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LAMA Memo No. 801

Joint Distribution of Atmospheric Transparency and Phase Fluctuations at Chatnantor

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2003-Nov-11

The cumulative joint probability distribution of path delay fluctuations and opacity at the ALMA telecsope site near Chatnantor, Chile has been derived from six years of site test data and is plotted in Figure 1.

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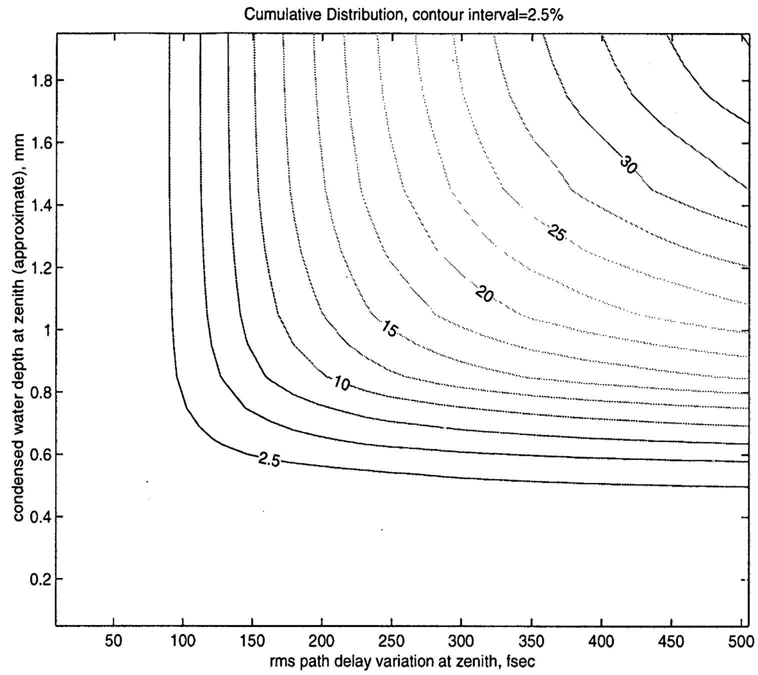

Figure 1: Cumulative joint distribution of path delay fluctuations on a 300 m baseline (abscissa) and condensed water depth (ordinate). Contour labels are in percent. At each point, the plotted value is the fraction of time that conditions are equal to or better than the values on the axes. For example, at each point on the contour labeled "10," the conditions to the left and below that point occupy 10% of all time.

The starting point for this is a data base consisting of the median values in each 1-hour interval of the rms phase at 11.2 GHz and opacity at 225 GHz, measured respectively by the test interferometer observing a geostationary satellite on a 300m baseline and the tipping radiometer. The data base was in turn derived from raw results of the instruments for 10 minute intervals. The hourly median values were then counted in bins of size 0.1 degrees of phase by .004 units of optical depth to obtain a matrix of samples of the joint frequency distribution. Next, the frequency distribution was integrated to obtain the cumulative distribution, and this is plotted as contours in Figure 1. Each value plotted is then the fraction of all time that the phase fluctuation was less than the abscissa and the opacity was less than the ordinate.

The abscissa and ordinate values have been scaled from the measurements so as to express them in an instrument-independent way. Specifically, the abscissa scale is

$$\sigma_t = \sigma_\phi \sqrt{\sin(2\pi 29/360)}/(2\pi\, 11.2\,\mathrm{GHz}) = (9.89 \times 10^{-12}\,\mathrm{sec})\, \sigma_\phi$$

where σ_ϕ is the rms phase measured by the test interferometer. This scales from the measurement elevation of 29d to zenith (assuming that, in accord with theory, the observed fluctuation is proportional to square root of air mass); and converts the phase fluctuation at 11.2 GHz to delay fluctuation. The ordinate scale is

$$w = (24\,\mathrm{mm})\,\tau,$$

where τ is the measured zenith optical depth at 225 GHz. w approximates condensed water depth (also called precipitable water vapor) corresponding to the column density of water vapor vertically above the site.

Path delay fluctuations cause reduction in coherence and consequent loss of sensivitity, while opacity also causes loss of sensitivity from source extinction. Considering these effects alone (and neglecting other causes of sensitivity loss, including coherence reduction from instrumental phase fluctuations, system temperature increase from emission associated with the opacity, and loss of integrating time while calibrating), a sensitivity loss factor can be calculated as

$$\alpha = e^{-\tau_o} e^{-\sigma_o^2/2}$$

where τ_o is the optical depth and σ_o^2 is the variance in corrected interferometer phase, both in the observing direction at the observing frequency. We use

$$\tau_o = A\, r(f)\, \tau$$

where $A = 1/\sin e$ is the air mass at elevation e and $r(f)$ is the ratio of optical depth at frequency f to that at 225 GHz. The uncorrected rms interferometer phase is given by

$$\sigma_u = 2\pi f \sigma_t \sqrt{A}\, (b/300\text{m})^{0.58}$$

where b is the baseline length. The corrected value σ_o is derived assuming fast phase switching or water vapor radiometry, as explained below.

The loss factor α is estimated in Figures 2 and 3 for $e = 45$d and for frequencies of 225 GHz and 875 GHz, near the centers of atmospheric windows.

The opacity at 875 GHz is assumed to be 23.0 times that at 225 GHz [1]. Although the ordinate in Figures 2 and 3 is a rough estimate of w, just as in Figure 1, errors in this estimate cancel when the figures are compared, provided only that the assumed opacity ratio is correct.

In Figure 2, the path delay fluctuation is assumed to be reduced by fast phase switching, based on assumed parameters of

$$\begin{aligned} h_{\text{atm}} &= 500\,\text{m} \\ v_{\text{atm}} &= 12\,\text{m/sec} \\ T_{\text{cycle}} &= 15\,\text{sec} \\ \theta &= 2.5\,\text{deg} \end{aligned}$$

where h_{atm} is the height of the turbulent layer; v_{atm} is the wind speed component parallel to the baseline at height h_{atm}; T_{cycle} is the switching cycle time; and θ is the angular separation of target and calibration sources. These parameter values are considered conservative, so that it will often be possible to do better. This leads to an "effective baseline" (interferometer baseline on which the uncorrected fluctuations would equal the corrected fluctuations on the actual baseline) of

$$b_{\text{eff}} = (v_{\text{atm}} T_{\text{cycle}} + \theta h_{\text{atm}})/2 \approx 100\,\text{m}.$$

The fluctuations measured on the 300m baseline of the test interferometer are then scaled to b_{eff} by assuming that the phase screen may be described by a power-law structure function with an exponent of 1.16, which is the median observed value (so that the square root of the structure function has exponent 0.58). The resulting scaling factor is 0.53.

In Figure 3, the path delay variation is assumed to be reduced by use of water vapor radiometer measurements. The residual error for each antenna is given by the current ALMA specification,

$$\sigma_{\text{corrected}} = \max(.02\sigma_1 + (.01w + 10\,\mu\text{m})/c,\ 59\,\text{fsec}),$$

where w is the condensed water vapor depth along the signal path (scaling as air mass) and c is the speed of light. Here σ_1 is the single-antenna delay fluctuation along the signal path, which we estimate from the test interferometer data on 300m baseline by first extrapolating to 1000m baseline at the target elevation and then dividing by $\sqrt{2}$. The extrapolation assumes a power law structure function with exponent 1.16, as before. The fluctuations are assumed to scale as square root of air mass, also as before. The interferometer phase fluctuation variance used to calculate the coherence is then $\sqrt{2}$ times the antenna phase fluctuation variance at the target observing frequency, since most of the residual is independent of antenna. The residual error floor of 59 fsec $[= (25\,\mu\text{m})/\sqrt{2}/c]$, shown in the last equation, is not part of the ALMA specification but is included because no better correction has yet been demonstrated under any conditions. Figure 4 shows the sensitivity factor at 875 GHz without this floor.

REFERENCE

[1] S. Matsushita et al., "FTS measurements of submilliveter-wave atmospheric opacity at Pampa la Bola II: Supra-Terahertz windows and mdel fitting." *Pub. Astron. Soc. Pacific*, vol 51, pp 605 ff (1999).

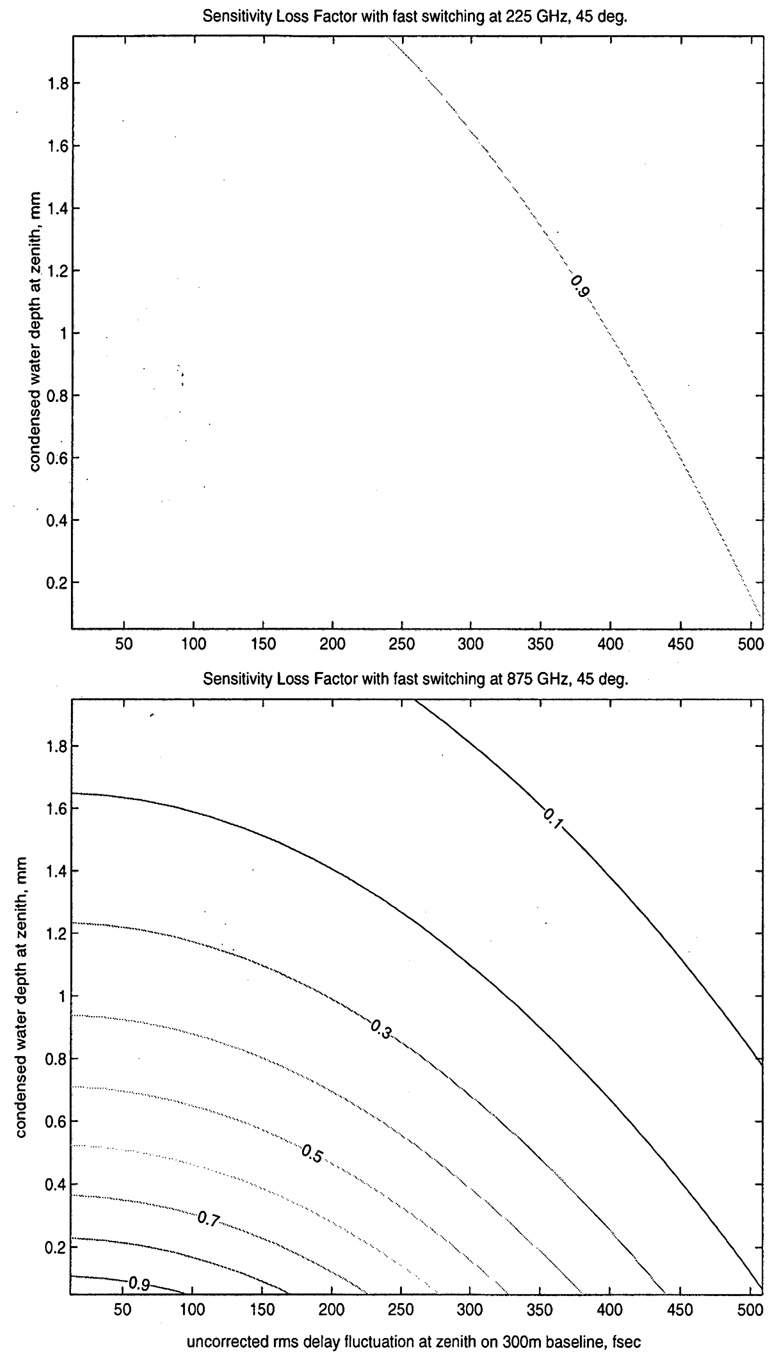

Figure 2: Sensitivity loss factor α at 225 GHz and 875 GHz and 45d elevation due to atmospheric path delay fluctuation (coherence loss) and opacity (extinction), with fast switching at 15 sec cycle time.

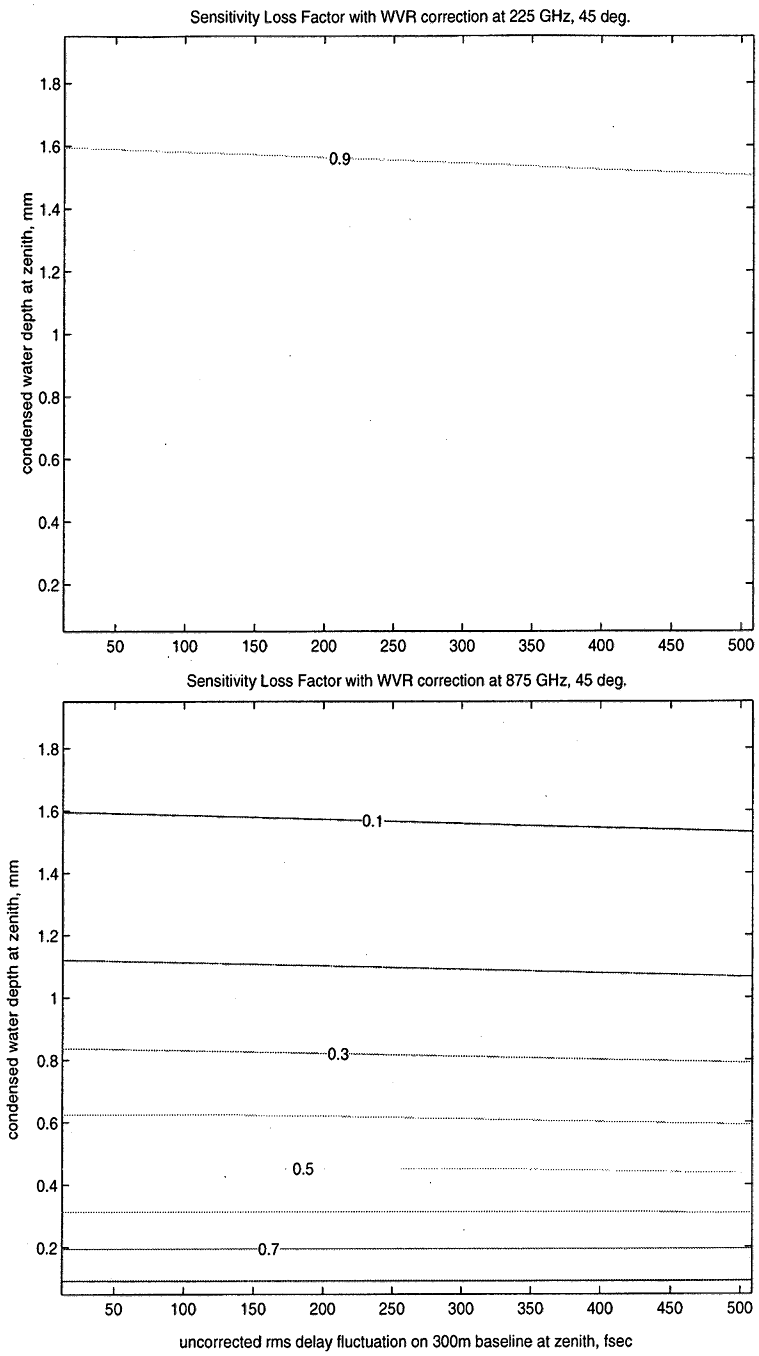

Figure 3: Sensitivity loss factor α at 225 GHz and 875 GHz and 45d elevation due to atmospheric path delay fluctuation (coherence loss) and opacity (extinction), with WVR corrections. The WVR residual error is calculated from the ALMA specification with a minimum of 59 fsec for each antenna.

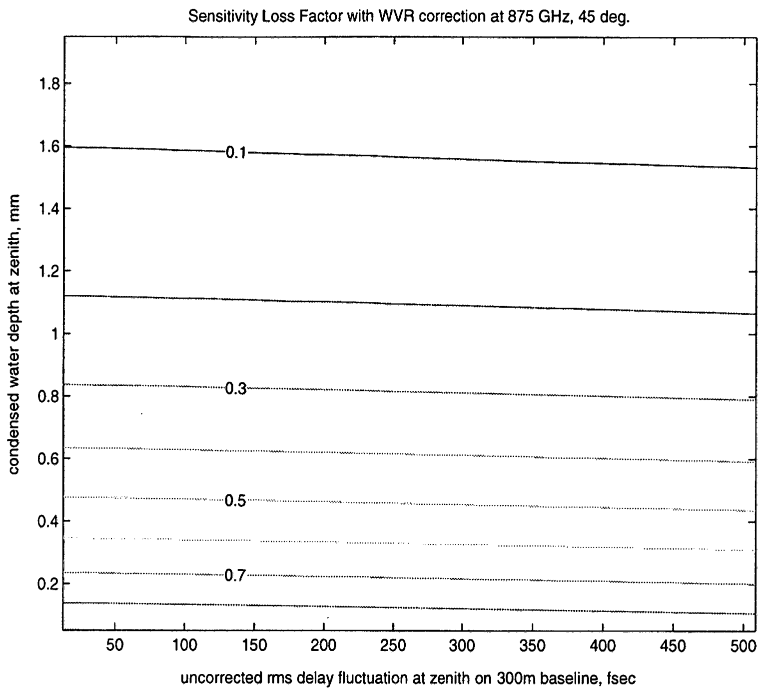

Figure 4: Sensitivity loss factor at 875 GHz with WVR corrections. Same as the lower part of Fig. 3, except that the WVR residual error is calculated from the ALMA specification without a minimum.