MILLIMETER ARRAY
MEMO NO. 20

Millimeter Array Memo 20

Some Initial Parameters of the Proposed mm Array

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I. INTRODUCTION

This memo is the first is a series dealing with the simulation and evaluation of various proposals for a mm array operated as a national faciity by NRAO.

We will assume that there are two major components of the proposed mm array: (1) an array of N antennas, of diameter D, arranged in configurations of diameter B (where B can range from roughly 90 m. to roughly 1 km); and (2) a multi-telescope system consisting of n antennas of diameter d (where 3 <= d <= 5m.) mounted on a structure of size scale b (where b is of the order of 25 m.), where the structure tracks the sources being observed in a manner which avoids the effects of one antenna shadowing another. In order to meet the specifications for a mm array as discussed in the Barrett report (cf. mm array memo #9), with a collecting area of the order of 2500 square meters, N needs to be in the range 15-27 and D needs to be in range of 9 to 13 m.

II. CONFIGURATIONS AND INTEGRATION TIME

The size of an array sets the minimum integration time that is required. This is determined by the criterion that only one "cell" in the u-v plane is "crossed" during a single integration time. For the VLA, with a maximum diameter of 36 km, the normal minimum integration time is 10 sec, with 3.3 sec being available for the worst case situations. Scaling from the VLA worst case situation the four "configurations" of potential interest for the mm array are listed in Table 1, with the associated minimum integraton times:

Table 1

mm Array "Configuration" Sizes and Integration Times

"Configuration"	B or b [m]	Integration Time [sec]
1km	1000	100
300m	300	340
90m	90	1200
M-T	25	4000

Let us designate the potential configurations of the N large antennas with the labels 1km, 300m, and 90m, and the proposed multi-telescope system of n small antennas as the M-T configuration. The minimum allowable integration time is an important parameter since the correlator capacity and the sizes of the data bases subsequently processed are proportional to this integration time for the worst case situation. However, since the mm array will be largely limited by the phase stability of the atmosphere, other effects may set the minimum integration time. For example, if one can develop a system for active correction of phase based upon either independent measurements (e.g. radiometer) or self-calibration techniques, the minimum time scale desireable for these methods may set the minimum integration time for the mm array.

III. OBJECTIVES OF mm ARRAY SIMULATION STUDIES

The objectives of the simulation studies that will be carried out for the mm array can be described as determination of:

1. N, the number of large antennas,
2. D, the diameter of the large antennas,
3. the two dimensional location of the N antennas in each configuration,
4. n, the number of multi-telescope elements,
5. d, the diameter of each multi-telescope antenna, and
6. the location of n antennas on a large support and tracking structure.

The evaluation of these possibilities will involve more than costs, collecting area, and technical feasability. Some of the other criteria are:

1. Beam sidelobe level
2. Desired sensitivity for point source observations
3. Desired surface brightness sensitivity.

While technical discussion of these issues will be largely postponed to other memos, there are a number of major parameters that can be easily determined.

IV. MAP AND BEAM SIZES

The area on the sky, or field of view, that is mapped with aperture synthesis observations is determined by the antenna beam width, while the resolution of each map, which can be described in terms of the synthesized beam size, is determined by the maximum separation between antennas. Assuming that all beam widths are half-intensity beam widths, the antenna beam width is

$$\theta_{ant} = 225''(\lambda_{mm}/D_m) \tag{1}$$

and the synthesized beam width is

$$\theta_{syn} = 0.19''(\lambda_{mm}/B_{km}) \tag{2}$$

for antenna diameters D (in m.) and maximum baselines B (in km). Note that Eqn. 2 is appropriate to the case of uniform weighting in the u-v plane; with natural weighting the sythesized beam widths are larger (in the case of the VLA by about a factor of 1.5).

The size of aperture synthesis maps is mainly determined by the ratio of antenna beam size and synthesized beam size. For the minimum acceptable sampling of two points per synthesized beam width, this means that the sampling map size for uniform weight, M_{un} is given by

$$M_{un} = 2400\ (B_{km}/D_m) \tag{3}$$

which is a result independent of wavelength. Taking a large antenna diameter of D = 10 m.and a small antenna diameter d = 3 m., we can make the following table of map sizes, fields of view, and synthesized beam sizes for the four configurations of Table 1:

Table 2

Fields of View, Synthesized beams, and Map Sizes

Configuration	B or b	θ_{ant}	θ_{syn}	M_{un}	M_{na}	M_{FFT}
1km	1000 m	$22''\lambda_{mm}$	$0.19''\lambda_{mm}$	230	150	128-512
300m	300 m	$22''\lambda_{mm}$	$0.65''\lambda_{mm}$	70	45	64-128
90m	90 m	$22''\lambda_{mm}$	$2.2''\ \lambda_{mm}$	20	13	16-64
M-T	25 m	$75''\lambda_{mm}$	$7.6''\ \lambda_{mm}$	20	13	16-64

In Table 2 we have also listed a sampling map size (M_{na}), for natural weighting analogous to that of the VLA, and the "practical" map sizes that would typically be used because the FFT algorithm requires map sizes that are powers of two.

There are many circumstances under which the map sizes would be different from the sampling map sizes. For map cleaning or map display one frequently wants to obtain 3-4 points per synthesized beam. If there is significant observable emission beyond the antenna half-power points, one may need to map fields of view that are a factor of two or so larger. Further weighting of data in the u-v plane, usually called tapering, in order to improve the signal to noise of larger scale structures, results in larger θ_{syn} and smaller M's. In cases where the sources are much larger that the antenna beams, which certainly will be true for many giant molecular cloud observations, this discussion of map sizes must be replaced by a dicussion of mosaicing fields which are many antenna beam widths in size, with concomitant increases in the size of the mosaic map.

V. SAMPLING AND SIDELOBE LEVELS

One important aspect of the map sizes in Table 1 is that the sampling in the u-v plane for the various mm array configurations is relatively crude by VLA standards. The gridded u-v plane that corresponds to two points per synthesized beam width (which is the gridded u-v plane with the same size as the portion of the u-v plane containing data) can be taken to correspond to M_{un}, that is, 230, 70, 20, and 20 for the 1km, 300, 90m, and M-T configurations. This is an important point in considering potential two-dimensional configurations of antenna elements. This is because the ultimate criteria for sidelobe level and surface brightness signal/noise is the desired occupancy of the cells in the gridded u-v plane, rather than a visual impression of uniform coverage. As discussed by T. Cornwell in mm array memo 18, the rms sidelobe level for a naturally weighted beam, σ_{na}, is given by

$$\sigma_{na}^2 = \sum_{j=1}^{M} \sum_{k=1}^{M} [N_{jk} T_{jk}]^2 / [\sum_{j=1}^{M} \sum_{k=1}^{M} N_{jk} T_{jk}]^2 \qquad (4)$$

where N_{jk} is the number of u-v data points within the j-k cell of an M X M grid and T_{jk} is an optional "tapering" function. For uniform weighting for which $N_{jk} = 1$ for every cell with one or more data points, and zero otherwise, and when $T_{jk} = 1$,

$$\sigma_{un}^2 = 1/N_{occupied} \qquad (5)$$

Thus $N_{occupied}$ is the (effective) number of cells containing at least one data point; because of this there is a theoretical limit to uniform weighting sidelobe levels. Out of each $M_{un}^2/2$ cells (factor of two removes effects of data appearing twice in the Hermitian u-v plane) in a u-v plane gridded for two points per synthesized beam, only a fraction $\pi/4$ (the circle within the square grid), can be occupied, so σ_{un}^2 is always greater than or equal to $(8/\pi)^{1/2}/M_{un} = 1.6/M_{un}$. This gives theoretical lower limits to the sidelobe levels for the 1km, 300m, 90m, and M-T configurations of 0.007, 0.023, 0.08, and 0.08, respectively, for uniformly weight beams and the results of Table 2.

One can evaluate the sidelobe levels for extended sources by making use of the "taper" function, T_{jk}, which we will take to be a Gaussian of the form

$$T(u,v) = \exp[-\alpha\theta^2(u^2 + v^2) \qquad (6)$$

where $\alpha = \pi^2/[4 \ln(2)]$ and θ is the half-power width of a Gaussian in the map plane given by

$$B(x,y) = [\pi/(\alpha\theta^2)] \exp[-\pi^2(x^2 + y^2)] \qquad (7)$$

The effect of a Gaussian taper function is to reduce the effective weight of data outside a particular radius in the u-v plane, which corresponds to a size scale θ in the map plane. Since a particular size scale, θ, in the map plane is effectively "seen" only by data for which $T(u,v) \le 0.5$, use of a taper function like Eqn. (6) means effectively using only the data inside the u-v plane radius

$$r_{uv} = (u^2 + v^2)^{1/2} = [\ln(2)/(\alpha\theta^2)]^{1/2} \qquad (8)$$

thus evaluation of σ_{na} and σ_{un} as a function of taper corresponds to determining the sensitivity of the u-v plane to surface brightness components Q. In the simulations we will discuss in later memos we will express surface brightness in terms of the brightness temperature, Tb. In order to accomplish this we need to relate brightness temperature to the noise level of inidividual data points. Let us take

$$\sigma_v = 0.78/D_m \quad \text{Jy} \qquad (9)$$

(cf. Eqn. 2-48 in "An Introduction to the NRAO VLA") which is the rms noise fluctuation for a single pair of antennas of diameter D_m, aperture efficiency of 0.5, bandwidth of 1 GHz, and a system temperature of 100 K. The surface brightness sensitivity formula given by mm Array Memo 18,

$$S(\theta) = \sigma_v/[(1.1331\ \theta^2)[\sum_{j=1}^{M} \sum_{k=1}^{M} N_{jk}T^2{}_{jk}]^{1/2} \quad \text{Jy/beam} \qquad (10)$$

may be turned into units of brightness temperature by

$$T_b/\lambda^2_{mm} = 3.61 \text{ X } 10^{-10}\ S(\theta) \quad \text{degrees Kelvin} \qquad (11)$$

assuming the units of θ are radians. Our basic evaluation of surface brightness sensitivity will be in terms of tables or plots of T_b/λ^2_{mm} as a function of θ.

In subsequent memos we will evaluate potential configurations in terms of Eqns. (4), (10), (11), and their equivalents when further tapering is applied. However, we can expect that for the 90m and M-T configurations one will need closely packed antennas in order to achieve sufficient collecting area in small diameter configurations, while an approach more like that used with VLA configurations will achieve the best results for the 1km and 300m arrays.

VI. FURTHER ASSUMPTIONS CONCERNING SIMULATION

The parameter space involved in simulating an array can be unwieldy and large unless on makes some restrictive assumptions. Let us discuss those that we will make during the initial stages of simulation of the proposed mm array. The principal assumptions are:

A. Fix the field of view as corresponding to θ_{ant}, the antenna half power beam width.

B. Fix u(max) = v(max) as corresponding to the maximum antenna separation, B_{km}, for all u-v plane operations such as gridding.

C. Do all simulations with the same frequency, 100 GHz, corresponding to 1 mm wavelength.

D. Even though the antennas diameters should not be thought of as fixed, for the purposes of array simulation assume D = 10 m. and d = 3 m.

Taking M X M to be the size of the array to be gridded, assumption B means

$$u_{ns}(max) = B_{cm}/29.9725 = B_{km}/(0.0003) = M\ \Delta u_{ns} \qquad (12)$$

and taking M as specified by Eqn. (3) (for the maximum baseline fixed by Assumption B), Eqn. (12) specifies Δu_{ns} to be

$$\Delta u_{ns} = 1.407\ D_m \qquad (13)$$

or, with u_{max} and D_m as constants during the simulation process,

$$M = u_{max}/(1.407*D_m) \qquad (14)$$

One can regard u_{max} as specified one the configuration of antennas in an array is set, then the value of D_m means M and Δu are specified.

With these assumptions the resulting cell size in the map plane, $\Delta x = \theta_{syn}/2.0$ for uniform weighting, or two points per beam. If natural weighting is applied for a VLA-like antenna distribution these assumption will give about three points per beamwidth. If tapering is applied the number of points per resulting synthesized beam will be $\theta_{taper}/\Delta x >= 2$.