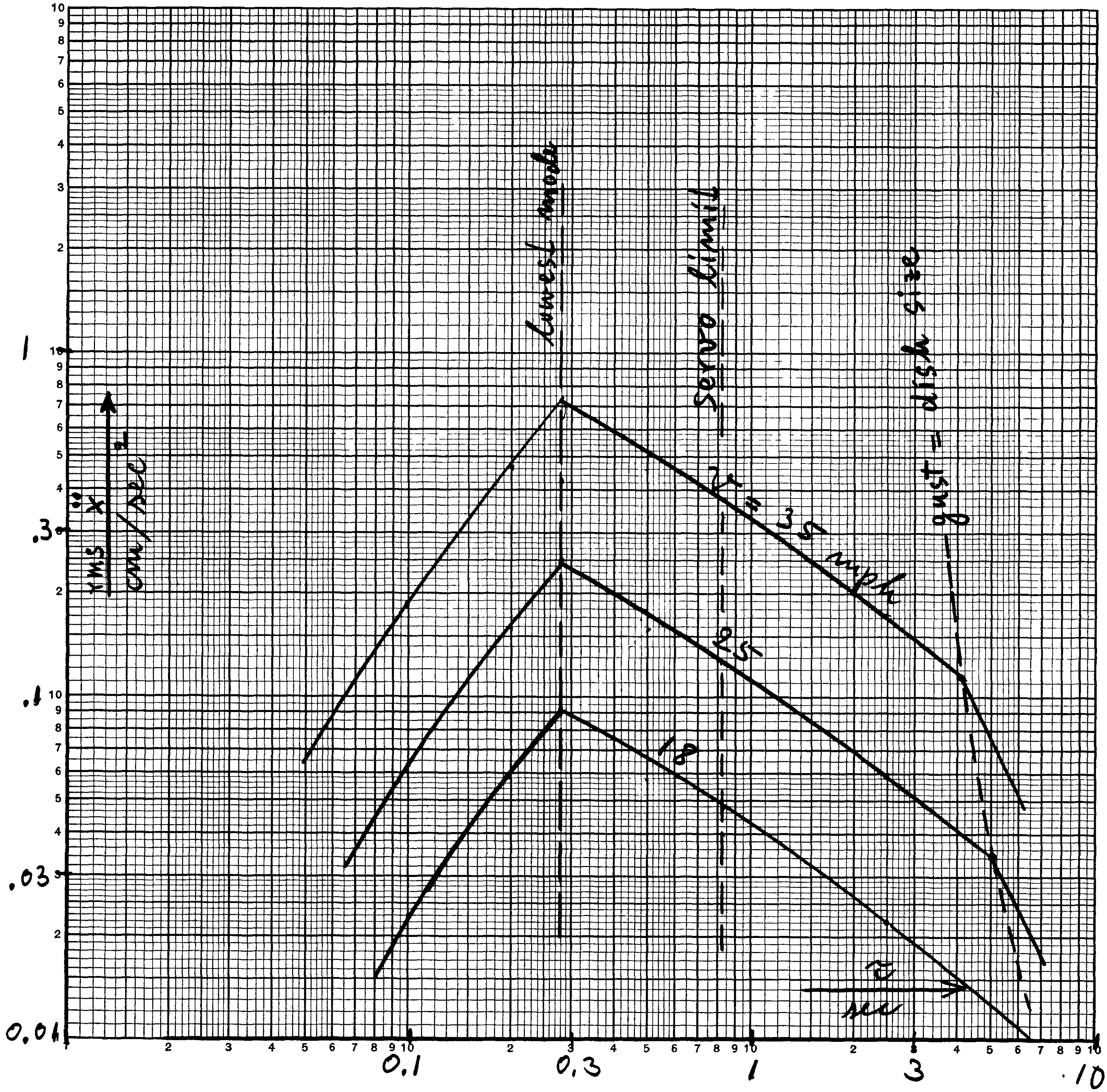



**Fig. 5.** The rms horizontal acceleration at the platform, from wind gusts of duration $\tau$, at average wind velocity v.