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Conditions for Pipe Diameters

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I. Member Types

All members of the telescope structure are divided in four types:

Type	Explanation	Number in one quadrant			
1. Normal pipe	commercially available	127	168	229	(1)
2. Special order pipe	pipes welded from plates	29			
3. Special design	feed legs, suspension, wheel, cabin	12			
4. Fictitious	replacing weight and stiffness of surface panels	61			

The first two types have two subtypes:

Subtype	Used, if	Regarded as	
a. straight ends	$d < 14$ inch	one end hinged, one fixed	(2)
b. both ends tapered	$d \geqslant 14$ inch	both ends hinged	

The Homology Program yields for each member the bar area A_h needed for homology. It also gives its length L, and the maximum stress S_m for any load condition. Members of type 3 and 4 are specially marked and are omitted from the following.

The Replace Program handles all pipes. It decides whether a member is type 1 or 2, and it replaces all those of type 1 by commercially available pipes given in a table. Each member must fulfil a set of conditions regarding its diameter; the program selects that pipe from the table which fulfills all conditions and whose area A comes closest to A_t. The resulting deviation we call $\Delta A = A - A_t$.

Since the table is only loosely packed above 10 inch^2, see Fig. 1, the type decision is done the following way:

area (inch^2)	Type 1	Type 2
$A < 10$	all	none
$10 \leqslant A \leqslant 16$	$\Delta A < 4\%$	$\Delta A \geqslant 4\%$
$A > 16$	none	all

(3)

II. Conditions Independent of Length or Load

The following conditions are best taken care of by just canceling all violating pipes from the table, leaving a Select Table on which the Select Program works for type 1 without further regard to those conditions.

1. Welding and Rolling

For welding the members to the joints, it was agreed to demand a minimum wall thickness t of

$$t \geqslant 0.1 \text{ inch, for type 1 only.} \tag{4}$$

For type 2, flat plates are rolled cylindrical and welded along the seam, for which

$$t \geqslant 0.25 \text{ inch, for type 2 only.} \tag{5}$$

In order to use cold rolling only, one should demand (O. Heine) a diameter - wall thickness ratio of $d/t \geqslant 20$, or with $A = \pi dt$:

$$d \geqslant 2.52\sqrt{A}, \text{ for type 2 only.} \tag{6}$$

2. Local Buckling

According to O. Heine, we use the condition $0.24\ Et/d \geqslant S_m = 20$ ksi; or, with $E = 29{,}000$ ksi, and $t = A/\pi d$,

$$d \leqslant 10.52\sqrt{A}. \tag{7}$$

As seen from Fig. 1, condition (7) actually is never active for type 1.

3. Thermal Lag

Report 17 (Jan. 3, 1967) gives for steel pipes with white protective paint a thermal time-lag of τ = 1.73 hours per inch of wall thickness. What matters is the difference, $\Delta\tau = 1.73\ (t_{max} - t_{min})$, where t_{min} = 0.1 inch from (4). Demanding $\Delta\tau \lesssim 0.5$ hours then gives

$$t \lesssim 0.4 \text{ inch.} \tag{8}$$

4. Pipe Table

All available Corten pipes were compiled in a table. Then all pipes violating (4), (7) or (8) were omitted. The remaining Select Table is shown in Fig. 1 for $2 \lesssim A \lesssim 16$ inch^2.

The table actually starts at A = 0.1767 inch^2, for panel members. The vertex cabin starts at 0.873 inch^2. The telescope starts at 1.570 inch^2, but all except six bars have $A \geqslant 2$ inch^2 (see Report 34). The Select Table contains 318 pipes for $2 < A < 16$ inch^2, and 598 pipes in total.

III. Remaining Conditions

The following conditions are incorporated in the Select Program. Where their numerical values depend on the subtype (2), we give both values as:

$$\begin{Bmatrix} \text{type 1, straight} \\ \text{type 2, tapered} \end{Bmatrix} \tag{9}$$

The program first calculates d assuming straight ends; if $d > 14$ inch, it assigns tapered ends and recalculates d for that case.

In the following formulas we mostly use, instead of length L,

$$\ell = L/1000 \text{ inch.} \tag{10}$$

1. Lateral Dynamical Frequency

Since the lowest dynamical frequency of the combined structure (telescope, towers, drives, soil) will be 1.6 - 1.8 cps, it was decided that each single member must be somewhat higher, say

$$\nu \gtrsim 2.5 \text{ cps.} \tag{11}$$

The lowest lateral vibration is given by

$$\nu = \frac{a}{2\pi}\sqrt{\frac{gEI}{\rho A L^4}}\ , \quad \text{with} \quad a = \begin{Bmatrix} 15.40 \\ 9.87 \end{Bmatrix}, \tag{12}$$

where $g = 386$ inch/sec^2, $E = 29{,}000$ ksi, $I = r^2A$, $r = d/\sqrt{8}$ for thin-walled pipes, and $\rho = 0.283$ lb/inch^3. This yields

$(10^6$

$$\nu = \begin{Bmatrix} 0.1724 \\ 0.1105 \end{Bmatrix} d/L^2 . \quad \left| \begin{array}{l} \nu \text{ in cps} \\ d,\ L \text{ in inch} \end{array} \right. \tag{13}$$

Demand (11) then results in

$$d \geqslant \begin{Bmatrix} 14.5 \\ 22.6 \end{Bmatrix} \ell^2 . \tag{14}$$

2. Maximum Slenderness Ratio

In previous discussions it was decided to use a maximum of

$$KL/r \lesssim 200 \tag{15}$$

with

$$K = \begin{Bmatrix} 0.8 \\ 1.0 \end{Bmatrix} \tag{16}$$

which yields

$$d \geqslant \begin{Bmatrix} 11.3 \\ 14.1 \end{Bmatrix} \ell . \tag{17}$$

3. Stress Ratio Q

For each member we consider the combination of the following two stresses:

S_m = maximum axial stress from survival loads on whole telescope (output of Homology Program); (18)

S_g = maximum bending stress from sagging of member under its own weight (horizontal, worst case). (19)

We further call

$$\Lambda = KL/r = \text{effective slenderness ratio,} \tag{20}$$

$$S_y = \text{yield stress of material,} \tag{21}$$

$$S_E = \pi^2 E/\Lambda^2 = \text{Euler stress of buckling,} \tag{22}$$

$$S_{Es} = S_E/1.92 = 149{,}000/\Lambda^2, \text{ (safety factor 1.92),} \tag{23}$$

$$S_\Lambda = \text{maximum allowed axial stress,} \tag{24}$$

$$C = \sqrt{2\pi^2 E/S_y}, \tag{25}$$

$$q = \Lambda/C = \sqrt{S_y/2S_E}\ . \tag{26}$$

We then have from the Steel Manual, page 5-16, formulas (1) and (2),

$$S_\Lambda = \begin{cases} \dfrac{1 - q^2/2}{5/3 + 3q/8 - q^3/8}\, S_y & \text{for } q \leqslant 1, \\[2ex] S_{Es} & \text{for } q \geqslant 1. \end{cases} \tag{27}$$

And for the combination of S_m and S_g we find from the Steel Manual, page 5-20 formula (7a) and the second formula on page 5-17, with $C_m = 0.85$,

$$Q = \frac{S_m}{S_\Lambda} + \frac{1.288\ S_g/S_y}{1 - S_m/S_{Es}} \tag{28}$$

with the demand

$$Q \leqslant 1. \tag{29}$$

For thin-walled pipes $S_g = Md/2I$, with $M = \rho AL^2/8$ for both subtypes (2), and $I = d^2A/8$, thus

$$S_g = 0.1415\ L^2/d \quad \left| \begin{array}{l} L,\ d \text{ in inch} \\ S_g \text{ in lb/inch}^2 \end{array} \right. \tag{30}$$

or

$$S_g = 141.5 \text{ ksi } \ell^2/d. \tag{30a}$$

Finally, with $S_y = 50$ ksi for Corten steel, we have the condition (S_m in ksi)

$$Q = \frac{S_m}{S_\Lambda} + \frac{2.83\ \ell^2/d}{1\text{-}53.7\ S_m K^2 \ell^2/d^2} \leqslant 1. \tag{31}$$

Because of the complicated form of (27), equation (31) could be solved for d only numerically, which would yield a minimum diameter d_m:

$$d \geqslant d_m(S_m, L, K) \tag{32}$$

Actually, the Replace Program calculates (31) for the nearest pipe in the Select Table. If fulfilled, the pipe is accepted; if not, the next one is tried, and so on.

4. Wind-Induced Vibrations

Wind-induced vibrations could result in fatigue and destruction. Vibrations do not occur if (a) the critical wind velocity for resonance is above the highest velocity ever expected, and/or if (b) the air flow is turbulent. If vibrations do occur, (c), the alternating stress must be below the fatigue limit of the material.

a) Critical Velocity

For laminar flow across cylindrical bars, the von Karman vortices cause a lateral vibration with a frequency

$$\nu_w = 3.52\ v/d. \quad \left|\begin{array}{lll} \nu & \text{in} & \text{cps} \\ v & \text{in} & \text{mph} \\ d & \text{in} & \text{inch} \end{array}\right. \tag{33}$$

Large amplitudes occur only in resonance, when (33) equals (13), which results in a critical velocity for resonance

$$v_{cr} = 25.1 \text{ mph } (100/\Lambda)^2. \tag{34}$$

Using 85 mph as the highest expected wind, resonance is excluded if $v_{cr} \geqslant 85$ mph, which yields the condition

$$\Lambda \leqslant 54.4 \tag{35}$$

or

$$d \geqslant \begin{Bmatrix} 41.6 \\ 52.0 \end{Bmatrix} \ell \ . \tag{36}$$

b) <u>Turbulent Flow</u>

If the Reynold number

$$R_e = 780 \ vd \quad \left| \begin{array}{l} v \ \text{in mph} \\ d \ \text{in inch} \end{array} \right. \tag{37}$$

is above a critical value of 2×10^5, the flow is turbulent and no vibrations occur. We ask only for the resonant case and demand

$$780 \ v_{cr} \ d \geqslant 2 \times 10^5 \tag{38}$$

or, with (34)

$$d \geqslant \begin{Bmatrix} 17.4 \\ 20.1 \end{Bmatrix} \ell^{2/3} \tag{39}$$

Since it is enough to fulfill either (36) or (39), we can disregard (36) because (39) is more favorable if (which holds for all members)

$$L > \begin{Bmatrix} 73 \text{ inch} \\ 58 \text{ inch} \end{Bmatrix} . \tag{40}$$

c) <u>Fatigue</u>

If vibrations do occur, the wind lift (perpendicular to both pipe axis and wind) is

$$F_w = 8.40 \times 10^{-9} \ v^2 L d \quad \left| \begin{array}{l} F \ \text{in kip} \\ v \ \text{in mph} \\ L, d \ \text{in inch} \end{array} \right. \tag{41}$$

and at resonance the dynamical force is

$$F_d = \frac{\pi}{\eta} F_w \tag{42}$$

with a damping factor η which is linear with stress except close to the yield point. We use (O. Heine Report, March 11, 1969)

$$\eta = 0.00033\ S. \qquad \Big|\ S \text{ in ksi} \tag{43}$$

Load (42) distributed over length L gives for thin-walled pipes and for both subtypes (2) a maximum alternating stress of

$$S_a = \frac{Mz}{I} = \frac{F_d L}{2Ad} = \frac{\pi}{2\eta}\frac{F_w L}{Ad} \quad . \tag{44}$$

F_d itself depends on S_a via η. At the onset of wind the amplitude increases until η becomes large enough for a saturation. This defines the final S_a, which is obtained by inserting v_{cr} from (34) into (41), and then solving (41) - (44) for S_a. This yields the alternating stress

$$S_a = \begin{Bmatrix} 0.310 \\ 0.198 \end{Bmatrix} \frac{d^2}{\ell\sqrt{A}} \text{ ksi.} \tag{45}$$

We now demand $S \lesssim 25$ ksi, yielding

$$d \lesssim \begin{Bmatrix} 8.98 \\ 11.23 \end{Bmatrix} \ell^{1/2} \ A^{1/4} \ . \tag{46}$$

With respect to wind-induced vibrations, the diameter thus must be either larger than (39) preventing vibration, or it must be smaller than (46) preventing fatigue.

IV. Summary

The following table contains all conditions. For the sake of consistency also the conditions (4), (5) and (8), which concern the wall thickness, are rewritten as diameter conditions using $t = A/\pi d$. Conditions in parentheses are already taken care of in the Select Table of Corten pipes, Fig. 1.

The most critical condition is that for the stress ratio, (31). Therefore this is already incorporated in the Homology Program, in an approximate way where $r = kA^{2/3}$ is assumed with $k = 0.73$ for lighter pipes and $k = 1$ for heavier ones. Condition (17) is also included in the program.

	Demand	Equation	$d \leqslant$	$d \geqslant$
Type 1 only				
welding joints	$t \geqslant 0.1$ inch	4	$(d \leqslant 3.18\ A)$	
Type 2 only				
welding seams	$t \geqslant 0.25$ inch	5	$d \leqslant 1.273\ A$	
cold rolling	$d/t \geqslant 20$	6		$d \geqslant 2.52\ \sqrt{A}$
Both types				
local buckling	$.24Et/d \geqslant 20$ ksi	7	$(d \leqslant 10.52\ \sqrt{A})$	
thermal lag	$\Delta\tau \leqslant 0.5$ hours	8		$(d \geqslant .795\ A)$
dynamical frequency	$\nu \geqslant 2.5$ cps	14		$d \geqslant \begin{Bmatrix} 14.5 \\ 22.6 \end{Bmatrix} \ell^2$
slenderness ratio	$KL/r \leqslant 200$	17		$d \geqslant \begin{Bmatrix} 11.3 \\ 14.1 \end{Bmatrix} \ell$
stress ratio	$Q \leqslant 1$	31		$d \geqslant d_m(S_m, L, K)$
fatigue: either	turbulence	39		$d \geqslant \begin{Bmatrix} 17.4 \\ 20.1 \end{Bmatrix} \ell^{2/3}$
fatigue: and/or	$S_a \leqslant 25$ ksi	46	$d \leqslant \begin{Bmatrix} 8.98 \\ 11.23 \end{Bmatrix} \ell^{1/2} A^{1/4}$	

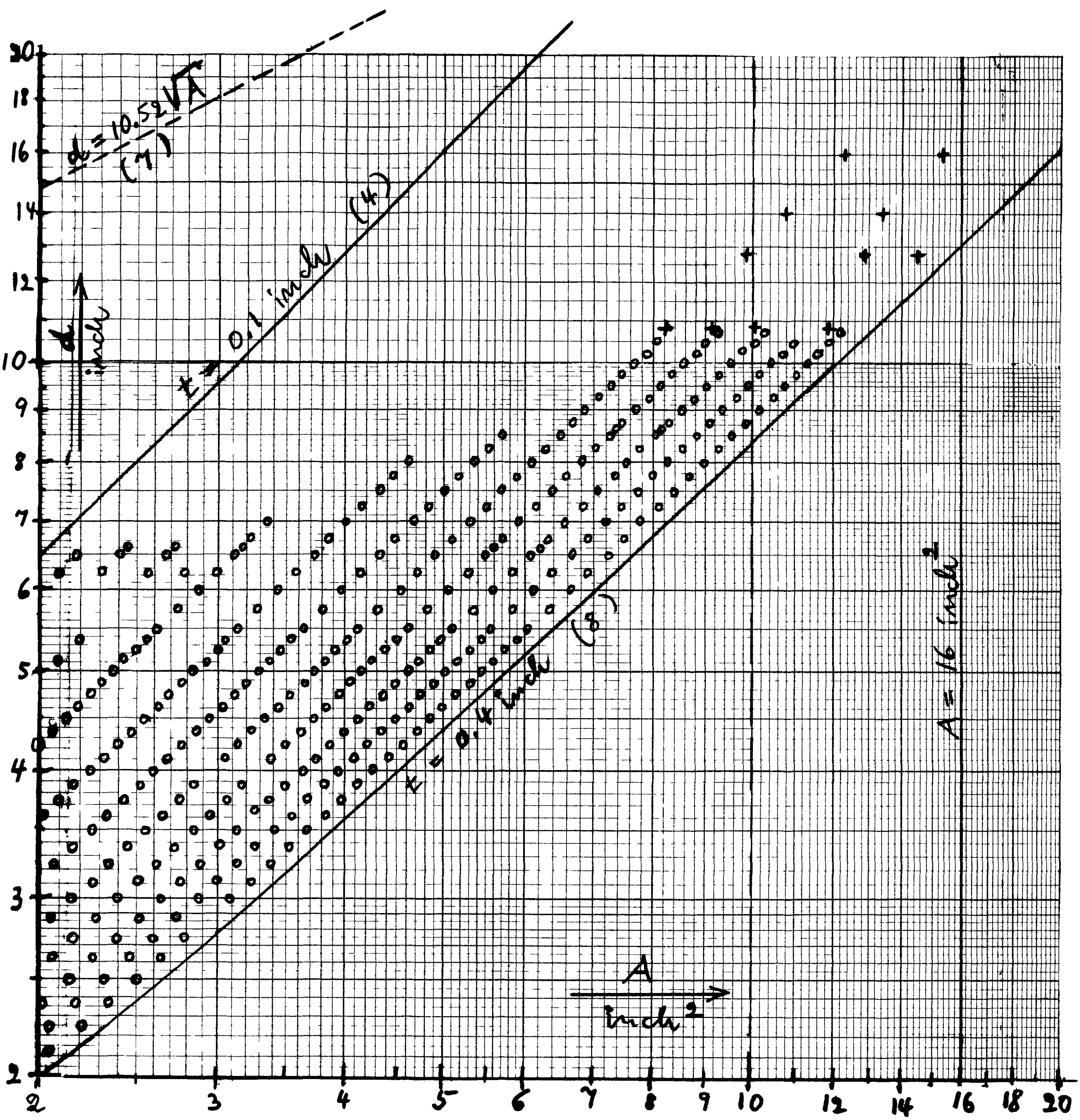

Fig. 1. Available Corten pipes for $2 \lesssim A \lesssim 16$ inch^2, fulfilling conditions (4), (7) and (8).
A = bar area, d = outer diameter, t = wall thickness, o = stock items, + = on request.