Report 33
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Non-Homologous Deformations from Tolerances, Joints and Rails

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Summary

Tolerances and Deviations. The actual telescope will deviate from the telescope as developed by the homology program in several ways: (1) the spherical joints, in the average, reduce the stiffness of all bars by 5% and increase the weight by 3.2%. (2) Stiffness and (3) weight of the individual joints scatter about these averages by about the same amounts. (4) The calculated bar areas are replaced by commercially available ones, 0.8% rms, which have also manufacturing tolerances, 2.2% rms. (5) The accuracy of erecting the telescope will be 0.25 inch rms for the positions of the joints. (6) The azimuth rails will settle and deviate by 0.25 inch rms, yielding a force of 50 tons along the elevation axis.

General Study. Some good original structures are taken, and changes of given amount and random sign are applied to each bar area, or each density, each stiffness, or each coordinate. The changed structure then is analyzed for gravitational deformations, finding the rms deviation ΔH from homology. A total of 16 changed structures, from 3 different original ones, was analyzed. The influences of area, weight and stiffness are very similar and amount to ΔH = .00085 inch per % change. The coordinate change gave ΔH = .0051 inch per inch change. A force along the elevation axis gave ΔH = .000019 inch per ton.

Application of the general study to the expected tolerances and deviations shows that the largest contribution results from the individual scatter of joint stiffness, with ΔH = .0043 inch. The total of all six contributions is ΔH = .0064 inch. These values apply for zenith angles between -30° and +90°. For the range from 0° to +60° the total is ΔH = .0034 inch = .086 mm.

I. Expected Tolerances and Deviations

1. Joints

The Homology Program regards each joint as a pin joint, with each member having its full length from one joint center to the other. Actually, the joints are spherical shells, as suggested, designed and analyzed by O. Heine, and the members get cut-off at both ends at the sphere radius. These spheres have a larger weight but a smaller stiffness than the cut-off parts of the members. The sphere diameters are defined by the angles between, and the diameters of, the members. The wall thickness of the members must fulfill two demands: stability under maximum loads, and a stiffness decrease of not more than 5% in the average for the members. Mostly, the second demand is more crucial.

Calling W the weight and K the stiffness of members, O. Heine's letter of Oct. 21, 1970, gives in the average

$$\overline{\Delta W} = +3.2\ \%, \tag{1}$$

$$\overline{\Delta K} = -5.0\ \%. \tag{2}$$

O. Heine's computer output gives all individual values of W and K; and a check sample shows that the individual deviations from these averages are about the same size as the averages:

$$\mathrm{rms}(\Delta W - \overline{\Delta W}) = 3.2\ \%, \tag{3}$$

$$\mathrm{rms}(\Delta K - \overline{\Delta K}) = 5.0\ \%. \tag{4}$$

2. Bar Areas

All members of the telescope are single pipes (except a few with special design, like feed legs and suspensions, which have almost no influence on homology). The smallest area is $A = 1.5$ inch^2. For $A > 10$ inch^2 (1/4 of all members), we use special order pipes, manufactured from plane sheets by rolling and welding.

In the range $1.5 \leq A \leq 10$ inch^2 (3/4 of all members) we use commercially available pipes which come in three types: (1) plain end pipes, (2) line pipes,

(3) round tubing. We have compiled a Combined Table, ordered with increasing A; it contains a total of 271 pipes within $1.5 \leq A \leq 10^2$inch .

From the Combined Table, a Selected Table is obtained by the following three conditions:

1.	Radius of gyration	$r \leq 3.72\sqrt{A}$,	against local buckling;	(5)
2.	Wall thickness	$t \geq .100$ inch,	for welding;	(6)
3.	Wall thickness	$t \leq .400$ inch,	against thermal lag.	(7)

The Selected Table contains 256 pipes for $1.5 \leq A \leq 10$ inch^2.

The Homology Program delivers all bar areas A needed for homologous deformations, as well as the length L and the maximum stress S_m of each bar. A Replacement Program then replaces each A by the nearest one from the Selected Table, under the following four conditions, actually expressed in terms of A, r and L (with S_o = maximum allowed stress for given L/r ratio, including sag):

4.	Wind-induced vibrations:		$\nu \geq 2.5$ cps;	(8)
5.	Fatigue:	either	$R_e \geq 2 \times 10^5$ (turbulence)	(9)
		and/or	alternate stress ≤ 25 ksi;	(10)
6.	Maximum slenderness:		$L/r \leq 200$;	(11)
7.	Stress ratio:		$S_m/S_o \leq 1$.	(12)

This Replacement Program has been run on several structures, and it gave in the average a change of only

$$\mathrm{rms}(\Delta A) = 0.8\ \%. \qquad (13)$$

In addition to this change from replacement, there is another one from manufacturing tolerances. According to W. Horne and a letter from O. Heine of June 25, 1970, we have the tolerances as given in Table 1. In cases 3 and 5, we will ask the manufacturer to weigh short samples and to deliver only those pipes where $\max(\Delta A) = \pm\ 5\ \%$, which should give only a small cost increase. In the average we adopt

$$\max(\Delta A) = \pm\ 4.8\%. \qquad (14)$$

Table 1. Manufacturing tolerances ΔA of bar areas A.
(d = diameter, t = wall thickness, both in inches)

Case	Type	max(ΔA)	Remark
1	plain end pipe, standard	± 5 %	
2	extra strong	± 5 %	
3	double extra strong	±10 %	specify ± 5 %
4	line pipe	± 5 %	
5	tubing, d = 3 ... 5.5	±10 %	specify ± 5 %
6	d = 5.5 ... 11	± 5 %	
7	special order, t = .25 ... .38	± 4.7 %	
8	t = .38 5	± 4.4 %	
9	t > .5	± 4.0 %	

To find the rms from the maximum, we ought to know the distribution function. A flat distribution would give a factor .577; a parabolic distribution gives .447, and a triangular one gives .408. We adopt rms(ΔA) = .450 max(ΔA) and obtain from (14)

$$\mathrm{rms}(\Delta A) \quad = \quad 2.16\ \%. \tag{15}$$

Including (13), the total of replacement and tolerances then is

$$\mathrm{rms}(\Delta A) \quad = \quad 2.30\ \%. \tag{16}$$

3) Coordinates of Joints

For the erection of the telescope, we will specify an accuracy of $\pm$ 0.5 inch max, for each coordinate of each joint, which can easily be met. Adopting 1/2 of that for the rms deviation, we have

$$\mathrm{rms}(\Delta x,\ y \text{ and } z) \quad = \quad 0.25 \text{ inch}. \tag{17}$$

4) Azimuth Rails and Soil

Because of our special pointing system, we need no high accuracy for rails and foundations. If the deviations get too large by soil settlements after some years of use, the rails will be readjusted. We adopt max(Δz) = 0.5 inch, and 1/2 of that for the rms:

$$\mathrm{rms}(\Delta z) = 0.25 \text{ inch.} \tag{18}$$

These rail deformations given an axial force at the elevation bearings if the average height of the four tower bases is different from the height of the central pintle bearing. Adopting the worst case, that all four tower legs are down by 0.25 inch while the pintle bearing has not moved at all, the geometry of the towers would make the tower tops move outward by Δy = 0.255 inch, if this were not restricted by the dish structure in-between the tower tops. The stiffness in y-direction of one tower top is $K_t = 4.09 \times 10^5$ lb/inch including bearings, trucks and soil; and the stiffness of the dish is $K_d = 13.4 \times 10^5$ lb/inch. The resulting force at the bearings then is

$$F = \Delta y \frac{K_t K_d}{K_t + K_d} = 98{,}300 \text{ lb} = 49.2 \text{ tons.} \tag{19}$$

II. General Study

1) Calculations

In order to investigate the influence of changes in bar area, weight, stiffness and coordinates separately, we have taken three good original structures (small deviations ΔH_o from homology). Using random numbers, changes of given amount but random sign are applied to one of these quantities for all bars (or joints). After the change, the structure is analyzed for gravitational deformations, and the deviation ΔH_c from a best-fit paraboloid is obtained. The additional deviation ΔH due to the change then is

$$\Delta H = \sqrt{\Delta H_c^2 - \Delta H_o^2} \; . \tag{20}$$

Table 2. Non-homologous deformations $\Delta H (10^{-3}$ inch), as caused by random changes of bar area A, density ρ, stiffness K, and joint coordinates x, y and z. Case 15 treats the average joint, case 16 an axial force at the elevation bearings.

Case	ΔH_o (before)	change	ΔH_c (after)	ΔH (additional)
1	3.2	$\rho \pm 5$ %	5.9	5.0
2		K ± 5 %	3.6	1.7
3	1.1	A ± 2 %	1.9	1.5
4		A ± 5 %	3.0	2.8
5		A ±10 %	3.6	3.4
6		A ±20 %	11.2	11.1
7		$\rho \pm 10$ %	3.2	3.0
8		K ± 4 %	3.8	3.6
9	1.5	A ± 5 %	3.4	3.1
10		A ±10 %	11.2	11.1
11		K ± 5 %	2.8	2.4
12		K ±10 %	8.1	8.0
13		x, y, and z ± 3 inch	14.9	14.8
14		x, y, and z ± 10 inch	50.1	50.1
15		all K - 5 %; ρ + 3.2 %	2.5	2.0
16	-	F = 50 tons	-	.95

2) Results

Table 2 shows the results of 16 calculations. Cases 1 to 14 are also plotted in Fig. 1. There is no significant difference between ΔA, $\Delta\rho$ and ΔK. We might use the median of the common distribution, but to be more on the safe side we use the third quartile, yielding

$$\Delta H / \Delta(A, \rho \text{ or } K) = 0.00085 \text{ inch per \% change.} \qquad (21)$$

From case 13 and 14 we find for the coordinate change of the joints

$$\Delta H / \Delta(x, y, \text{ and } z) = 0.0051 \text{ inch per inch change.} \qquad (22)$$

Case 15 uses values (1) and (2) for the average influence of the joints:

$$\Delta H \text{ (from average joints)} = 0.0020 \text{ inch.} \tag{23}$$

Case 16 treats an axial force at the elevation bearings, without any gravity, and it results in ΔH = .000935 inch. In addition, we have 0.0022 inch displacement between cabin and best-fit-paraboloid prime focus, which formally can be expressed (same gain loss) as a surface deformation ΔH = 0.000140 inch. Both ΔH together add up quadratically to ΔH = 0.000945 inch, or

$$\Delta H / F = 1.9 \times 10^{-5} \text{ inch per ton.} \tag{24}$$

III. Application

1) The Total ΔH

Table 3 shows seven single items and their contributions to ΔH. As to the first item, the Homology Program should reduce ΔH_o to zero, but it is limited by the calculating accuracy of the computer. The other items represent the application of the formulas of the general study to the values of tolerances and deviations obtained in the first section.

Table 3. The total deviation ΔH from homology

Item	change	amount	ΔH (10^{-3} inch) single	totals	
1	(original)	ΔH_o	2.0		
2	joints, average	K - 5%, ρ+3.2%	2.0		
3	indiv. joint stiffness	K $\pm$ 5%	4.3		
4	indiv. joint weight	ρ $\pm$ 3.2%	2.7		
5	areas, replace + toler.	A $\pm$ 2.3%	2.0		
6	coordinates	x, y and z $\pm$ .25 inch	1.3	6.28	
7	rails and soil	z - .25 inch	.9		6.35

If the telescope is moved in both directions, item 7 does not change with elevation, and the value given in Table 3 applies to the worst case in azimuth, see text before equation (19).

Items 1 to 6 depend on elevation angle but not on azimuth. The values ΔH given in Table 3 (and in the previous sections) are defined as follows. Let the telescope be adjusted to a paraboloid in the absence of gravity. Then switch gravity on. Call ΔH_ϕ the rms surface deviation from a best-fit paraboloid, as a function of zenith distance angle ϕ. Define

$$\Delta H = \mathrm{rms}(\Delta H_\phi), \quad \text{for all} \quad 0 \leq \phi \leq 360°. \tag{25}$$

Actually, the Homology Program yields $\Delta H_1 = \Delta H_{\phi=0}$ and $\Delta H_2 = \Delta H_{\phi=90°}$, and it computes

$$\Delta H = \sqrt{(\Delta H_1^2 + \Delta H_2^2)/2} \tag{26}$$

which meets definition (25). For any given zenith angle ϕ, the surface deviation then is

$$\Delta H_\phi = \sqrt{(\Delta H_1 \cos \phi)^2 + (\Delta H_2 \sin \phi)^2} \, . \tag{27}$$

2) Adjustment Angle θ

If the telescope is adjusted to a perfect paraboloid for some zenith angle θ, and then observes at zenith angle ϕ, the surface deviation is

$$\Delta H_{\phi\theta} = \sqrt{\Delta H_1^2 (\cos \phi - \cos \theta)^2 + \Delta H_2^2 (\sin \phi - \sin \theta)^2} \tag{28}$$

which can be written as

$$\Delta H_{\phi\theta} = G(\phi, \theta; g) \, \Delta H \tag{29}$$

with

$$g = \Delta H_2 \, / \, \Delta H_1 \tag{30}$$

and

$$G(\phi, \theta; g) = \sqrt{\frac{(\cos \phi - \cos \theta)^2 + g^2 (\sin \phi - \sin \theta)^2}{(1 + g^2)/2}} \tag{31}$$

The telescope can be adjusted to any zenith angle θ wanted. Either, we measure the telescope in zenith position and calculate the necessary offset of all points at zenith needed for reaching a perfect fit at angle θ, using our gravitational deformation analysis data; or, more directly, we measure the telescope at the angle θ (which for example is possible with the Zeiss method).

Next, we ask for the best adjustment angle θ, and we define it by demanding equal deviations $\Delta H_{\phi\theta}$ at both ends of the most important observing range of ϕ. Since points beyond the zenith can be reached by turning 180° in azimuth, we start this range at $\phi = 0$. And since observations will be limited to a good extent by the atmosphere, and nobody can observe at the shortest wavelengths close to horizon, we end this range at $\phi = 60°$ where the atmosphere is twice as thick as at zenith. The best θ then is defined as a function of g by

$$G(0, \theta; g) = G(60°, \theta; g) \tag{32}$$

or

$$g^2 = \frac{\cos\theta - 3/4}{\sqrt{3}\sin\theta - 3/4} . \tag{33}$$

The result is plotted in Fig. 2, together with the resulting G from (31). We see that G(g) increases only slow with g.

The value of g depends on the final structure, which we cannot analyze since it will contain all the random changes discussed earlier. For the examples of Table 2, the value of g scatters from 0.3 to 1.3 with an average of 0.7; small ΔH usually give large g and vice versa. It will be on the safe side if we adopt

$$g = 1 \tag{34}$$

which yields

$$\theta = 30° \tag{35}$$

and

$$G(\phi = 0) = G(\phi = 60°) = 0.518. \tag{36}$$

For other ϕ we have

$$G(\phi) = \sqrt{(\cos\phi - \sqrt{3/4})^2 + (\sin\phi - 1/2)^2} \tag{37}$$

or

$$G(\phi) = \sqrt{2[1 - \cos(\phi - 30°)]}. \tag{38}$$

Finally, we obtain from Table 3 and equations (29) and (38) the total deviation as a function of the zenith angle ϕ as

$$\Delta H_\phi = 10^{-3} \text{ inch} \sqrt{[6.28\ G(\phi)]^2 + 0.9^2} \tag{39}$$

which is plotted in Fig. 3. In the interesting range of ϕ, we have

$$\left.\begin{array}{ll} & 0.9 \leq \Delta H_\phi \leq 3.38 \times 10^{-3} \text{ inch} \\ \text{or} & .023 \leq \Delta H_\phi \leq .086 \text{ mm} \end{array}\right\} \quad \text{for} \quad 0 < \phi < 60° \tag{40}$$

This then is the total surface deviation of the telescope structure, due to joints, inaccuracies and tolerances. For obtaining the total gravitational deviation, we should add the deformations of panels and plates, which will be done in a following report.

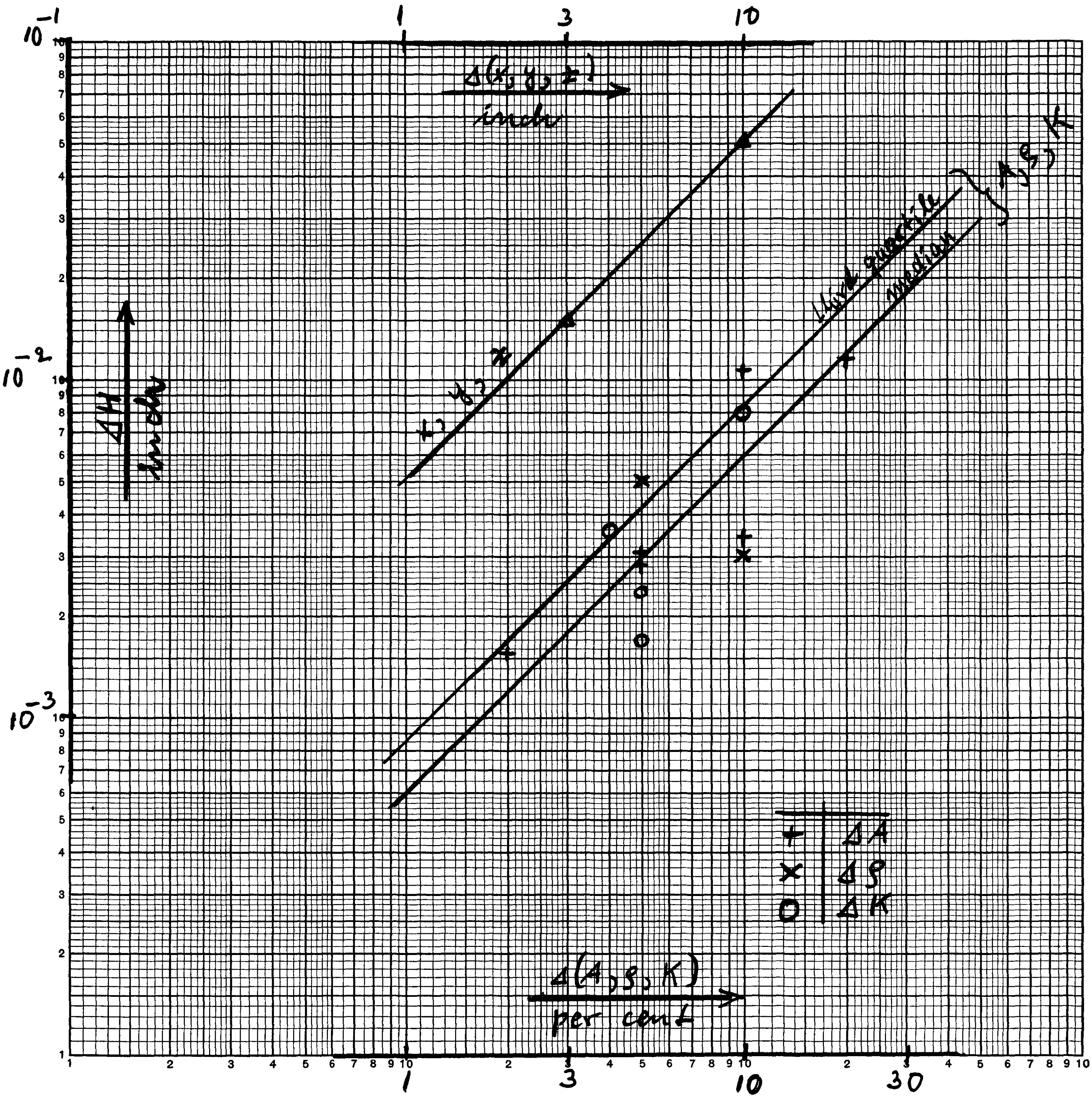


Fig. 1. Non-homologous deformation ΔH, caused by random changes of bar area A, density ρ, stiffness K, and joint coordinates x, y, z.

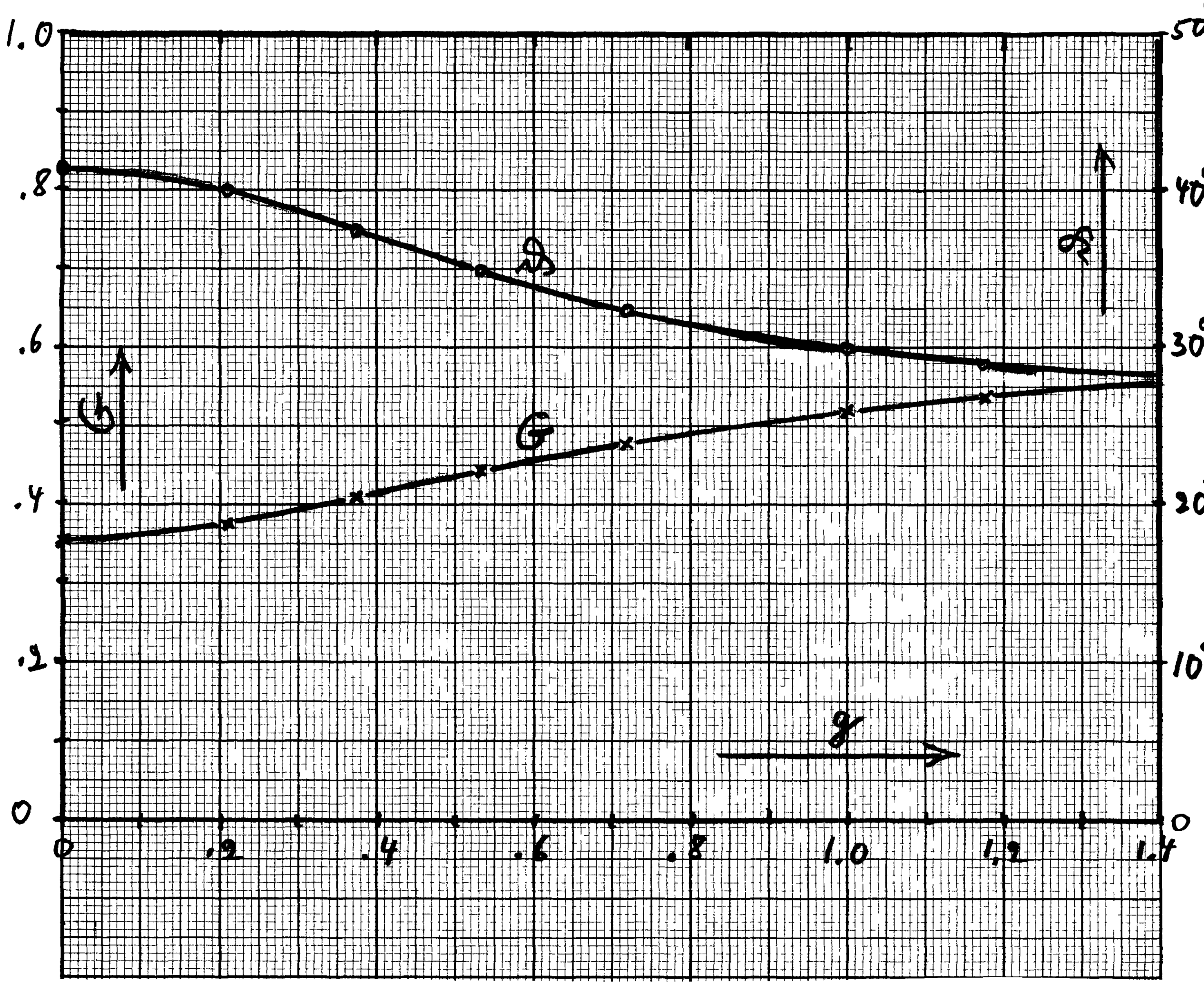
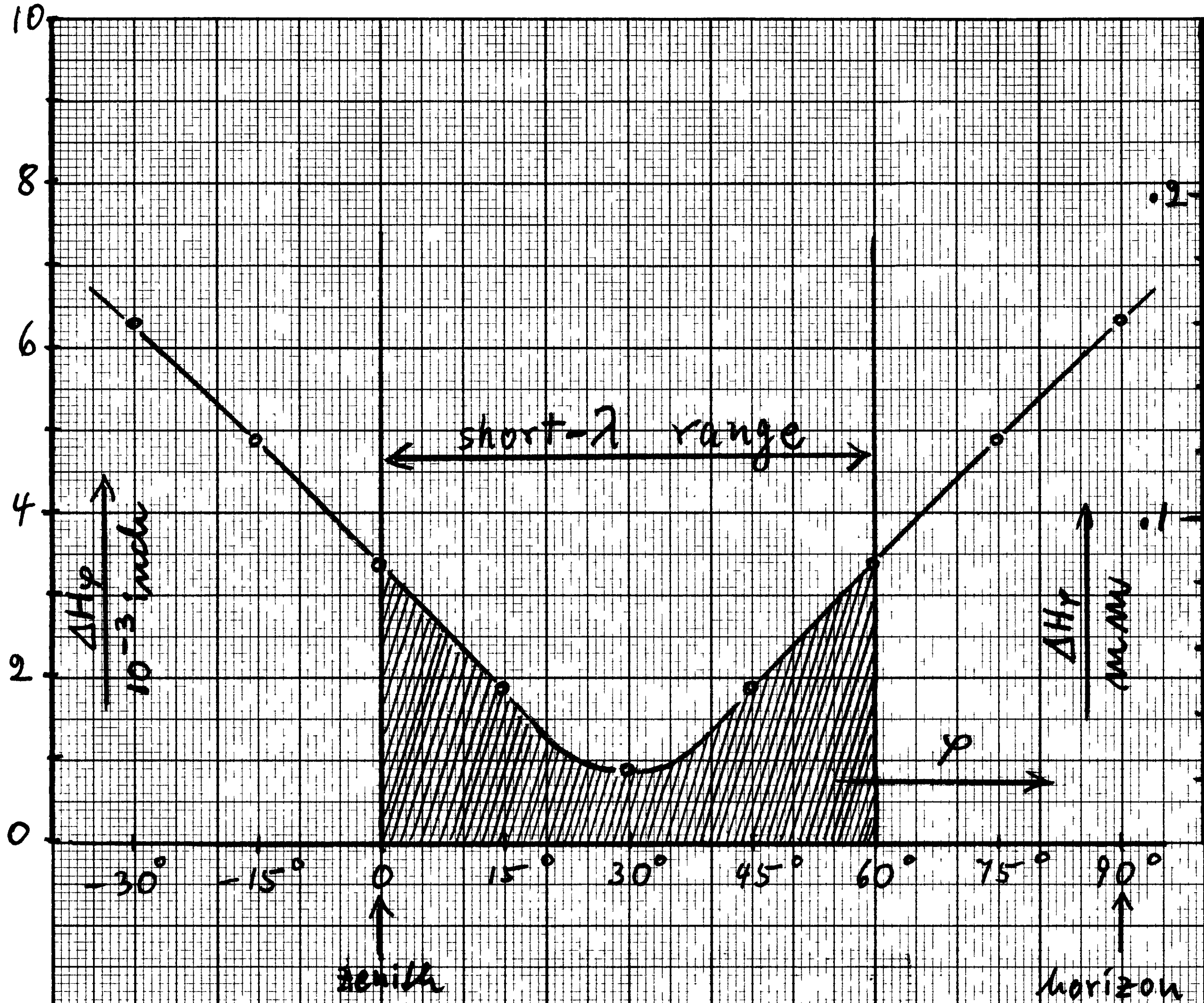


Fig. 2. The best adjustment angle ϑ, as a function of $g = \Delta H_2/\Delta H_1$, the ratio of the deformations in horizon and zenith position. For G see (38).



<u>Fig. 3.</u> Non-homologous deformation ΔH_{φ}, as a function of zenith angle φ, if telescope is adjusted for zenith angle 30°. Including all inaccuracies of the dish structure and its joints, and the influence of rail deviation and soil settlement.