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Thermal Deformations of the 65-m Telescope

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Summary

Thermal measurments as a function of the hour are compiled from seven Reports and Memos, taken at the 140-ft, its spare panel, the 36-ft, and a surface plate for the 65-m telescope. On 95 % of all clear, calm days, the vertical structural temperature difference ΔT is, in full sunshine, below 12 ^{o}F for the surface plates and below 9 ^{o}F for panels and back-up structure; at night, ΔT is below 2.0 ^{o}F and 1.5 ^{o}F, respectively. The time-derivative $\dot{T}$ of the ambient air tempereature is below 8.6 ^{o}F/h on sunny days and below 1.5 ^{o}F/h at night.

Thermal deformations of the plates are found by direct measurments, and those of panels and back-up structure by various computer analyses. Gain loss by defocussing is formally converted into a surface error, to be added quadratically to the other errors.

Including plates, panels, back-up structure and defocussing, the total thermal rms surface error of the 65-m telescope is 0.0169 inch = 0.43 mm at noon in full sunshine, and 0.0029 inch = 0.073 mm during 8 hours at night. During 10.3 hours of each night, the error is below 0.004 inch = 0.10 mm.

I. Measurments of ΔT and $\dot{T}$

We call ΔT the temperature difference of any two structural parts, preferably at different vertical locations (z-gradient) since these give the largest deformations; and we call $\dot{T} = dT/dt$ the time-derivative of the ambient air temperature which causes a thermal lag of the heavier members. Both values are wanted separately for clear nights (shortest wavelengths) and sunny, calm days (worst case). For both cases we want, if possible, the distributions of ΔT and $\dot{T}$, and we decided to use the 95% level. This means that the actual thermal deformations will not surpass the calculated ones for 95% of all days. The following summarizes the available measurments.

1. Report 17 (Jan.3, 1967)

a) A spare panel of the 140-ft was painted white and mounted on a south slope; ΔT was measured between the skin and a low pipe of the panel structure always in the shadow of the skin. Readings were taken 1966 on clear summer days at noon, with the result

$$\Delta T = \begin{cases} 16\ ^{\circ}F & \text{maximum,} \\ 9\ ^{\circ}F & \text{average.} \end{cases} \qquad (1)$$

Part of this (about 2 °F) may be measuring errors, since the thermistors were not calibrated.

b) Values $\dot{T}$ were obtained from Sugar Grove, W.Va., where the air temperature was measured each hour during the year 1962. The maximum hourly rise and drop of each day was taken and their distribution is plotted in Fig. 1. On 95% of all days, the maximum rise or drop is below

$$\dot{T} = 8.5\ ^{\circ}F/h\ . \qquad (2)$$

c) The time-constant τ of the thermal lag was found experimentally as

$$\left.\begin{array}{l} \tau\ (\text{steel}) = 1.73 \text{ hours} \\ \tau\ (\text{aluminum}) = 1.14 \text{ hours} \end{array}\right\} \text{per inch of wall thickness,} \qquad (3)$$

for pipes with white paint; open shapes like angles or I-beams have 1/2 these values. Temperature differences go down with $e^{-t/\tau}$. In case of a constant rise or drop $\dot{T}$, the temperature difference between two members of wall thickness w_1 and w_2 is

$$\Delta T = \tau \dot{T} (w_1 - w_2)\ . \qquad (4)$$

2. Memo July 17, 1970; V. Herrero

Thermistors had been installed 1967 at various members of the 140-ft. The maximum temperature difference ΔT between any two thermistors was read every two hours during 29 days, and 2°F were subtracted for instrumental errors. The cumulative distribution $F(\Delta T)$ was

$F(\Delta T)$	ΔT
.50	4.4 °F
.75	8.7
.95	18.2

(5)

These values include the time-lag of heavier members. The wall thickness of the 140-ft ranges from 0.25 to 1.00 inch, which would reduce the 95 % level from 18.2 °F to about

$$\Delta T = 12\ ^{\circ}F\ . \qquad (6)$$

3. Report Oct. 6, 1970; V. Herrero

On 21 days in Aug. and Sept. 1970, seven thermistors were monitored every two hours at various points of the basic tower structure of the 85-ft-1 telescope after repainting it. The distribution of the maximum difference is

F	ΔT
.50	1.0 °F
.75	2.5
.95	5.4

(7)

4. Memo Nov. 4, 1970; V. Herrero

On the 140-ft, two thermistors (representing a vertical gradient) were monitored during seven nights, including four very clear ones. Calibration was done by subtracting the average. The result was $\overline{\Delta T} = 1.4$ °F and rms(ΔT) = 1.6 °F. The peak-to-peak was 4.5 °F over the whole period, and 3.6 °F within a single night. For the 95 % level deviation from the average we may use

$$\Delta T = 2.2\ ^{\circ}F. \qquad (8)$$

5. Memo Sept. 28, 1970; E. Conklin

The temperature of the 36-ft at Kitt Peak was measured at the surface and at a

lower point of the back-up structure (vertical gradient). Simultaneously, the best focal length f was obtained by observations of radio sources. Fig.2 shows Δf plotted against ΔT. First, the maximum of ΔT is 12.8 ^{o}F, and the 95 % level is

$$\Delta T = 12.5\ ^{o}\mathrm{F}. \tag{9}$$

Second, there is a good correlation between ΔT and Δf, which suggests to use the measured value of ΔT for an automatic correction of the focal length. This reduces the total spread of Δf from 36.5 mm to 8.5 mm, or by a

$$\text{reduction factor for defocussing} = 0.233\ . \tag{10}$$

6. Memo Jan. 15, 1971; V. Herrero

In September and December 1970, a number of clear and calm nights at the 36-ft were selected under the conditions: cloud cover $< 1/4$, $\dot{T} < 1\ ^{o}\mathrm{F/h}$, wind $<$ 10 mph. Measured was again ΔT between skin and lower back-up structure, subtracting the average of the whole period. The result is

F	ΔT
.50	.62 ^{o}F
.75	1.07
.95	1.90

(11)

7. Report 36, Jan. 20, 1971; S.v.Hoerner

A surface plate of the 65-m design was manufactured at Green Bank work shop. It was painted white, mounted 5 ft above ground, and the temperature of skin and lowest rib (vertical gradient) was measured, as well as that of the air and of a small blank aluminum sheet. We found that the white paint improves ΔT by a factor 5.3 during sunshine, but makes it worse by a factor 1.4 during clear nights. During these measurments we had an extremely clear and calm period. All details are given in Report 36. The results are for the 95 % level:

$$\Delta T = \begin{cases} 2.0\ ^{o}\mathrm{F}, & \text{clear nights,} \\ 9.2\ ^{o}\mathrm{F}, & \text{sun at noon,} \end{cases} \tag{12}$$

and

$$\dot{T} = \begin{cases} 1.5\ ^{o}\mathrm{F/h}, & \text{clear night,} \\ 8.6\ ^{o}\mathrm{F/h}, & \text{after sunset.} \end{cases} \tag{13}$$

8. Summary and conclusion

Table 1. Measurments of ΔT and $\dot{T}$, 95% level.
(Including: s = skin, p = panels, b = back-up structure.)

Table 1,a (measured) No.	Item	including	ΔT (°F) clear night	ΔT (°F) noon sun	$\dot{T}$ (°F/h) clear night	$\dot{T}$ (°F/h) after sunset
1.	140-ft spare panel, ΔT Sugar Grove, $\dot{T}$	s,p		14.0		8.5
2.	140-ft, lag subtr.	p,b		12.0		
3.	85-ft, tower	b		5.4		
4.	140-ft, calibrated	p,b	2.2			
5.	36-ft, with lag	s,p,b		12.5		
6.	36-ft, no lag, clear	s,p,b	1.9			
7.	65-m surface plate	s	2.0	9.2	1.5	8.6
Table 1,b (to be used)						
	surface plates	s	2.0	12.0	-	-
	panels	p	1.5	9.0	1.5	8.6
	back-up structure	b	1.5	9.0	1.5	8.6

Table 1 summarizes the previous results of measurments, and gives the values to be used for the thermal deformation analysis of the 65-m telescope. In z-direction, the telescope is divided in three parts: surface plates (s), panels (p), and back-up structure (b). Each of these parts will have its own vertical difference ΔT, and the total will add up like

$$\Delta T_{total} = \Delta T_b + (\Delta T_p + \Delta T_s) / 2$$

since the upper bars of any part should have the average temperature of the part above it. Since there is some uncertainty in this division as well as in the measurments, we have chosen the values of Table 1,b such that they should be all on the safe side.

As to the 24-hour period, we use Fig. 6,b of Report 36 unchanged for $\dot{T}$, to be used for panels and back-up structure. Fig. 6,a of Report 36, for ΔT of the plates, is left unchanged during the night; but its noon amplitude is increased from 9.2 to

12.0 °F according to Table 1,b; also the maximum has been considerably broadened, assuming the telescope is always pointed at the sun (worst case), using atmospheric transmission data as a function of zenith angle given in Allen, "Astrophysical Quantities", page 127. The result is shown in Fig. 3,a, to be used for the surface plates; for panels and back-up structure we multiply ΔT of Fig. 3,a by 3/4 according to Table 1,b.

II. Thermal Deformations, Single

1. Surface Plates

The thermal deformation of the surface plates has been measured at Green Bank; Details are given in Report 36. The result is, with ΔT from Fig. 3,a:

$$\text{rms}\ (\Delta z) = 1.87 \times 10^{-3}\ \text{inch}\ \Delta T/^{\circ}F, \tag{14}$$

$$\text{rms}\ (\Delta z - \overline{\Delta z}) = .67\ \text{"}\ \text{"}\ \text{"}\ \text{"}. \tag{15}$$

If the telescope surface were flat, we should use (15). The worst case of a curved surface is when half of it is shadowed by its own rim, the other half then being illuminated with an angle of 40° between rays and skin. For this case we use (14) multiplied by a factor

$$(1/\sqrt{2})\ \sin 40^{\circ} = 0.4545, \tag{16}$$

as the rms over the whole surface. This gives

$$\Delta z = 0.850 \times 10^{-3}\ \text{inch}\ \Delta T/^{\circ}F, \quad .039\ \text{mm}/6^{\circ} \tag{17}$$

which is

$$\Delta z = \begin{cases} 0.0017\ \text{inch, at night,} \\ 0.0102\ \text{inch, full sun.} \end{cases} \tag{18}$$

2. Panels

For each of the four panels (A, B, C, D), four computer runs have been made: for vertical gradient ΔT and for thermal lag $\dot{T}$, both with fixed and with gliding restraints at the holding points (taking the average of both since the actual case is in between).

Table 2. Thermal deformations of the panels (in 10^{-3} inch).

Panel	number in telescope	$\Delta T = 1\ ^{o}F$		$\dot{T} = 1\ ^{o}F/h$
		rms(Δz)	rms($\Delta z - \overline{\Delta z}$)	rms(Δz)
A	16	1.47	0.70	0.150
B	16	1.19	.25	.250
C	8	1.19	.56	.167
D	4	.89	.39	.194
weighted rms	(44)	1.28	.52	.198

Table 2 shows the results. For the gradient ΔT, we multiply rms(Δz) again with (16) yielding

$$\Delta z = 0.00582 \text{ inch } \Delta T/^{o}F \tag{18}$$

or with Table 1,b

$$\Delta z = \begin{cases} 0.00087 \text{ inch, at night,} \\ 0.00524 \text{ inch, full sun.} \end{cases} \tag{19}$$

For the thermal lag we obtain

$$\Delta z = 0.000198 \text{ inch } \dot{T}/(^{o}F/h) \tag{20}$$

or with Table 1,b

$$\Delta z = \begin{cases} 0.00030 \text{ inch, at night,} \\ 0.00170 \text{ inch, after sunset.} \end{cases} \tag{21}$$

3. Back-up Structure

Table 3 gives the results of seven computer runs, with thermal loads as listed in the first column. A STRUDL analysis yields the deformations of all surface points and of the prime focus cabin. An additional program makes a least-squares fit of a paraboloid of revolution (no contraints) to the surface, yielding the residual surface rms error listed in the fifth column of Table 3. It also yields the changes of focal length, of vertex position, and of axial direction.

We call:

		symmetric	antisymmetric
focal change	best-fit	Δf	-
axis tilt	best-fit	-	$\Delta\alpha$
vertex shift	best-fit	Δz_v	Δx_v
shift of equipment cabin		Δz_c	Δx_c
distance focus - cabin		$\mathcal{E}_s$	$\mathcal{E}_a$
pointing error		-	$\Delta\varphi$

(22)

Table 3. Thermal deformations of back-up structure.

(ΔT = peak-to-peak; s = symmetric, a = antisymmetric)

(1) Thermal load	(2) symm.	(3) $\Delta\varphi$ pt.err. arcsec	(4) $\mathcal{E}$ defoc. 10^{-3}in.	surface rms(Δz), 10^{-3}inch: (5) from surface deform.	(6) σ from defoc.	(7) total uncorr.	(8) total corrected
1. Thermal lag, $\dot{T} = 1^{\circ}F/h$	s	0	16.4	1.05	1.04	1.48	1.08
2. z-gradient ($\Delta T = 1^{\circ}F$)	s	0	34.4	.74	2.19	2.31	.90
3. x-gradient ($\Delta T = 1^{\circ}F$)	a	.19	.53	.01	.01	.02	.01
4. y-gradient ($\Delta T = 1^{\circ}F$)	a	.18	.52	.01	.01	.02	.01
5. radial grad. ($\Delta T = 1^{\circ}F$)	s	0	11.2	.47	.71	.85	.50
6. all suspension ($\Delta T = 1^{\circ}F$)	s	0	.11	.23	.01	.23	.23
7. center susp.only ($\Delta T = 1^{\circ}F$)	s	0	.15	.31	.01	.31	.31

With notations (22), and for f/D = .425, the pointing error is

$$\Delta\varphi = 1.843\,\Delta\alpha + 0.843\,(\Delta x_v - \Delta x_c)\,/\,f\,. \tag{23}$$

The focal offset or defocussing, $\mathcal{E}$, is listed in the fourth column of Table 3 and is obtained from

$$\mathcal{E}_s = \Delta f + \Delta z_c - \Delta z_v\,, \tag{24}$$

$$\mathcal{E}_a = f\,\Delta\alpha + \Delta x_v - \Delta x_c\,. \tag{25}$$

This offset causes a gain loss, in addition to the one from the surface deformation. Since our final error budget is done in terms of surface errors, we convert the gain loss from defocussing (formally) into a surface error σ which would give the same gain loss. The following conversion factors are derived from formulas and graphs given by J. Baars (Int. Report 57; August 1966):

$$\sigma_s = 0.0589\ \varepsilon_s \tag{26}$$

$$\sigma_a = 0.0176\ \varepsilon_a\ . \tag{27}$$

Values σ are listed in the sixth column of Table 3. They are added quadratically to the fifth column, and the resulting total surface error is listed in the seventh column.

As measured by E. Conklin at the 36-ft (Fig.2), the focal offset can greatly be reduced if ΔT is measured by two thermistors in the structure. For this case we multiply σ by 0.233 according to (10) and add the result quadratically to the fifth column. This corrected total surface error is listed in the last column of Table 3, to be used in the following.

For temperature differences ΔT, the worst case is a vertical z-gradient resulting in

$$\Delta z = 0.90 \times 10^{-3} \text{ inch } \Delta T/^{\circ}F \qquad .023^{mm}/^{\circ}F = .041^{mm}/^{\circ}C \tag{28}$$

or, with Table 1,b

$$\Delta z = \begin{cases} 1.35 \times 10^{-3} \text{ inch, at night,} \\ 8.10 \quad '' \quad '' \text{, full sun.} \end{cases} \tag{29}$$

$\dot{T} = 1^{\circ}F/h$ $\quad \dot{T} = 1^{\circ}C/hr = .049^{mm}$

For the thermal lag, we have

$$\Delta z = 1.08 \times 10^{-3} \text{ inch } \dot{T}/(^{\circ}F/h) \qquad .027^{mm} \tag{30}$$

and with Table 1,b

$$\Delta z = \begin{cases} 1.62 \times 10^{-3} \text{ inch, at night,} \\ 9.29 \times \quad '' \quad '' \text{, sunset.} \end{cases} \tag{31}$$

Both z-gradient and lag, from (29) and (31), add up quadratically to

$$\Delta z = \begin{cases} 2.11 \times 10^{-3} \text{ inch, at night,} \\ 12.33 \quad '' \quad '' \text{, full sun.} \end{cases} \tag{32}$$

III. Whole Telescope

1. Day and Night

Table 4. Thermal deformations, summary (10^{-3} inch).

	z-gradient		therm. lag		together	
	night	sun	night	sun	night	sun
surf. plates	1.70	10.2	-	-	1.70	10.2
panels	.87	5.2	.30	1.70	.92	5.6
back-up structure	1.35	8.1	1.62	9.30	2.11	12.3
together	2.34	14.0	1.65	9.45	2.86	16.9
					total	

Table 4 shows the single contributions and their total. First, we see that the plates and back-up structure give comparable contributions while the panels deform only half as much. Second, z-gradient and lag are comparable, the latter being about 30 % lower. In total we have

$$\Delta z = \begin{cases} 2.86 \times 10^{-3} \text{ inch, at night,} \\ 16.9 \quad \text{"} \quad \text{"}, \text{ sun.} \end{cases} \tag{33}$$

2. The 24 hours

For the z-gradient, we add quadratically (17), (18) and (28), but in order to apply ΔT of Fig. 3,a we multply (18) and (28) by 0.75 according to Table 1,b. The result is

$$\Delta z = 1.17 \times 10^{-3} \text{ inch } \Delta T \text{ (of Fig.3,a).} \tag{34}$$

For the thermal lag, we add quadratically (20) and (30) and obtain

$$\Delta z = 1.10 \times 10^{-3} \text{ inch } \dot{T} \text{ (of Fig.3,b).} \tag{35}$$

Then, we add quadratically (34) and (35) and obtain the total thermal deformation as shown in the last column of Table 5 and in Fig. 4.

Table 5. Thermal deformations as a function of the hour .

hour	ΔT	$\dot{T}$	surface rms(Δz), 10^{-3}inch		
	^{o}F	$^{o}F/h$	z-grad.	th. lag	together
18	5.4	7.6	6.32	8.36	10.48
19	4.9	7.3	5.73	8.03	9.87
20	4.4	5.3	5.15	5.83	7.78
21	3.6	3.2	4.21	3.52	5.49
22	2.7	1.6	3.16	1.76	3.62
23	2.1	1.5	2.46	1.65	2.96
24	2.0		2.34		2.86
1					
2					
3					
4					
5					
6					
7	2.0		2.34		2.86
8	2.7	1.5	3.16	1.65	3.56
9	6.3	2.6	7.37	2.86	7.91
10	10.4	6.4	12.17	7.04	14.06
11	11.7	8.4	13.69	9.24	16.52
12	12.0	8.4	14.04	9.24	16.81
13	12.0	6.4	14.04	7.04	15.71
14	12.0	3.6	14.04	3.96	14.59
15	11.7	2.3	13.69	2.53	13.92
16	10.4	2.8	12.17	3.08	12.55
17	7.4	5.4	8.66	5.94	10.50

Finally, Fig. 5 shows during how many hours of each day the surface error stays below a given value Δz.

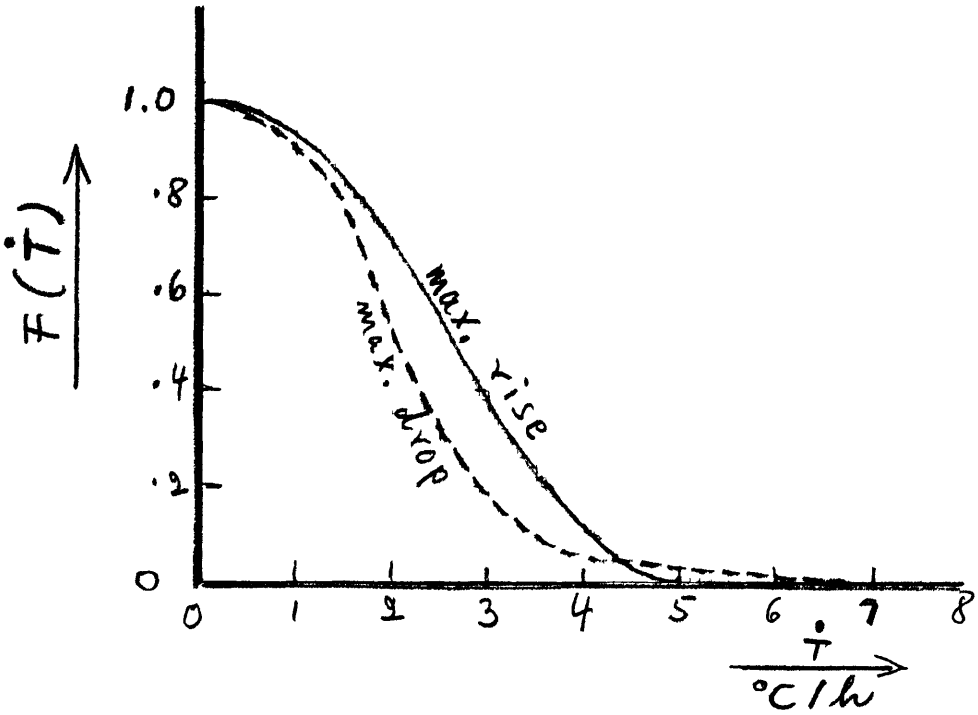

Fig. 1. Temperature change per hour, at Sugar Grove, W.Va, in 1962. Maximum rise and maximum drop of each day.

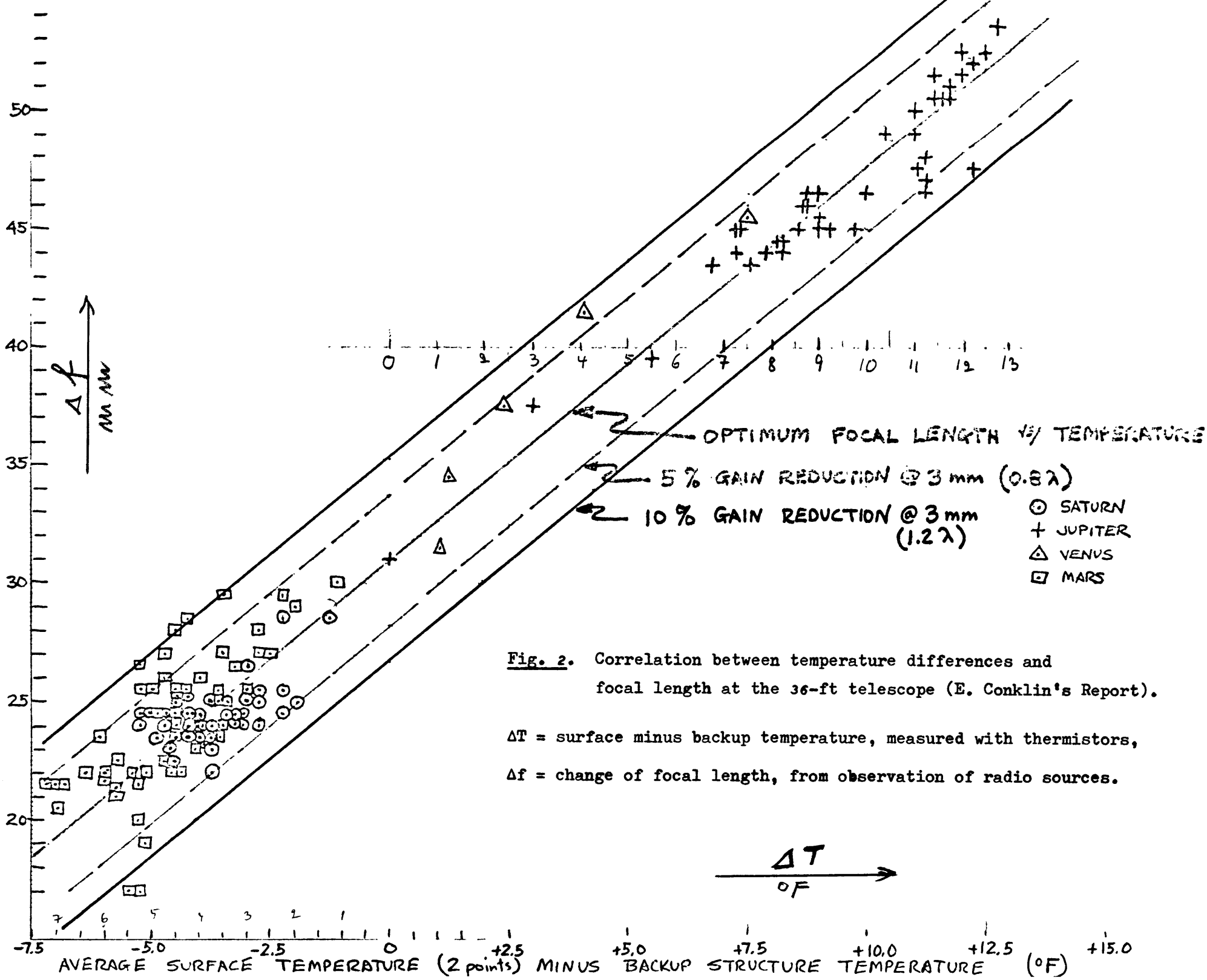

Fig. 2. Correlation between temperature differences and focal length at the 36-ft telescope (E. Conklin's Report).

ΔT = surface minus backup temperature, measured with thermistors,

Δf = change of focal length, from observation of radio sources.

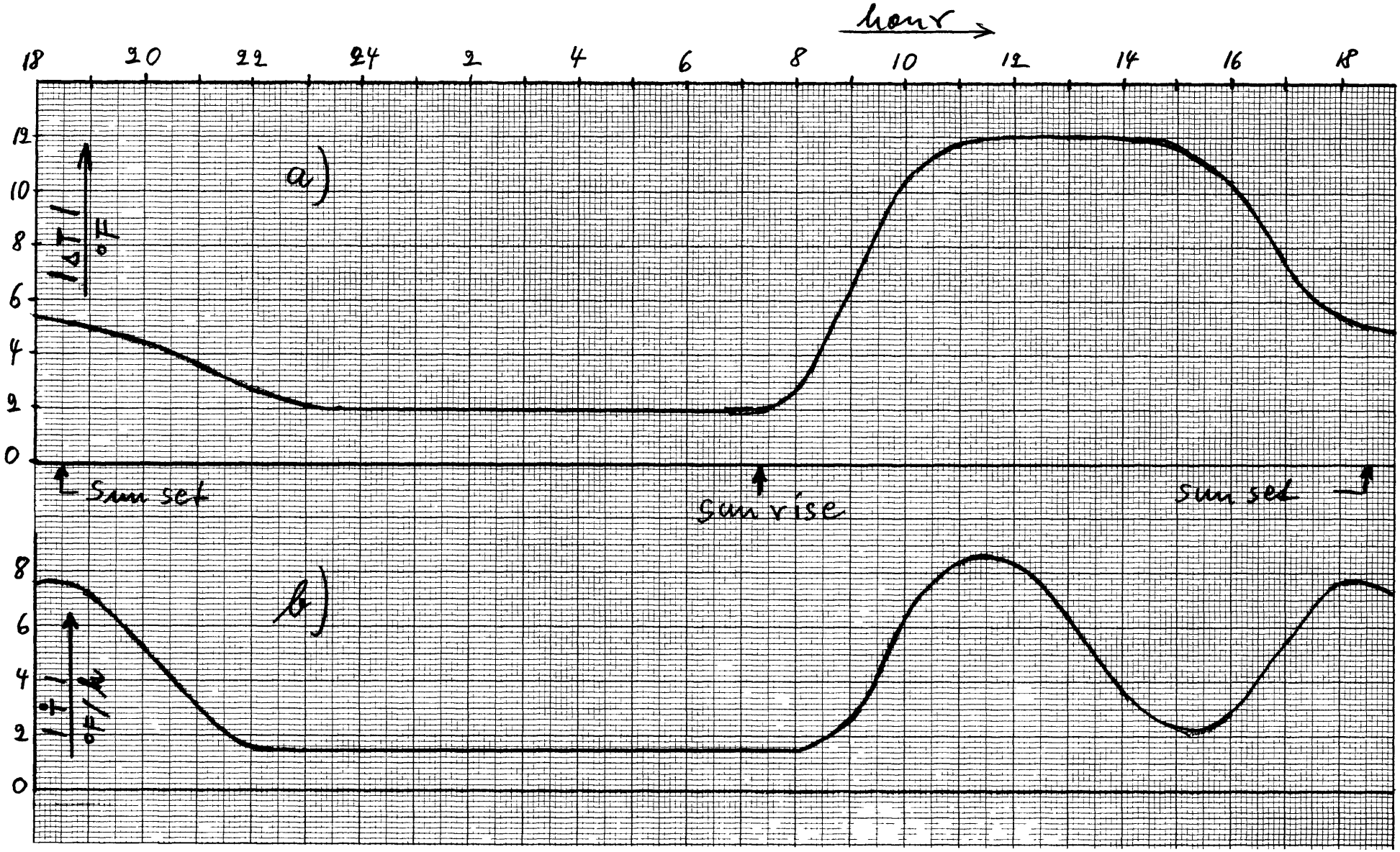

Fig. 3. Absolute values of a) ΔT = vertical temperature difference for surface plates (multiply by 3/4 for panels and back-up structure);

b) $\dot{T}$ = time-derivative of ambient air temperature.

Maximum values for 95 % of all clear, calm days. As a function of the hour.

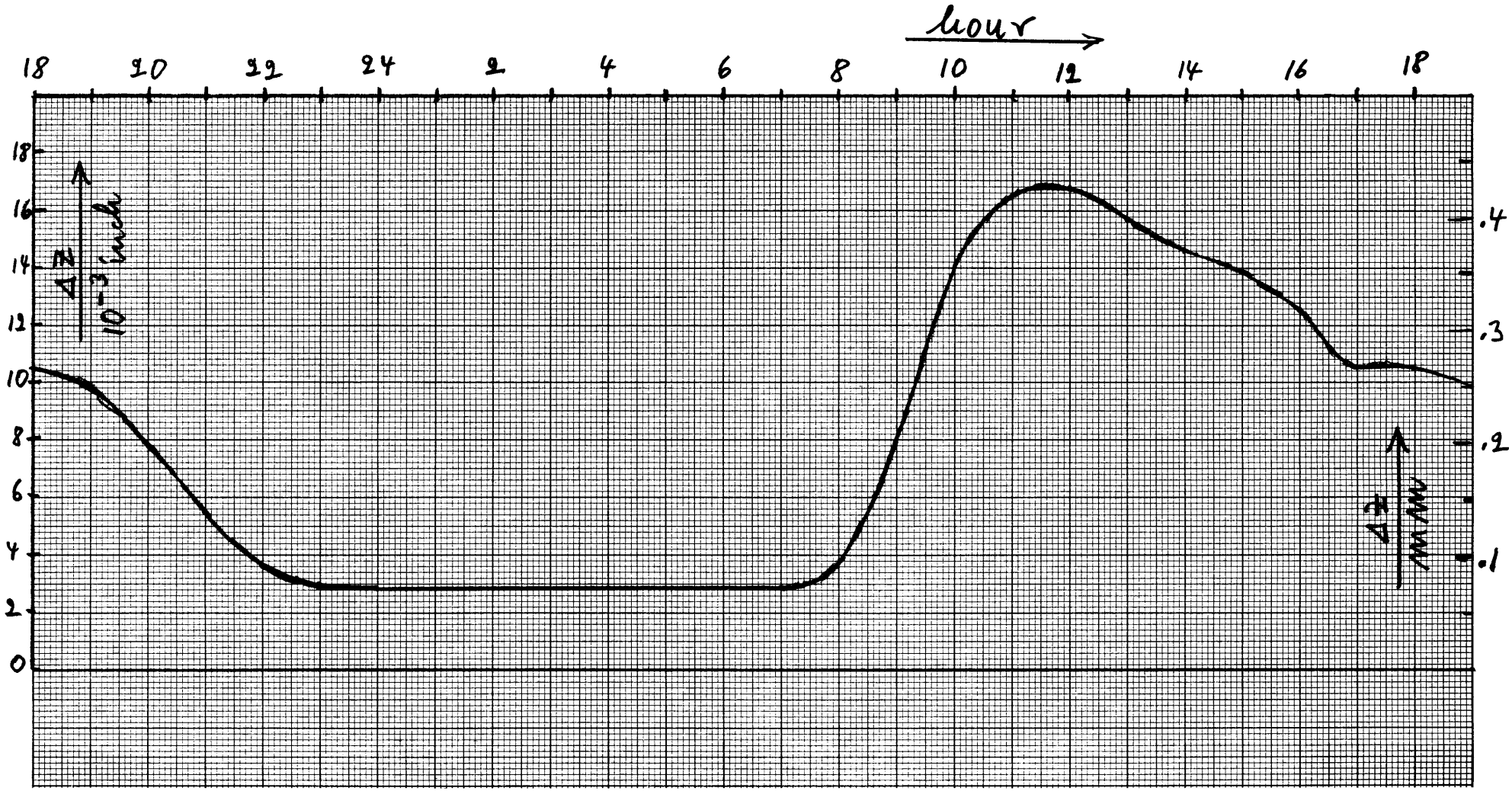

Fig. 4. Total rms surface error, Δz, from all thermal deformations of the telescope, as a function of the hour.

Maximum values for 95 % of all clear, calm days.

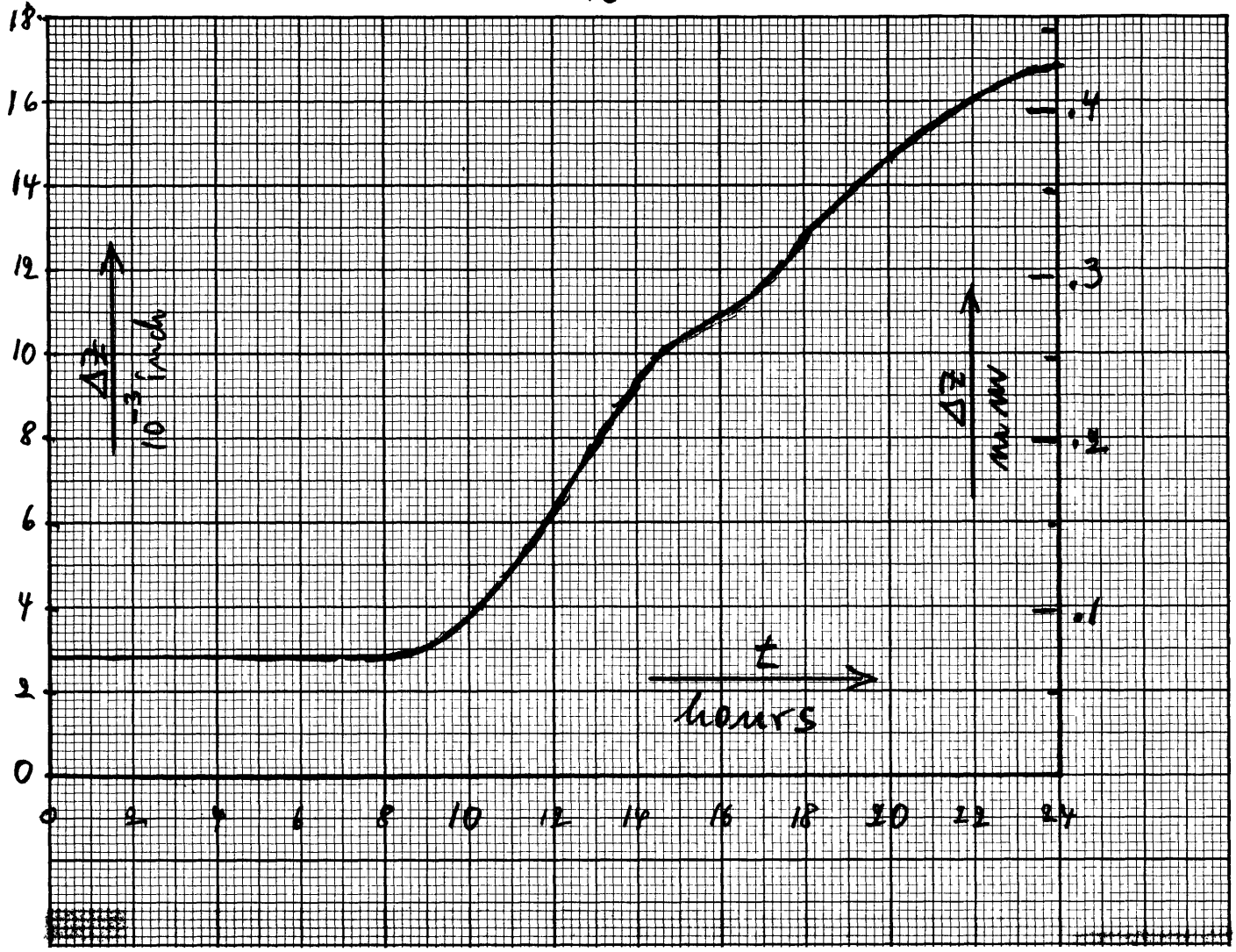

Fig. 5. During t hours of each day, the thermal surface error is below Δz.