11-9-00

Misalignment tolerance in ALMA optics
(Band 5)

B.L. 04-Nov-00

Use ABCD formalism of geometrical optics to propagate perturbation of chief ray caused by various misalignments:

- "break", i.e. a displacement of all elements from feedhorn up to "breakpoint" relative to the rest of the optical train beyond breakpoint;
- misalignment of single optical element w/r to the rest of the optical train

The optical path is assumed to be in a plane, and only misalignments in that plane are considered

Because absolute focussing accuracy is not required for ALMA optics, displacements of the focus point along the chief ray are not considered. The position of the chief ray is characterized by its position and angle at a given point along the propagation direction.

Propagation of rays (and errors) is conceptually "in emission", from the feed towards the sky

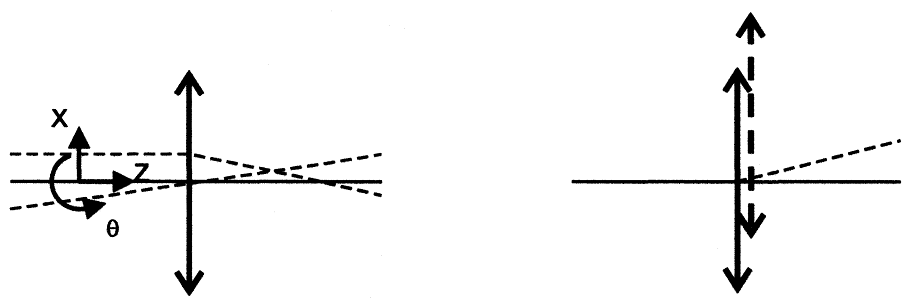


Fig.1. Illustrating the coordinate system and the misalignment of a) object space; b) optical element

Angular tolerance at Cass focus (from April-00 memo on alignment tolerances)

$\mathrm{Tol\theta C} := 0.006 \qquad \mathrm{deg} := \frac{\pi}{180}$

Data for the band under study

$d1 := 88.5 \qquad f1 := 42.3 \qquad d2 := 150 \qquad f2 := 62.4 \qquad d3 := 175 \qquad \alpha 1 := 33\cdot \mathrm{deg} \qquad \alpha 2 := 35.4\cdot \mathrm{deg}$

Set up generic ABCD matrices

$$\mathrm{Space}(d) := \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \qquad \mathrm{Lens}(f) := \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Note: these matrices act on vectors, having as 1st element the X-offset of the chief ray, and as 2nd element, its angular offset θ.

Part A: Sensitivity to breaks

Break at horn, measured by X_O and θ_O; O stands for "object" space; compute propagation matrix

$P := \mathrm{Space}(d3)\cdot \mathrm{Lens}(f2)\cdot \mathrm{Space}(d2)\cdot \mathrm{Lens}(f1)\cdot \mathrm{Space}(d1)$

We inspect only the second row of P, that gives angular offsets of the output beam; lateral offsets are negligible at that point

$P_{1,0} = 0.017$ radians at cass focus per mm at horn

$P_{1,1} = 0.115$ radians at cass focus per radian at horn

Linear and angular tolerances at horn

$\mathrm{TolXh} := \left|\mathrm{Tol\theta C}\cdot\left(P_{1,0}\right)^{-1}\right|$ $\mathrm{TolXh} = 0.35$ $\mathrm{Tol\theta h} := \left|\mathrm{Tol\theta C}\cdot\left(P_{1,1}\right)^{-1}\right|$ $\mathrm{Tol\theta h} = 0.052$

Break after lens/mirror 1

$P := \mathrm{Space}(d3)\cdot \mathrm{Lens}(f2)\cdot \mathrm{Space}(d2)$

$P_{1,0} = -0.016$ $P_{1,1} = -1.404$

$\mathrm{TolX1} := \left|\mathrm{Tol\theta C}\cdot\left(P_{1,0}\right)^{-1}\right|$ $\mathrm{TolX1} = 0.374$ $\mathrm{Tol\theta 1} := \left|\mathrm{Tol\theta C}\cdot\left(P_{1,1}\right)^{-1}\right|$ $\mathrm{Tol\theta 1} = 4.274\cdot 10^{-3}$

Break after lens/mirror 1

$P := \mathrm{Space}(d3)$

$P_{1,0} = 0$ $P_{1,1} = 1$

"Infinite" tolerance on lateral displacement. No intervening optical element means zero sensitivity of illumination angle to lateral displacement of whole receiver optics.

$\mathrm{Tol\theta 1} := \left|\mathrm{Tol\theta C}\cdot\left(P_{1,1}\right)^{-1}\right|$ $\mathrm{Tol\theta 1} = 6\cdot 10^{-3}$

And here we have directly the tolerance on the shooting angle towards the secondary.

Part B: Sensitivity to individual element position

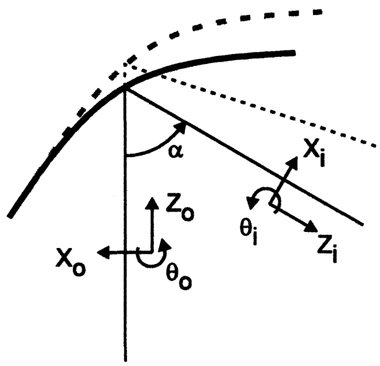
In part A, it did not matter whether the elements were lenses or mirrors. Here it does.
For a mirror, the angular position directly affects the output ray; not for a lens in the paraxial approx;
Fro a mirror, its displacement *along* the input ray direction affects the position of the output ray (except of course for normal incidence); for a lens it affects only the focus, which we do not consider here.

A little algebra gives the following matrix that relates the X, θ perturbations of the output ray reckoned in the image frame), to the X, θ, and Z displacements of the mirror (reckoned in the object frame):

$$S(\alpha, f) := \begin{pmatrix} 1-\cos(\alpha) & 0 & \sin(\alpha) \\ -\frac{1}{f} & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} X_i \\ \theta_i \end{pmatrix} := \begin{pmatrix} 1-\cos(\alpha) & 0 & \sin(\alpha) \\ -\frac{1}{f} & 2 & 0 \end{pmatrix} \cdot \begin{bmatrix} X_o \\ \theta_o \\ Z_o \end{bmatrix}$$

Inert equation, for illustration only. The vector on the rhs represents the mirror displacement; the vector on the lhs is the perturbation of the image ray



Geometry and frames of reference for mirror displacement; Note that the mirror displacement is reckoned in the object frame. "o" stands for object (not output); "i" stands for image (not input).

Displacement of first mirror

$P := \text{Space}(d3)\cdot\text{Lens}(f2)\cdot\text{Space}(d2)\cdot S(\alpha 1, f1)$

Here again, we inspect only the second row of P, which gives us the sensitivity coefficients of the beam direction at the Cass focus w/r to the X, θ, and Z displacements of the mirror under consideration.

$P_{1,0} = 0.031$	$TolXM1 := \left\lvert Tol\theta C\cdot\left(P_{1,0}\right)^{-1}\right\rvert$	$TolXM1 = 0.196$	mm, perp to beam
$P_{1,1} = -2.808$	$Tol\theta M1 := \left\lvert Tol\theta C\cdot\left(P_{1,1}\right)^{-1}\right\rvert$	$Tol\theta M1 = 2.137\cdot 10^{-3}$	radians
$P_{1,2} = -8.728\cdot 10^{-3}$	$TolZM1 := \left\lvert Tol\theta C\cdot\left(P_{1,2}\right)^{-1}\right\rvert$	$TolZM1 = 0.687$	mm, along beam

Displacement of second mirror

$P := \text{Space}(d3)\cdot S(\alpha 2, f2)$

$P_{1,0} = -0.016$	$TolXM2 := \left\lvert Tol\theta C\cdot\left(P_{1,0}\right)^{-1}\right\rvert$	$TolXM2 = 0.374$	mm, perp to beam
$P_{1,1} = 2$	$Tol\theta M2 := \left\lvert Tol\theta C\cdot\left(P_{1,1}\right)^{-1}\right\rvert$	$Tol\theta M2 = 3\cdot 10^{-3}$	radians
$P_{1,2} = 0$	"infinite" tolerance		

And, voilà. Zat's all folks