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MILLIMETER ARRAY
MEMO NO. 22

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N/Réf. : DD/PB
V/Réf. :

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Dear Frazer,

Thanks for sending me the MMA Memos. I worked out some cost-diameter curves based on the cost equations in the recent SAO proposal, and also on the cost equations given in the Memo by Sandy Weinreb. You may have done this exercise too, so the conclusions may not be news to you. The analysis follows closely that by Ron Ekers for the Australia Telescope, some years ago.

The first set of cost-diameter curves for constant sensitivity are based on the equations in the SAO proposal. I = Investment, in K$.

Antennas: $I(ant) = 9.3\ n\ D^2/\lambda^{0.7}$ p.115, eq.(39) SAO report.

n = number of antennas
D = dish diameter (meters)
$\lambda(min)$ = minimum wavelength = 20 x r.m.s. surface error.

Receivers: $I(rx) = 300\ (n + 1)$ p.95 & budget in Table 12 p.113, SAO Report.

Correlator: $I(BE) = 200 + 0.6\ J\ n + B\ M\ n^2/J$

B = total bandwidth (GHz)
J = number of filters/antenna
M = total number of spectral channels

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Assumed parameters:

$$B = 1 \text{ GHz}$$
$$M = 1000$$
$$J = 100$$
$$\lambda(\text{min}) = 1 \text{ mm}$$

Result:

$$I(\text{ant}) = 9.3\ n\ D^2$$
$$I(\text{rx}) = 300\ (n + 1)$$
$$I(\text{BE}) = 200 + 60\ n + 10\ n^2 \ .$$

Subtotal:

$$I(\text{tot}) = 500 + (360 + 9.3\ D^2)\ n + 10\ n^2$$

(All investment formulae are in K$)

Conclusions for a mm array:

The curves of investment vs. dish diameter, for a given sensitivity show that the costs rise drastically for small-dish solutions, and reach broad minima for large-dish solutions. The transition from the steep to the flat regime (in cost) shifts to ever-larger dish diameters as the total investment increases.

For example, at a level of 22.5 M$ for antennas, receivers & correlators (MM-Array Memo 1), the flat region of the curves is reached at dish diameters of about 20 meters.

That is, for 22.5 M$, one could afford 15 x 10-m dishes, but one would have about the same sensitivity, for less money, with 7 or 8 x 15-m dishes, with (5 or 6) x 17.5-m dishes, or (4 or 5) x 20-m dishes. Some of these solutions would not only save about 5 M$, without loss in sensitivity, but would also greatly ease the task of equipping the array, and greatly increase the flexibility for adding new receivers as the technology changes with time or as there are demands for new frequencies from observers.

Alternatively, for a fixed sum of 22.5 M$, one could have a 15 x 10-m array, an (8 or 9) x 15-m array, a 7 x 17.5-m array, or a 5 x 20-m array. All of these other arrays would have more sensitivity (and hence more speed to reach a given sensitivity) than a 15 x 10-m array, in addition to the advantages of flexibility noted above.

The crucial difference between the mm Array and the VLA is that a mm array will be sensitivity-limited for most applications, whereas the VLA is not, for many applications. Hence as sensitivity varies as $n D^2$ and since maximum usable baseline (for mapping resolved line sources) varies as $D n^{0.5}$, it seems vital to make the dishes as large as possible. Also note that because the array is sensitivity limited, the usual idea of interferometer speed as being given by the number of baselines, is irrelevant. The only time scale which is relevant is that required to reach a given sensitivity. Hence an array of a few big dishes can actually be faster than a less-sensitive array of many smaller dishes.

It does not seem to me to be a tragedy to reduce the number of dishes as long as one can still produce reasonable maps in one day (6 to 8 dishes might do), and excellent maps in two to four days. Since the results of Cornwell 1983 and Herring show that global fringe fitting improves the sensitivity only as $n^{0.5}$ (and hence the maximum usable baseline only as $n^{0.25}$, it may not hurt to reduce the number of dishes by a factor of two.

I think the field of view argument is also a red herring, for many reasons.

One comes to similar conclusions by using the equations by Sandy Weinreb in MMA Memo N°6: (all investment in K$):

Antennas $\quad I(ant) = 500 + 100\, n + 192\, (n/\lambda)\, (D/6)^{2.7}$

D = diameter in meters of dish with accuracy $\lambda/16$, with λ in mm

n = number of antennas

Front ends: $\quad I(Rx) : 200\, m + 40\, m\, n\, K^{0.5}$

m = number of frequency bands (each 20 % wide)

K = number of beams/antenna, dual polarization

Let m = 4, K = 4, so that coefficient agrees roughly with the front end cost per dish in the SAO proposal. Then:

$$I(Rx) = 320\, n + 800$$

LO system: $I(LO) = 70\ n + 100$

IF transmission:

$$I(IF) = 15\ (b + 2)\ n + 100$$

b = baseline, in km ($b < 3$ km)

Correlator Back end:

$$I(BE) = 200 + 0.6\ J\ n + (2CBM\ n^2)/J$$

With coefficient C = 0.5, as recommended by Weinreb, the formula is the same as in the SAO proposal. As in their proposal, take
B = 1 GHz
M = 1000 channels total
J = 100 channels per antenna

Hence,

$$I(BE) = 200 + 60\ n + 10\ n^2$$

For a minimum wavelength 1 = 1 mm, the total investment for these items is

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$$I(tot) = 1700 + (580 + 15\ b + 1.52\ D^{2.7})\ n + 10\ n^2$$

The curves of investment vs. dish diameter, for a given sensitivity again show that the costs rise drastically for small dish arrays, and reach broad minima in the range 10-17 meters. Again, these minima shift so as to favour the larger dishes, as the level of investment increases. Hence, if one is ever starting an array with hopes of expanding it later on, one should start with large dishes.

For the particular case of the MMA, for 15 x 10-m dishes, Sandy Weinreb's formulae predict a cost of 25 M$ for the items treated here (antennas, front ends, LO system, IF transmission and correlator). However for less money (a saving of 3 M$), one could have about the same sensitivity (and hence the same speed to map to a given detection level) with an array of 7 x 15-m dishes. Besides the saving of 3 M$, the task of equipping the latter array, would be much easier (28 receivers instead of 60 -- or 56 instead of 120, for dual polarization). It would also be easier to add new receivers, and would allow more flexibility in meeting observers' requests for new equipment, and in retro-fitting receivers as the technology improves.

Another way of looking at this is to say that for the same money (25 M$) as required for 15 x 10-m, you could also build 8 x 15-m, which would be more sensitive by a factor of (8.54/7.32) = 1.17, and hence faster (to map to a given detection level) by a factor of 1.36.

Note that although Sandy's formulae favour 15-m dishes (for a level of investment of 25 M$ for the specific items mentioned above), the formulae in the SAO proposal favour even bigger dishes (for millimeter wavelengths, anyway). It all depends on the power law for antenna cost as a function of dish diameter. Sandy gives I(ant) varying as $D^{2.7}$, whereas the empirical data in the SAO proposal give $D^{2.0}$. If the dependence were even shallower, as in the Mitsubishi study for the Australia Telescope, then there would be no minima, and the bigger the dishes, the better.

Based on this sort of analysis, the planners of the Australia Telescope decided (correctly, I think) to change their original concept of a 15 x 15-m array to one of a 6 x 22-m array (for the compact array at Culgoora).

It is probably worth thinking about these things some more, especially as the current plans of ((15-27) x 10-m + 24 x 3-m) imply 200 receivers (single polarization) or 400 receivers (dual polarization).
(MMA Memo N° 14, by J. Moran).

With best regards,

Dennis

cc: L. E. Snyder

D. Downes

P.S: Figs. 1 and 2 are for the cost range indicated in your MMA memo n°1 (about 22.5 M$ for antennas, frontends, LO system IF transmission, and correlator).

If on the other hand, one considers 27 x 10 m (see MMA memo no. 14), then the cost saving is much greater by going to bigger dishes (about 8 M$ -- see Fig. 3).

Figure Captions

Fig. 1 - Cost (for antennas, receivers, correlator) vs. dish diameter, for constant sensitivity (relative to 6 x 6-m). Labels next to points indicate number of dishes of a given diameter. The integration times, aperture efficiencies, bandwidths and system temperatures are assumed to be the same in all cases.

Other assumptions: Cost equations as in SMA Report
Surface accuracy = $\lambda/20$, with λ = 1 mm
1000 total spectral channels
100 filters per antenna
3 frequency bands, with one prototype receiver per band
1 GHz total bandwidth for the correlator

Sensitivity is meant in the sense of signal-to-noise ratio; higher numbers mean better sensitivity.
Relative speed is meant in the sense of 1/(time required to reach a given sensitivity) and hence varies as $(\text{sensitivity})^2$.

Fig. 2 - Cost (for antennas, front ends, L.O. system, I.F. transmission and Correlator) vs. antenna diameter, for constant sensitivity (relative to 6 x 6-m). Labels next to points indicate number of dishes of a given diameter. Integration times, aperture efficiencies, bandwidths and system temperatures are assumed to be the same in all cases.

Other assumptions: Cost equations as in MMA Memo N° 6 by S. Weinreb

Surface r.m.s. = $\lambda/16$, with λ = 1 mm

Maximum baseline = 1 km

Bandwidth = 1 GHz

1000 total spectral channels

100 filters/antenna

4 frequency bands

Sensitivity is meant in the sense of signal/noise ratio; higher values mean better sensitivity.

Relative speed is meant in the sense of 1/(time required to reach a given sensitivity), and hence varies as $(\text{sensitivity})^2$.

Fig. 3 (see P.S. to letter)

Cost vs. antenna diameter for constant sensitivity. Labels next to points indicate number of dishes of a given diameter. Cost equations as in Fig. 2.

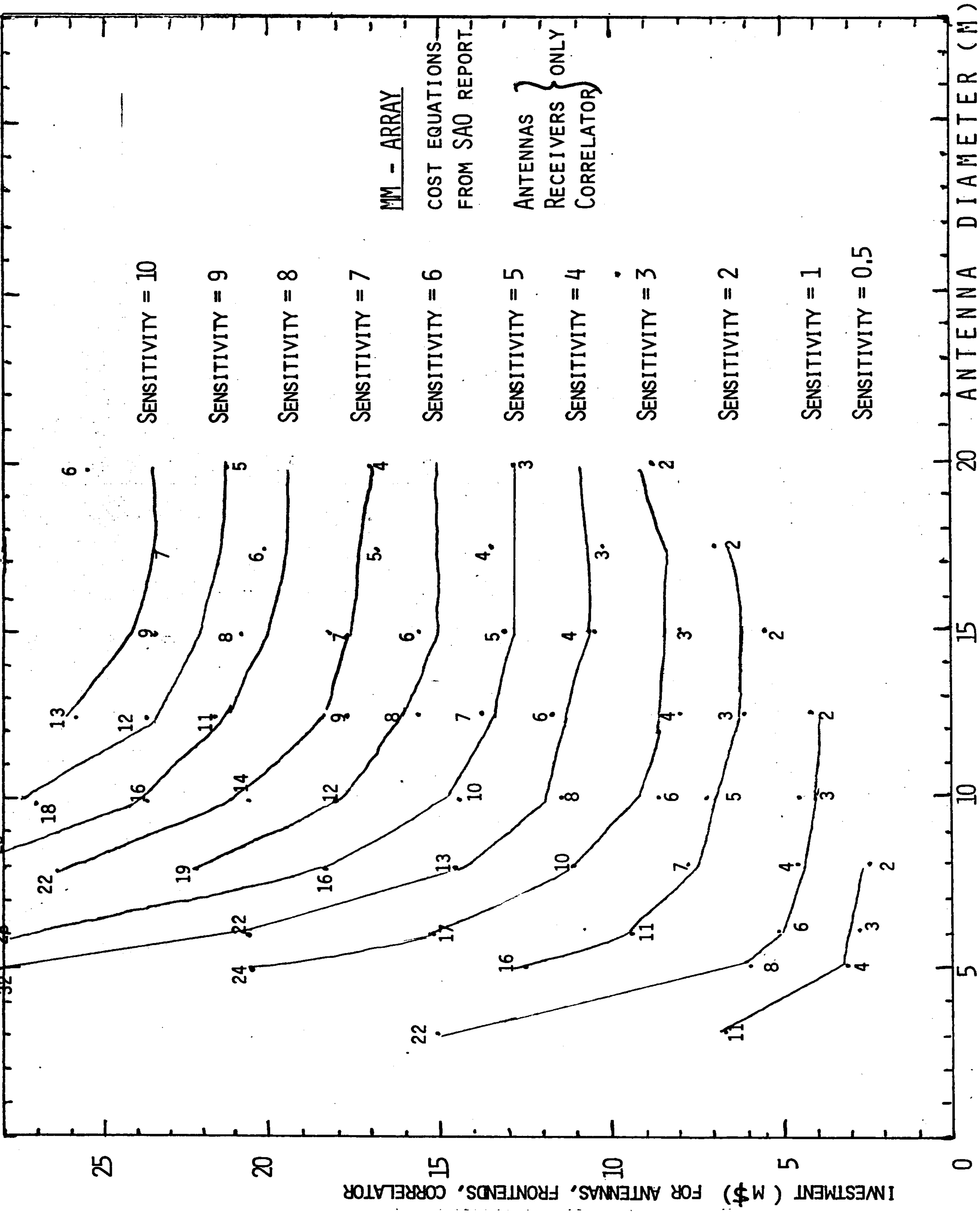

MM - ARRAY
COST EQUATIONS FROM SAO REPORT
ANTENNAS RECEIVERS CORRELATOR ONLY
SENSITIVITY = 10
SENSITIVITY = 9
SENSITIVITY = 8
SENSITIVITY = 7
SENSITIVITY = 6
SENSITIVITY = 5
SENSITIVITY = 4
SENSITIVITY = 3
SENSITIVITY = 2
SENSITIVITY = 1
SENSITIVITY = 0.5
ANTENNA DIAMETER (M)
INVESTMENT (M $) FOR ANTENNAS, FRONTENDS, CORRELATOR
0
5
10
15
20
25

FIG. 1

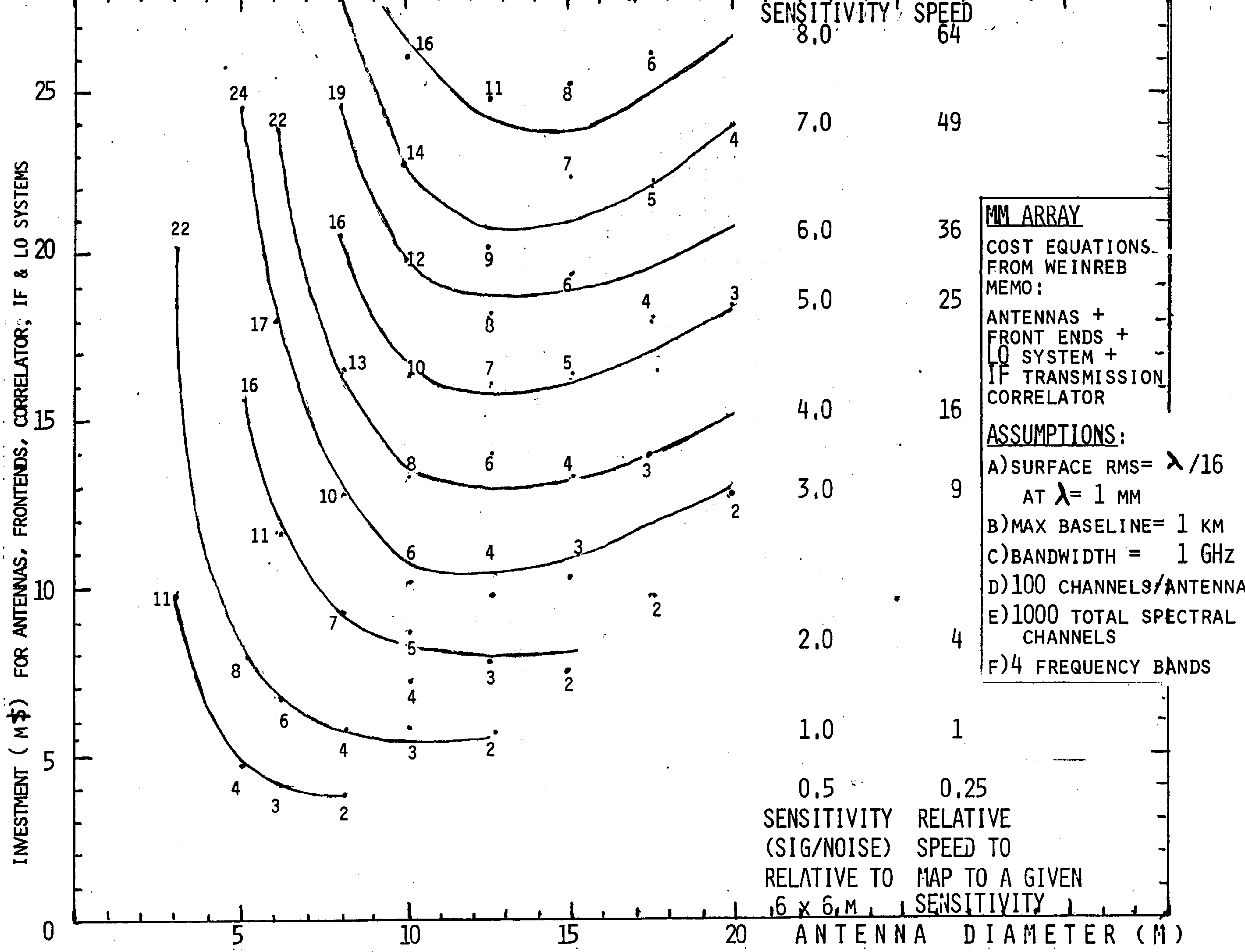
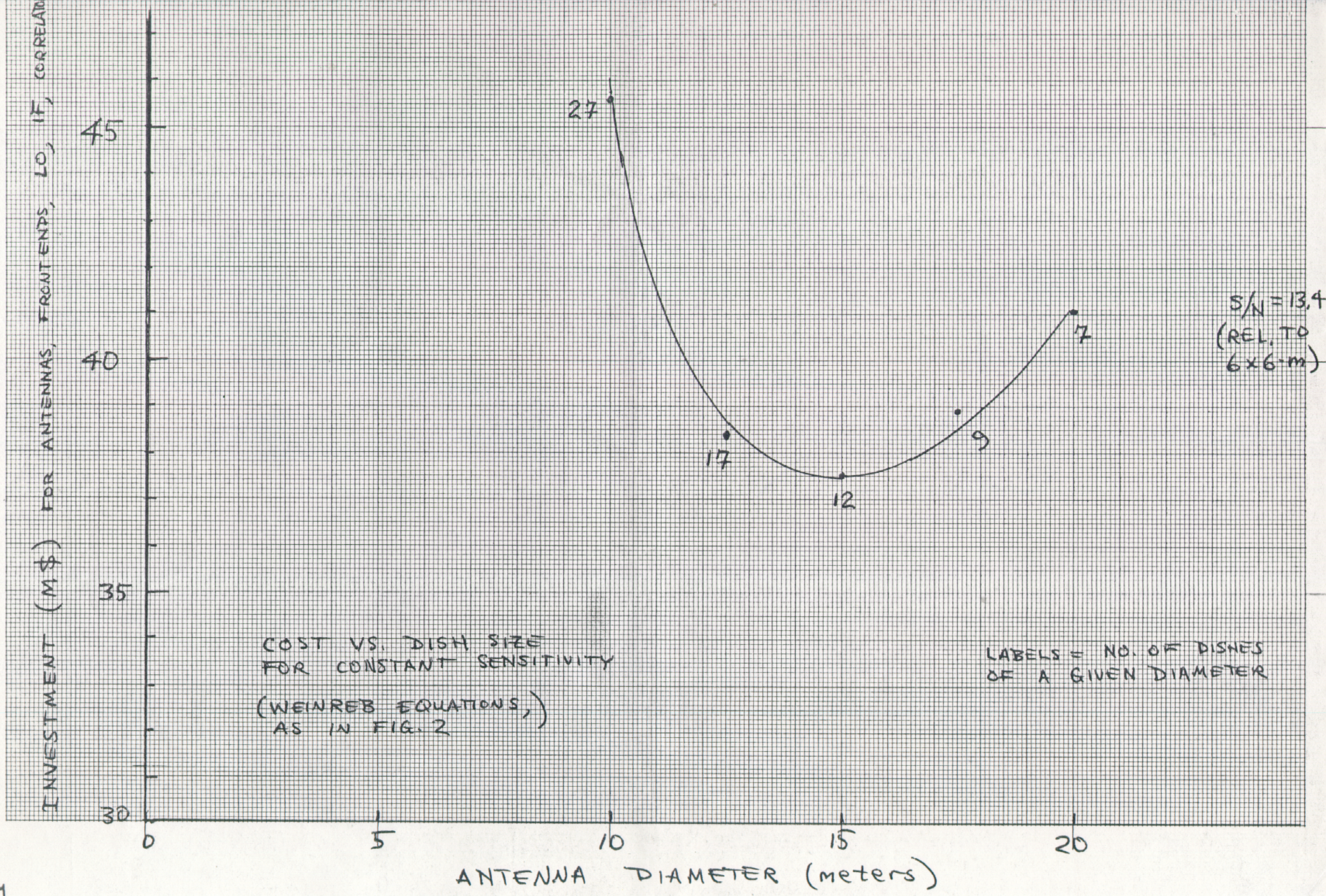

MM ARRAY
COST EQUATIONS FROM WEINREB MEMO:
ANTENNAS + FRONT ENDS + LO SYSTEM + IF TRANSMISSION CORRELATOR
ASSUMPTIONS:
A) SURFACE RMS= λ/16 AT λ= 1 MM
B) MAX BASELINE= 1 KM
C) BANDWIDTH = 1 GHz
D) 100 CHANNELS/ANTENNA
E) 1000 TOTAL SPECTRAL CHANNELS
F) 4 FREQUENCY BANDS
SENSITIVITY SPEED
8.0 64
7.0 49
6.0 36
5.0 25
4.0 16
3.0 9
2.0 4
1.0 1
0.5 0.25
SENSITIVITY (SIG/NOISE) RELATIVE TO 6 x 6 M
RELATIVE SPEED TO MAP TO A GIVEN SENSITIVITY
INVESTMENT (M$) FOR ANTENNAS, FRONTENDS, CORRELATOR, IF & LO SYSTEMS
ANTENNA DIAMETER (M)
0
5
10
15
20
25

FIG. 2



INVESTMENT (M$) FOR ANTENNAS, FRONTENDS, LO, IF, CORRELATO
45
40
35
30
0
5
10
15
20
ANTENNA DIAMETER (meters)
27
17
12
9
7
S/N = 13.4
(REL. TO 6×6-m)
COST VS. DISH SIZE
FOR CONSTANT SENSITIVITY
(WEINREB EQUATIONS, AS IN FIG. 2)
LABELS = NO. OF DISHES OF A GIVEN DIAMETER

FIG. 3