Pointing Errors and the Possibility of Pointing Calibration

M.A. Holdaway, NRAO

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This short document indicates the sorts of pointing errors which are harmful to mosaicing and how much time must be spent on pointing calibration.

As you can see, this note is formatted to be printed in TeX. If you'd prefer a printed hardcopy, it is Millimeter Array Memo #63 which can be obtained by contacting Ms. Meri Stanley (505-835-7310).

1 Characterization of Pointing Errors

What sorts of pointing errors should we be concerned with when mosaicing? Consider an imaging observation in which N pointings are observed each for t_{int}. The entire field is scanned M times, every $t_{scan} = N * t_{int}$. Pointing calibration is performed at intervals of t_{cal}. Assuming the time spent calibrating is negligible, the total observation time is $t_{obs} = M * t_{scan} = M * N * t_{int}$. The pointing errors will have some power spectrum. Pointing changes on time scales shorter than t_{int} will lead to a simple blurring of the beam. Pointing changes on time scales of t_{cal} or longer will be unimportant as they can be removed. Pointing errors which change on time scales less than t_{scan} will be unimportant if M is large.

While standard beam switching deconvolution routines require uniform pointing over some region, the mosaic algorithm can image interferometer or beam switched single dish data which is not observed over a uniform grid. Hence, it is not "good pointing", but "good pointing knowledge" which is important and linear drifts in the pointing can be accounted for.

If each pointing is observed only once (*ie*, M = 1), and pointing calibration is performed before and after the observation, then pointing changes on scales between t_{int} and t_{obs} will be important. If M is large, then pointing changes on time scales between t_{scan} and t_{cal} will be important. If M is large and $t_{cal} \simeq t_{scan}$, then pointing changes of all time scales will be calibrated out or will average out. Since this is only feasible for observations which require many pointings and high SNR on each pointing (large M), or when a particularly bright pointing calibrator is nearby, there will generally be some pointing changes on an intermediate scale which cannot be calibrated out and will not average out. We can imagine $t_{cal} \simeq$ 30-60 minutes and t_{scan} will vary between 5 and 200 minutes, depending on the field of view. In this case, the pointing errors (which may be better characterized as tracking errors) of greatest concern are those that occur on time scales between about 10 and 30 or 60 minutes. This then becomes a constraint to be applied to the permissible spectrum of errors in the antenna tracking and servo design.

One feature of the antenna pointing errors which does not fit into the framework of a power spectrum is the pointing errors which result each time the antenna moves to a different pointing on the sky, producing an additional (hopefully) random pointing error. If the "ambient" pointing error is larger than the "movement induced" pointing error, each new error will serve to randomize the pointing and will lessen the effects of the "ambient" error. If the "ambient" pointing error is small, then the "movement induced" pointing errors will hurt the quality of the mosaiced image. If each pointing is observed many times, even these errors will tend to average out. This is one of several reasons to observe with M large (i.e., repeated coverages of short duration).

While it is not very important for the array to have repeatable pointing across the sky at a very high accuracy, it *is* important that each antenna be able to hold its mean pointing over time scales short compared to t_{cal} and that any large amplitude fluctuations from this mean pointing should have short time scales ($<$ 1 minute).

2 Pointing Calibration with the MMA

Pointing calibration with the MMA will be done interferometrically. Since there are $N-1$ correlations which will include each antenna, an improvement of $\sqrt{(N-1)/2}$ in speed is achieved over treating the array as N independent single dishes. If the calibration source is resolved by the synthesized beam, a source model will need to be generated. The model may then need to be refined as the pointing errors are determined.

To simplify the arguments, consider a calibration source which is unresolved. A simple pointing calibration scheme is to make one observation pointing on the calibrator, two observations offset to $-x_\circ$ and $+x_\circ$ in azimuth, and two observations offset to $-y_\circ$ and $+y_\circ$ in elevation. The central observation is used to determine the antenna gains. If the pointing is bad, the antenna gains can be adjusted iteratively as the pointing errors are determined.

If we wish to achieve $1''$ pointing, we must achieve $0\farcs707$ in azimuth and elevation. Consider the pointing calibration in azimuth alone. The analytic voltage pattern for a uniformly illuminated circular aperture with 1/10 central blockage is nearly linear over a wide range centered about $\lambda/2D$ (see Figure 1):

$$V(x) \simeq 1.25 - \Gamma D\nu|x|$$

where $V(x)$ is the antenna voltage pattern, D is the dish diameter in cm, ν is the observing frequency in Hz, x is the distance from the pointing center in arcseconds, and Γ is a constant of proportionality: $\Gamma = 1.77 \cdot 10^{-16} Hz^{-1} cm^{-1} arcsec^{-1}$. The analysis of pointing calibration is simplified by assuming this linear relationship.

Consider the special case where antenna 1 has a pointing error of Δx and antenna 2 has no pointing error. When observing a calibrator of flux density S, the difference in the powers at the two offset pointings is

$$\begin{aligned} P_- - P_+ &= S\left(V_1(-x_\circ + \Delta x)V_2^*(-x_\circ) - V_1(x_\circ + \Delta x)V_2^*(x_\circ)\right) && (1)\\ &= SV_2^*(x_\circ)\left(V_1(x_\circ - \Delta x) - V_1(x_\circ + \Delta x)\right) && (2)\\ &= SV(x_\circ)2\Gamma D\nu\Delta x. && (3) \end{aligned}$$

The noise in the measurement of $P_- - P_+$ is

$$\sigma = \frac{8kT_{sys}10^{23}}{\epsilon_a \epsilon_q \pi D^2 \sqrt{2\Delta\nu\, t}} Jy \qquad (4)$$

where ϵ_a is the aperture efficiency, ϵ_q is the 3-level quantization efficiency, $\Delta\nu$ is the frequency in Hz, t is the time spent on *one* of the offset pointings, and N is the number of elements in the array. There is an extra factor of $\sqrt{2}$ in the numerator because of the differencing of two noisy signals. Hence, the pointing error is given by

$$\Delta x = \frac{P_- - P_+}{SV(x_\circ)2\Gamma D\,\nu} arcsec. \qquad (5)$$

For $N-1$ correlations with an antenna, the noise in the pointing determination will be

$$\sigma_{pointing} = \frac{4kT_{sys}10^{23}}{\Gamma S V(x_\circ) D^3 \nu \epsilon_a \epsilon_q \pi \sqrt{2\Delta\nu\, t\,(N-1)}} arcsec. \quad (6)$$

The following numbers are appropriate for the MMA operating at 115 GHz:

$$\begin{aligned} \Delta\nu &= 10^9 Hz \\ N &= 39 \\ \epsilon_a &= .8 \\ \epsilon_q &= .82 \\ D &= 800cm \\ T_{sys} &= 150K \\ V(x_\circ) &= .8 \end{aligned}$$

and we find that the error in the pointing calibration is

$$\sigma_{pointing} = \frac{1.73}{S\sqrt{t}} arcsec. \quad (7)$$

We wish to know the pointing to 0.″707. Requiring the error in the pointing determination to be 0.″707/3, we find this can be achieved with a 1 Jy calibrator in 54 seconds per pointing, or about 5 minutes for the entire five point calibration.

While discrepancies from rotationally symmetric primary beams are cumbersome to account for with AZ-EL mount antennas in the *imaging programs*, such discrepancies can easily be dealt with in a *pointing calibration*. Holographically determined beams may be used for each element.

3 Pointing Calibration With a Single Dish

It has been shown that a pointing determination will be less efficient with a single dish than for an array because of the large number of correlations to each antenna. At millimeter wavelengths, beam switching will be required to make radio pointing determinations with a single dish, increasing the required time by a factor of four (a factor of two from the differencing of noisy signals, and another factor of two because the calibrator is only observed for half the time). Hence, the uncertainty of a pointing measurement with a single dish will be

$$\sigma_{pointing} = \frac{4\sqrt{2}kT_{sys}10^{23}}{\Gamma S D^3 \nu \epsilon_a \pi \sqrt{2\Delta\nu\, t}} arcsec \tag{8}$$

Here, Γ is the slope of the *primary beam* rather than the voltage pattern, and is about $2.55 \cdot 10^{-16} Hz^{-1} cm^{-1} arcsec^{-1}$. Since $\sigma_{pointing} \propto D^{-3}$, large single dishes are calibrated more easily than small ones. Let us take as an example the NRAO 12 M antenna. The following numbers are appropriate for the 12-meter operating at 115 GHz:

$$\begin{aligned} \Delta\nu &= 6 \cdot 10^8 Hz \\ \epsilon_a &= .5 \\ D &= 1200cm \\ T_{sys} &= 150K \end{aligned} \tag{9}$$

and we find that the error in the pointing calibration is

$$\sigma_{pointing} = \frac{4.82}{S\sqrt{t}} arcsec. \tag{10}$$

Using the same criteria as for the array, a 1 Jy calibrator should be observed for 420 seconds at each of the five pointings (35 minutes total). For single dish pointing calibration, one will want to use something bright like CTA 21 or 1641+399 for the pointing calibrator to bring the required observing time down.

4 At what frequency should pointing calibration be performed?

For the moment, neglect all arguments about how many pointing calibrators there are on the sky as a function of frequency. From the frequency dependence in Equation 6, we see

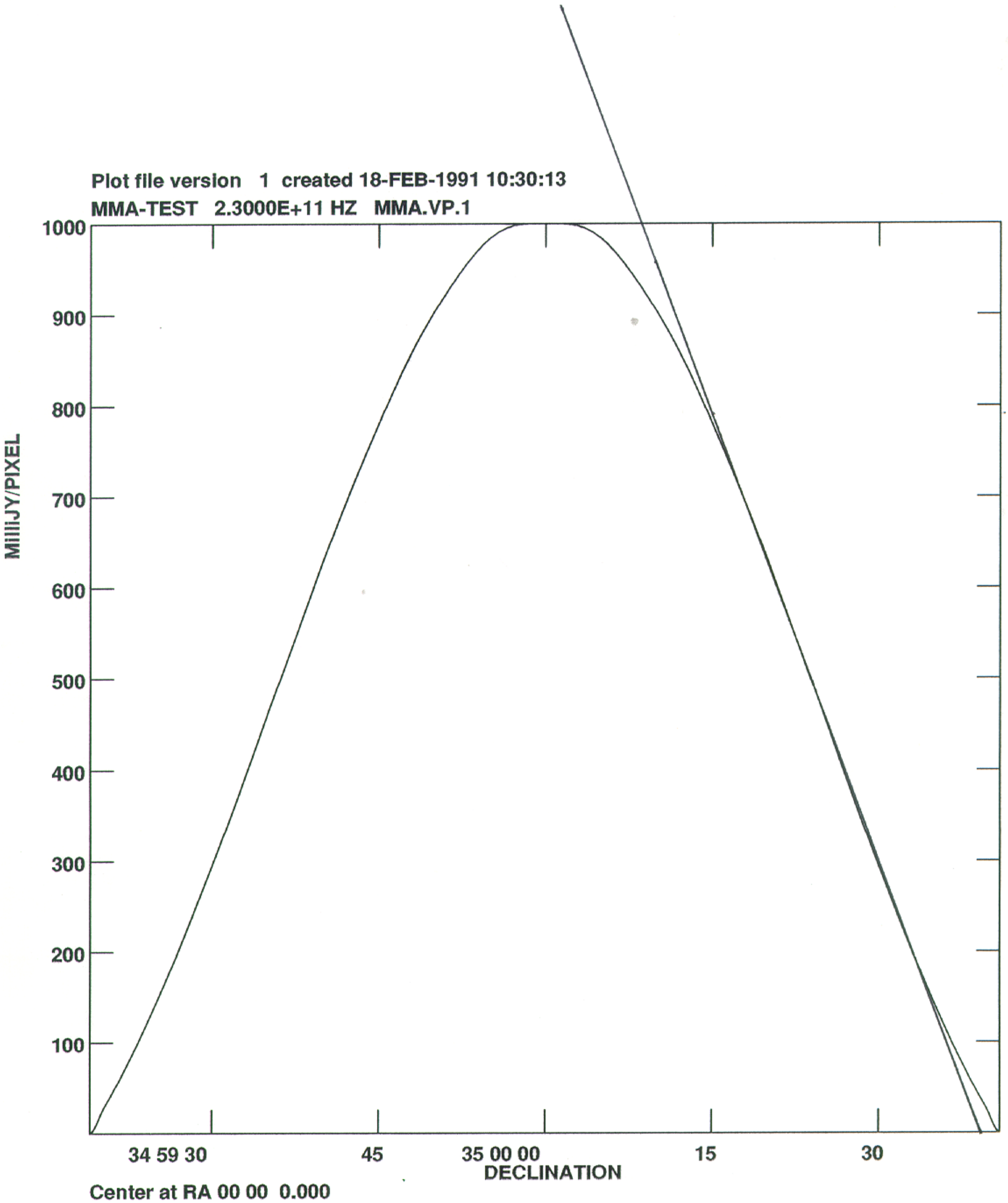
$$\sigma_{pointing} \propto \frac{T_{sys}(\nu)}{\nu \epsilon_a(\nu) S(\nu)}. \tag{11}$$

If the quantity $T_{sys}/\epsilon_a S(\nu)$ increases less than linearly with ν, then it is advantageous to perform pointing calibration at higher frequencies. Restated:

if the receiver performance, atmosphere, and aperture efficiency do not degrade too quickly as the frequency increases, then the increased slope of the primary beam at higher frequencies will enable pointing calibration to be performed more quickly at higher frequencies. This only works if there just happens to be a flat spectrum point source nearby.

On most present single dish millimeter-wave telescopes the system temperature increases with frequency more rapidly than linearly thereby offsetting the gain achieved in pointing with a smaller beam at high frequency. Pointing of these single dish telescopes is better done at lower frequencies.

Voltage Pattern



$$V(x) \simeq 1.25 - \pi D \nu |x|$$

Figure 1