MILLIMETER ARRAY
MEMO NO. 86

# POSSIBILITIES FOR WIDE-ANGLE BEAM-SWITCHING

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26 June, 1992

## 1 Introduction

One possible scheme for calibrating atmospheric phase variations on short time scales is to switch between the source being observed and a near-by calibration source [1]. In essence, it is assumed that the pathlength variations are due to variations in the water vapor in a thin layer in the atmosphere. The temporal variation is due to this layer being blown over the array. If the irregular layer is blowing by at some velocity $v$ then length scales greater than $\ell$ can be calibrated if the calibrator is observed with a period $T \lesssim \ell/v$. This means that calibrators which are further from the source will not give such good phase measurements, but it also means that they do not need too be observed as frequently and the speed of switching may be lower. In fact the speed at which the telescope has to move is dependent on $v$ but not on the distance to the calibrator. If there is a calibrator at an angular distance $\alpha$ from the source being observed and the irregular layer is at a height $h$ then the corresponding length scales are $\ell = h/\alpha$.

Parameters which have been proposed are to switch by up to about 1° from the source to the calibrator with a 2 s integration time. This magnitude of beam switching was considered be difficult to do by moving the antenna so some optical switching scheme, such as a focal plane chopper or nutating secondary, has been proposed. Any such arrangement should not compromise the overall sensitivity of the instrument. In this note we compare some methods and conclude that moving the whole telescope is probably the best solution.

## 2 Comparison With Single-Dish Requirements

Beam-switching has been used for many years on millimeterwave telescopes and several techniques have been devised. Although these methods may serve as a useful basis for developing the required switching mechanism it should be recognized that the use of it proposed for phase calibration differs in several ways from traditional applications. The following points should be considered (for position switching as well as beam switching in the optics)

### *2.1 Beam Throw*

The beam throw is very large compared to traditional beam-switching. Typically throws of a few beamwidths are required, but in this case the throw may be as large as 100 beamwidths. There are several reasons that the throw has been limited. Large throws generally lead to poor beam shapes, reduced aperture efficiency, and increased spillover noise. In addition, when the switching angle is relatively large, the two beam do not overlap in the atmosphere and the desired cancellation of atmospheric fluctuations is not achieved. Deterioration of cancellation can be evident at quite small throws (for example, Jewell [2]). For the proposed mmA antennas the beams become completely separate in the atmosphere after less than 500 m for a 1° throw. This means that the small scale fluctuations in phase will not be calibrated out, but presumably most of the effect should be in the larger scale structure.

### 2.2 *Switching Speeds*

The 2 s integration time is long compared to the rates used in single-dish astronomy, but is commensurate with the larger beam throw required.

### 2.3 *Beam Equality*

Usually single dish switching requires that the "signal" and "reference" beam be very similar in terms of beam shape, efficiency, and spillover. In the case being considered here the two beams will be looking at different sources with different requirements. It will be necessary to maintain optimum beam properties for the source being observed but the conditions may be relaxed for the calibrator. Because of the large offsets being considered between the calibrator and the source, it is probable that the source beam would be on or close to the antenna boresight and the calibrator beam would be considerably offset and the attendant degradations accepted. The frequency of observation need not even be the same in the two positions and it may be that the calibrator is observed at a lower frequency than the source.

### 2.4 *Beam Direction*

Since the calibrators will be at arbitrary positions relative to the source being studied the beam switching system must be able to move in any direction. This is not usually a requirement for single-dishes, though it is be coming more common (e.g. JCMT, SMT, CSO).

### 2.5 *Phase stability*

For the calibration to be effective the phase difference between the source and calibrator beams needs to be stable.

## 3 Focal Plane Switching

The telescope beam may be switched between the source and calibrator by moving the feed or an image of the feed in the focal plane. Generally the aberrations resulting from a displacement of the feed are smaller than those caused by nutating the secondary mirror to give the same beam displacement, but there is a problem with vignetting by the hole in the primary (assuming that the feed is behind the primary vertex). If the telescope effective focal length is $F$ then the displacement $\Delta x$ of the feed required to displace the beam by $\theta$ is

$$\Delta x = \theta F \tag{1}$$

For an antenna with a diameter $D = 8$ m, primary focal length $f = 3.2$ m, and secondary focal ratio $F/D = 12$, a 1° beam displacement requires a feed displacement of 1.68 m. This is impractical since an excessively large hole in the primary would be required. Even if the beam were switched by ±0.5° the vignetting would be excessive and the beam degradation in the source beam would be unacceptable.

Another problem with focal plane switching is that the direction the beam must be switched is variable, which could lead to a rather complicated mechanical arrangement.

## 4 Chopping Secondary

Chopping secondaries have been used on many telescopes because of their versatility, low loss, and wide bandwidth. The primary performance specifications are switching speed, switching angle, and beam degradation. A typical scheme is shown in Figure 1, with the notation used in this document.

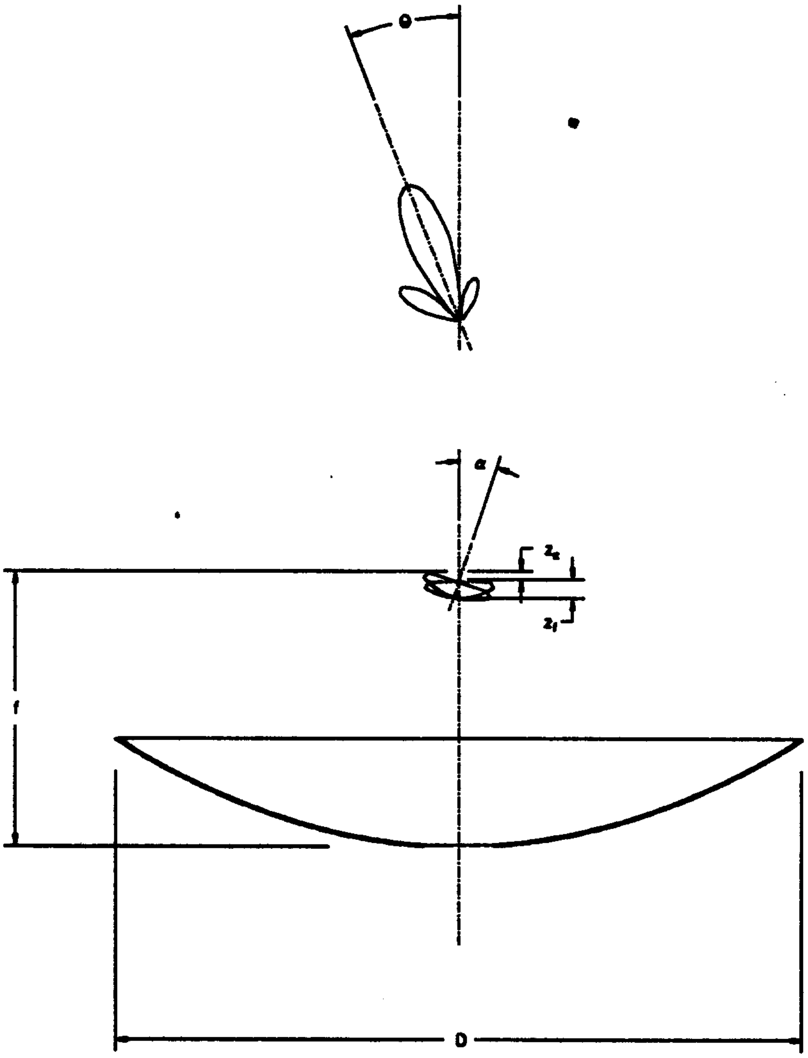

**Figure 1** Geometry of the nutating secondary reflector.

### *4.1 Optical Performance*

When the secondary is tilted (Figure 1) there will be aberrations introduced into the beam, and some spillover round the edge of the primary mirror. From the perspective of the aberrations the optimum rotation center for the secondary is at the prime focus ($z_c = 0$), as discussed in detail by Lamb and Olver [3] and van de Stadt [4]. When this point is used the comatic aberrations are very small and the dominant term is the astigmatism. Astigmatic path length errors increase as the square of the secondary reflector rotation angle so that the gain is reduced in proportion to the fourth power of the beam throw (for small angles). When the center of rotation is about any other point, the errors are mainly comatic. The phase errors are then linear in the rotation angle so that the gain varies as the square of the beam throw. For small beam throws the prime focus center gives the best beam quality, but at larger angles the astigmatism grows too fast. At large angles the higher order aberrations need to be considered also and numerical studies are required to give reasonably accurate results.

Some preliminary studies have been made using ray tracing in the plane of tilt. The cases which were investigated had rotation centers at the prime focus, the secondary vertex, and halfway between these two points. The phase of the aperture field is shown in Figure 2 where a linear term corresponding to the desired beam squint has been removed. For the situation where the prime focus is the rotation center the astigmatic phase error can be seen while the other two cases show the characteristic phase errors for comatic aberrations. It may be seen that the phase error is rather large for the three cases. The pathlength errors were calculated for a number of offset angles and the rms error along the cut through the aperture was calculated — the results are given in Figure 3*a*. As predicted by the first order theory, for small angles the error is quadratic in angle for $z_c = 0$ and linear otherwise, but for large angles the increase is more rapid. It should be noted that the effective pathlength errors will be smaller than these numbers, firstly because the rms over the circular aperture will be less than that along a line through the center, and secondly because the amplitude taper will reduce the weighting at the edges of the dish. However, the numbers are still not negligible in proportion to the wavelengths involved.

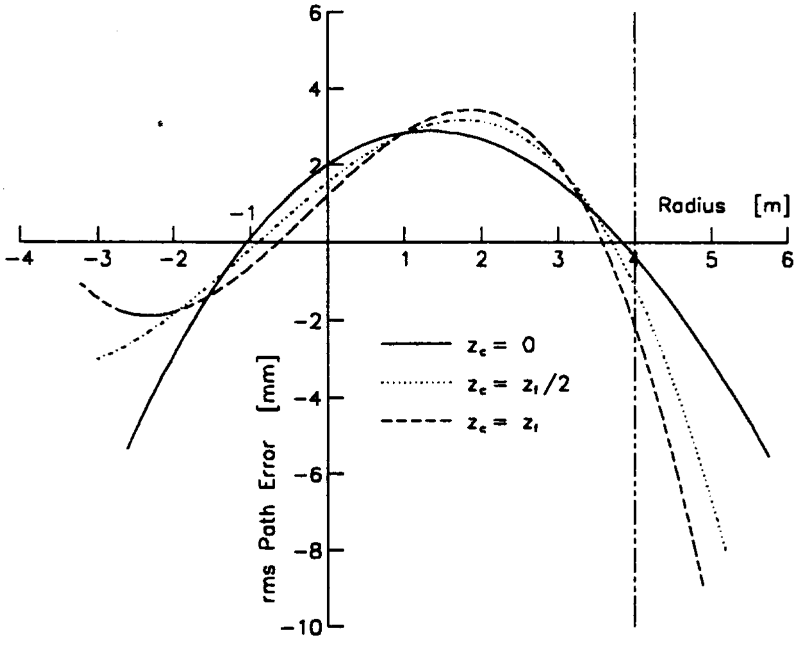

**Figure 2** Phase errors in the plane of tilt induced by rotating secondary about different points. The beam squint is 1°.

The other effect which degrades the system is the displacement of the amplitude illumination pattern on the aperture. This leads to reduced gain and an increase in the spillover noise. Figure 3*a* shows an estimate for the magnitude of the pattern displacement for the different cases. It is worst when the rotation is about the focus and reduced by about a factor of two when the rotation axis passes through the vertex. The shift in the pattern is linearly proportional to the beam squint and the spillover noise and gain degradation will both be proportional to this.

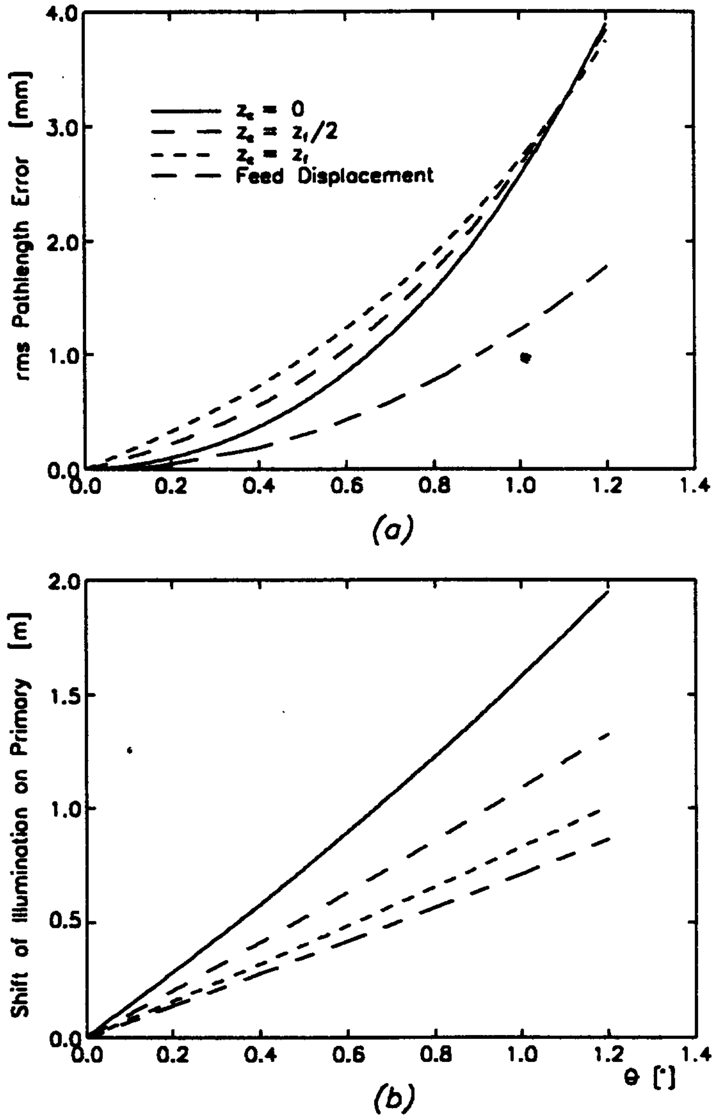

**Figure 3** *(a)* Estimated path length error along principal axis in aperture plane. *(b)* Shift of illumination pattern on aperture.

To get a more accurate estimate of the degradation, some beam pattern calculations were made by constructing an aperture field with a Gaussian amplitude shifted by the appropriate amount, and a phase varying in one direction according to the ray tracing results. The beam is broadened in the direction of tilt as expected, and the gain is reduced by about 4.8 dB at 30 GHz. Some of the gain may be recovered by re-focusing the secondary reflector to give about equal phase errors in the two orthogonal directions. This was calculated by adding a defocusing term to the phase which was adjusted to optimize the gain, but the improvement was only about 0.4 dB because of the higher order aberrations which are present. If asymmetrical beam switching was used the secondary would need to be re-focused between positions, but this could be accomplished by moving the rotation center off axis.

The extra spillover noise was determined by integrating the power which spills past the aperture, and it was found that the additional noise temperature for a reasonable range of parameters is given approximately by

$$T_{spill} = 8.7s^2 + 9.3s + 0.23 \qquad [kelvin] \tag{2}$$

where s is the shift of the amplitude pattern in m. $s$ is proportional to the beam throw as shown in . This could easily add 10 - 50 K to the antenna temperature, a range comparable to the receiver noise temperatures. The reduction in sensitivity would reduce the number of sources detectably in a given time or increase the integration time required on the source by significant amounts.

For comparison, the pathlength error and the shift in the illumination pattern are both shown for a feed displaced in the focal plane, assuming no vignetting. Both are about half the values for nutation of the secondary about the prime focus.

### *4.2 Choice of Parameters*

The antenna parameters are not yet fixed so it may be that there is a better set of parameters than those chosen here. Variation of the rms pathlength error with so the antenna parameters is shown in Figure 4. The results are insensitive to the value of the secondary focal ratio, but depend strongly on the secondary size and the primary focal ratio. The pathlength error is inversely dependent on the secondary size, but a significant improvement could be achieved only with an unrealistically large secondary. Similarly, only marginal improvements can be obtained by increasing the primary focal length within a reasonable range as the path error is also inversely dependent on the primary focal ratio.

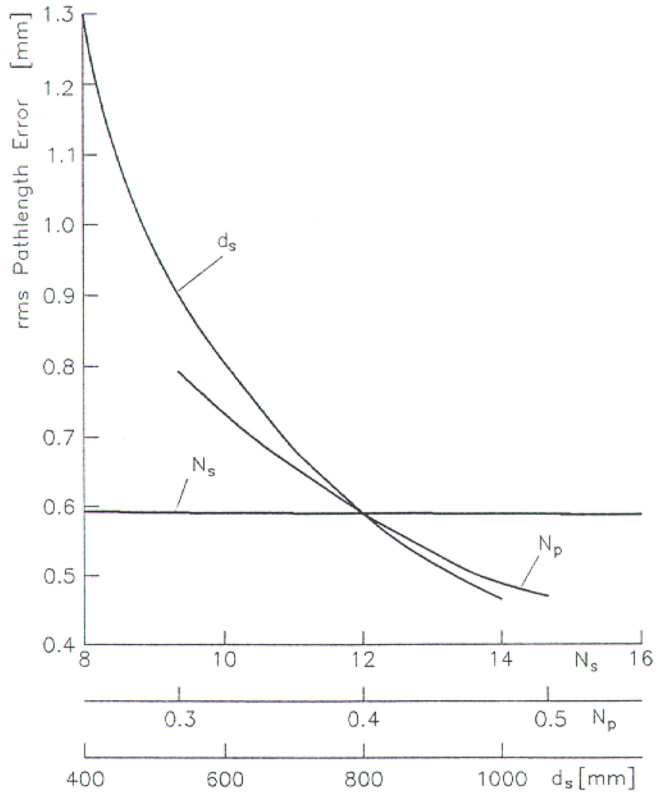

**Figure 4** Pathlength error *vs* $d_s$. $N_s = Mf/D$, and $N_p = f/D$. Nominal: $d_s = 800$ mm, $N_s = 12$, and $N_p = 0.4$.

### *4.3 Mechanical Considerations*

For a constant actuator force the switching time varies as

$$t \propto \sqrt{\alpha} \tag{3}$$

where $\alpha$ is the angle of rotation of the secondary mirror. On the 12-m the switching frequency may be up to 5 Hz and the beamthrow on the sky is ±3 arcmin. In principle, a 1° throw could be achieved with a frequency of about 1.3 Hz for the same system. In practice, the larger beam throw will probably require that the secondary be rotated about the prime focus (rather than a point closer to the secondary vertex) which makes the rotational inertia larger and the required secondary rotation greater for the same beam throw. Additionally, the necessity of two-axis switching will require a more sophisticated mechanism. Use of a relatively light CFRP secondary mirror and a well designed servo system could make this feasible.

## 5 Ritchey-Chretien Design

The Ritchey-Chretien (RC) design [5] has sometimes been mentioned as a way of obtaining a larger field of view, but this is generally not relevant to millimeterwave antennas. The RC telescope uses two reflectors with equal and opposite coma aberrations to produce an acomatic final design. However, a long focal ratio telescope has very little comatic aberration and astigmatism is dominant at millimeter wavelengths. This is because the path errors increase linearly for the coma term and quadratically for the astigmatism term as a function of the off-set distance in the focal plane. There is therefore a distance where the astigmatic term becomes larger than the comatic term, and it turns out that when this occurs the path errors are still negligibly small compared to a wavelength — when the off-set is large enough to produce appreciable phase errors the astigmatism overwhelms the coma.

It is possible that some comparable scheme could be used to correct astigmatism, but two-reflector schemes which have been devised have the secondary focus in an inaccessible place. A multi-reflector solution may be possible. A further complication is that the RC design is to increase the field of view at the secondary focus whereas the requirement here is to be able to nutate the secondary so that the aberrations are different.

## 6 Position Switching

Position switching by moving the whole antenna should not be ruled out. Some simple estimates can be based on existing antennas. If the drive system is capable of exerting a torque $T$ and the rotational inertia of the antenna is $I$ then the antenna can be moved to a new position an angular distance $\theta$ away in a time $t$ given by

$$t = \sqrt{\frac{2\theta I}{T}} \tag{4}$$

For example, for the 45-ft antenna in Green Bank, the motors can deliver a torque of $T = 1.5 \times 10^4$ kg m, and the moment of inertia is about $3.8 \times 10^4$ kg m s$^{-2}$, giving a switching time of 0.42 s. A number for the mmA may be obtained by scaling the values in some appropriate way. For a given geometry, a direct scaling of the antenna diameter leads to a change in the moment of inertia in proportion to $D^5$ and the switching time scales according to $D^{2.5}$, giving a switching time of *0.11 s for a 1° position change*. (Note that for now we have neglected the secant elevation term which actually requires that the antenna should rotate more than this for a 1° movement on the sky.) This scale factor may be optimistic since the higher precision requirements of the mmA antennas may require a proportionately greater amount of material in the backing structure to minimize the effects of wind loading. On the other hand, a higher torque motor could be designed in from the start, and the use of a material with a higher stiffness-to-weight ratio, such as CFRP, could be used. It appears, then, that this would be a feasible technique provided that the necessary specifications were placed on the servo performance from the start. To put things in perspective, a 1° movement of the antenna will move the prime focus position or the edge of the dish by ~100 mm.

The switching time required depends on the off-set to the calibrator. If these are linearly related, a constant velocity movement will give equal performance for all off-sets, but the constant acceleration implied in (4) actually favors the larger off-sets.

An advantage of moving the antenna as opposed to the feed or secondary is that there are no additional spillover noise or aberrations.

## 7 Other Techniques

Beam switching with the secondary or focal plane switching were proposed because of the difficulty of position switching the whole antenna, although it appears that the latter may not be as much of a limitation as originally thought. It may be possible to use a combination of these methods by position switching the antenna, but using a nutating secondary to "track" the source or calibrator at the ends of the switching half-cycles.

Another possibility is to attempt to correct the aberrations in the feed system. Several implementations could be used. If a single frequency was used the secondary could be nutated symmetrically. If the center of rotation is the prime focus, the phase errors are reduced by a factor of four and as the astigmatism errors are the same for the plus and minus rotations so that a single correction is required for both beams. (Coma is asymmetric so that the optics would need to be rotated by 180° between beams if a rotation center other than the prime focus was used.) If two frequencies were used, the source beam could be on-axis and a corrector could be used for the calibrator beam only. Further work, would need to be done to demonstrate that adjustable correcting optics could be designed and constructed.

## 8 Calibration Frequency

The frequency at which the calibrator is observed need not necessarily be the same as that at which the source is observed. When there are aberrations present the gain will be reduced when the phase errors are a significant fraction of a wavelength so that a lower frequency is preferred.

Another important consideration is the optics needed to diplex the two signals. Dichroic mirrors tend to be relatively lossy, limited in bandwidth, and difficult to construct at short wavelengths. A better solution may be to use a Martin-Puplett Diplexer [6]. Even such a diplexer could be difficult to fabricate if the two frequencies are significantly different since the high frequencies require fine wires and the low frequencies require large grids. The spot size at the Cassegrain focus is about

$$s = 4F/D\lambda \tag{5}$$

For the antenna parameters considered here this is about 480 mm at 30 GHz. Although a smaller secondary *f*-ratio would apparently reduce the optics size, this is not so in practice since the size of the diplexer also depends on the beam divergence which becomes greater for smaller *f*-ratios. The minimum diplexer size is around 400 mm and the length of the wires in the grid is almost 700 mm.

Since the beams two frequencies need not be coincident the feeds could be located in different places in the focal plane. This favors higher frequencies since the off-set need not be as large.

## 9 Conclusions

Simple application of traditional focal plane or subreflector chopping techniques of beam switching will not be adequate for large beam throws required for switching to calibration sources. The beam degradation and system temperature increase could be severely limiting factors. If the calibrator is at a lower frequency then the degradation may be acceptable provided that the beam for the source at the observing frequency is not significantly affected. A frequency of 30 GHz may be acceptable, but the design and layout of the optics would be greatly eased if a higher frequency (90 GHz or greater) can be used.

Position switching the antenna seems to be quite feasible, possibly in conjunction with "fine tuning" the pointing in real time with the secondary mirror. This would need to be included in the specifications for the servo drive system. Dual-frequency operation could be done by offsetting the feeds for the source and calibrator by some small distance in the focal plane. We favor moving this as being the simplest solution. Initial calculations indicate that the switching times could be made very short even for a degree. In addition, the unlimited switching angle means that there is less reliance on having a calibrator within a degree or so of the source.

Since the aberrations are strong functions of the beam throw it is important that some realistic information on the density of suitable sources is obtained — small changes could have a large impact on the design. In addition, the switching time could affect the choice of method considerably. To get a better idea of the requirements we need to know more precisely the distribution of possible calibration sources on the sky, and the length of integrations on the calibrator. A comparison with the possible merits of the calibration source scheme with an alternative method using continuous measurements of the atmospheric emission as measure of the pathlength change should be made.

Although this preliminary analysis shows that there are probably no trivial solutions, it seems likely that some method of switching can be implemented. The optimum solution currently appears to be to move the antenna and possibly to have a calibration feed off-set by a suitable amount in the focal plane. The calibration receiver could be the 65-90 GHz band which would be at the observing frequency while that band was used, or tuned to, say, 90 GHz if a higher band was used.

## 10 References

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