11-9-00

Alignment tolerances for ALMA optics

Version 1.9

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Preliminary draft

Abstract

The purpose of this report is to define alignment tolerances for ALMA optics. A short note was circulated in April-2000 (hereafter referred to as version 1.0). It gave results for the alignment tolerances for what was then the optical train for band 10, believed to be the most critical w/r to alignment.

What is new compared with the version 1.0?

- Results are presented for bands 1-7, based upon the "final" optical design elaborated by the optics design workshop (Tucson, 25-29 Sep 2000), and the numbers provided by M.Carter (ALMA-Optics-13Oct00.xls).
- A second type of misalignment is considered, i.e. displacement of an individual element, as opposed to a "break" in the optical train.
- Refocussing off-axis mirrors are treated specifically for geometrical misalignment (were previously treated as in-line thin lenses).

What is still missing

- A check of the tolerance w/r to aberrations with the present "final" configuration. Version 1.0 of this note note showed that the tolerances w/r to aberrations were, in the case of band 10, significantly larger than those w/r to geometrical alignment in the aperture plane. It seems a safe assumption that this conclusion will hold for modified versions of band 10 optics, and a fortiori for optical trains operating at longer wavelengths.
- Bands 8-10, that were not defined by the September optical workshop, and whose optical design was left to the respective groups in charge of those bands.

Although incomplete, the present report is circulated to provide input to the dewar design.

Method, assumptions

In each of the 10 ALMA bands, an optical train is designed to couple the feed (horn, QO radiator) to the telescope. Its goal is to provide maximum coupling of the feed to a point source in the sky. Mechanical misalignments cause the parameters of the beam illuminating[1] the secondary to deviate from their nominal values. Such deviations can be classified as:

1) Displacement
   a) Along propagation axis
   b) Lateral shift in focal plane (= tilt in aperture plane, = pointing offset on the sky)
   c) Tilt in focal plane (= lateral shift in aperture plane, =loss of aperture efficiency)
2) Distorsion (coupling to higher order modes, if the launched beam is fundamental gaussian)

Effect 1a is not considered here, because even in the nominal design, the various bands are not constrained to have a common focus. Effect 1b is not considered either; lateral shifts of the beam illuminating the secondary would be at most of the order of a few mm, and cause a negligible loss of

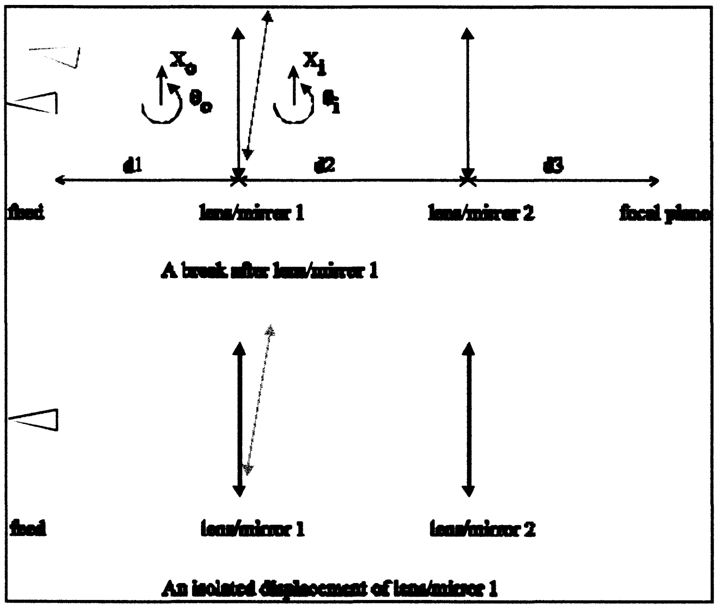
[1] As is common practice, I regard the optics and telescope working as a transmitter.

efficiency; they also cause a pointing offset which needs to be calibrated for each aband anyway. Effect 1c is what is considered in the present draft version, and is believed to be the main driver for alignment tolerances. A calculation of the loss of on-axis efficiency versus aperture plane misalignment was made in version 1.0, concluding to a tolerance of *6mrad* in the focal plane for 1.3% loss of efficiency. Effect 2 was considered in version 1.0, and will be computed for the sake of completeness in the final version.

I use the ABCD equations of geometrical optics to propagate the perturbation of the chief ray; one can show that the same equations apply to the main axis of a gaussian beam. It is assumed that the unperturbed optical path is contained in a plane — which is true for the present ALMA design —, and only perturbations of the chief ray within that plane are computed, which simplifies the work while still providing a valid estimate for tolerances.

Perturbation matrices

Two types of perturbations to perfect alignment are considered: breaks and isolated displacements. The difference between the two is illustrated in the figure below, in the case of an inline lens system.



Note: The displacement (shift+rotation) of an optical element is reckoned in its image (**i**) space in the case of a break, and in its object (**o**) space in the case of a displacement. The center of rotation is defined in either case at the optical center of the element. These two frames are distinct only in the case of mirrors, of course.

In the case of a break, the (X,θ) displacement is propagated through the rest of the system, for example, in the case of a break after lens/mirror 1:

$$P := \mathrm{Space}(d3)\cdot\mathrm{Lens}(f2)\cdot\mathrm{Space}(d2)$$

where Lens(f) is the usual ABCD matrix for a thin lens/mirror, and Space(d) the matrix for free propagation over a distance d.

In the case of an individual displacement of an optical element, we need the perturbation of the image ray as a function of the displacement of that element:

$$\begin{pmatrix} X_i \\ \theta_i \end{pmatrix} := \begin{pmatrix} 1-\cos(\alpha) & 0 & \sin(\alpha) \\ -\frac{1}{f} & 2 & 0 \end{pmatrix} \cdot \begin{bmatrix} X_o \\ \theta_o \\ Z_o \end{bmatrix}$$

in the case of a mirror (the only case where a Z-displacement along the ray path is significant); and:

$$\begin{pmatrix} X_i \\ \theta_i \end{pmatrix} := \begin{pmatrix} 0 & 0 \\ \frac{1}{f} & 0 \end{pmatrix} \cdot \begin{pmatrix} X_o \\ \theta_o \end{pmatrix}$$

in the case of a lens (insensitive to lens tilt in the paraxial approximation); in either case, the perturbation is then propagated through the rest of the system.

The end result is a 2×2 matrix (2×3 for the individual displacement of a mirror) that relates the X,θ displacement of the beam in the focal plane to the misalignment parameters. Only the second line of that matrix (that provides the θ-displacement of the beam) is used, together with the tolerance on the shooting angle $\theta_C = 6\times10^{-3} rd$, to derive the positioning tolerance for the misalignment under consideration:

$$Tol(X) = |P_{2,1}|^{-1} \times \theta_C$$

$$Tol(\theta) = |P_{2,2}|^{-1} \times \theta_C$$

$$Tol(Z) = |P_{2,3}|^{-1} \times \theta_C$$ (only in the case of a mirror's individual displacement)

Results

I give for each band the tolerance (lateral shift X and tilt θ) for breaks and individual displacements. I also give the relevant elements of matrix P, that can be useful in the case of simultaneous and deterministic shift and tilt displacement, where one should combine algebraically the effects on the output beam before estimating the tolerance. Units are mm, radians. The tolerance for a break after the last element is trivial, and is shown only for completeness.

Band		Feed break	Displ element #1	Break after element #1	Displ element #2	Break after element #2
1	$Tol(x)$ $Tol(\theta)$ $[Tol(Z)]$	1.14 ∞	1.14 ∞	∞ 6×10^{-3}	N.A.	N.A.
1	$P_{2,1}$ $P_{2,2}$ $[P_{2,3}]$	-5.3×10^{-3} *0*	5.3×10^{-3} *0*			
2	$Tol(x)$ $Tol(\theta)$ $[Tol(Z)]$	0.58 ∞	0.58 ∞	∞ 6×10^{-3}	N.A.	N.A.
2	$P_{2,1}$ $P_{2,2}$ $[P_{2,3}]$	*-0.010* *0*	*0.010* *0*			

Band		Feed break	Displ element #1	Break after element #1	Displ element #2	Break after element #2
3	$Tol(x)$ $Tol(\theta)$ $[Tol(Z)]$	0.47 0.34	0.27 ∞	0.63 4.3×10^{-3}	0.63 3×10^{-3} ∞	∞ 6×10^{-3}
3	$P_{2,1}$ $P_{2,2}$ $[P_{2,3}]$	*0.013* *-0.017*	*-0.022* *0*	-9.5×10^{-3} *-1.38*	-9.5×10^{-3} *2* *0*	
4	$Tol(x)$ $Tol(\theta)$ $[Tol(Z)]$	0.437 0.749	0.25 ∞	0.58 3.7×10^{-3}	0.58 3.0×10^{-3} ∞	∞ 6×10^{-3}
4	$P_{2,1}$ $P_{2,2}$ $[P_{2,3}]$	*0.014* -8.0×10^{-3}	*-0.024* *0*	*-0.010* *-1.59*	*-0.010* *2.0* *0*	
5	$Tol(x)$ $Tol(\theta)$ $[Tol(Z)]$	0.35 0.052	0.20 2.2×10^{-3} 0.687	0.37 4.3×10^{-3}	0.37 3.0×10^{-3} ∞	∞ 6×10^{-3}
5	$P_{2,1}$ $P_{2,2}$ $[P_{2,3}]$	*0.017* *0.115*	*0.031* *-2.808* -8.7×10^{-3}	*-0.016* *-1.404*	*-0.016* *2* *0*	

Since the optical trains for bands 5, 6, and 7 have nearly identical parameters (from the point of view of geometrical optics), the tolerances for band 5 apply also to bands 6 and 7, and I have not performed separate calculations.

When the optical train comprises a final planar mirror (bands 3 and 4), an angular tolerance of 3×10^{-3} rd applies to that mirror.

If you have read so far, you have certainly noticed that, for bands 3, 4, 5, the matrix element $P_{2,1}$ has the same value for "Break after element 1" as for "Displacement of element 2", instead of having opposite signs. That is because I have been lazy: for the *propagation* of the ray, I used the matrix $Lens(d)$ for a mirror while I should have used $-Lens(d)$. This affects neither the values of the tolerances, nor the *relative* signs of matrix elements in the same box of the table, which are of concern when combining deterministic X, θ displacements in the optical train.

The symbol ∞ should not be understood literally, it just means that the considered displacement produces a pure lateral shift of the beam, and within a few mm, such a shift is negligible. Large angular tolerances are found for the feed, especially in bands 1–4; this is due to the feed being imaged approximately to the aperture plane (in fact, with the numbers supplied by M.Carter, that condition is not always met exactly). There again, that large tolerance should not be interpreted literally: beyond a certain point, the beam from the feed might spill off the finite aperture of the mirrors; but with easily achievable angular tolerances (like 0.01rd) for feed placement, this should not be a concern.