Exploring the Clustered Array Concept for the Atacama Array

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Abstract

One mode of collaboration between the LMSA and the MMA is to form a 10 km array using all 90 antennas to gain the sensitivity required for such high resolution observations. This array has been called the "Atacama Array". One possible configuration of the 90 element Atacama Array is to arrange the antennas in clusters of 2-6 antennas, scattered in a ring-like shape. The signals from the antennas in a cluster would be phased up, and the phased cluster signals would be correlated, some by the LMSA correlator and some by the MMA correlator. The advantage of this scheme is that the number of correlations which must be performed is greatly reduced, permitting much wider bandwidth per visibility, and hence, continuum sensitivity which is a factor of 2-5 higher (depending upon the number of clusters) than using the two arrays separately. With 90 antennas, we can afford to trade Fourier plane coverage for bandwidth, but this trick would not work very well for the arrays individually. One antenna from each cluster could also be used for phase calibration. The disadvantages of this configuration include a reduced field of view, a reduced number of baselines resulting in poorer image quality, the possibility of atmospheric phase fluctuations across the cluster dephasing the signal, and greatly increased correlator complexity. The field of view problem can best be overcome by correlating all antennas (with a decrease in

bandwidth), but most astronomical targets of the Atacama Array are very compact objects and would not be troubled by the field of view restrictions. The long track Fourier coverage of an array with 20-30 clusters is still quite good (comparable to the VLA, which produces excellent images), and we expect most astronomical targets of the Atacama Array will be sensitivity limited rather than limited by the quality of the Fourier plane coverage. The atmosphere at Chajnantor is good enough to permit the clusters to phase up at 950 GHz with less than 15% loss in sensitivity about 50% of the time. In fact, the phasing works much better than fast switching phase connection between the clusters. If radiometric phase correction worked better than fast switching, it would also work well enough to improve the phasing of the clusters.

In order to correlate signals from both the LMSA and the MMA in the guise of the Atacama Array, the two instruments must interface cleanly, which will require extensive design work. We feel that this early design work, drawing upon the expertise of both the NRO and the NRAO, should result in strengthening the designs of both instruments. The clustered array should be considered as a viable way to get the full collecting area of the Atacama Array while gaining, rather than losing, continuum bandwidth.

1 Introduction

While there is a fantastic amount of new science which the proposed MMA could perform, there are several interesting problems which are just at the sensitivity threshold of the instrument. This is especially true at the highest resolutions where the small beam results in rather poor brightness sensitivities in feasible integration times. We are currently discussing possibilities for combining the LMSA and MMA antennas into a 90 element, 10 km configuration, called the Atacama Array, to improve the sensitivity at very high resolution. If the LMSA and MMA are used as separate arrays, and signals from the two arrays are not cross correlated, the sensitivity is improved by something like $\sqrt{2}$. But if all the elements are correlated, we gain by a factor of about 2 (assuming equal collecting areas for the two arrays, which is not quite true). Hence, cross correlating all elements results in an array which is twice as fast as the two arrays observing separately. However, since the number of baselines is approximately proportional to the number of antennas squared, cross-correlating all antennas requires a correlator which is about twice as large as the LMSA and MMA correlators combined.

It was pointed out early on that if we only correlated the total intensity visibilities and threw away the polarization, we could fit all (or almost all) baselines into the correlator at full bandwidth. However, the current LMSA and MMA antenna designs rotate differently on the sky, and hence all four polarization correlations must be performed to obtain total intensity.

Another way to overcome this limitation is to reduce the bandwidth of the observations to fit the increased number of baselines into the correlator, but this also loses sensitivity, cancelling the increase in sensitivity gained by cross-correlating the two instruments in the first place. In other words, the continuum sensitivity of the Atacama Array, made by naively combining the MMA and the LMSA, is limited by the correlator and not by the collecting area. Since the brightness sensitivity of the Atacama Array precludes observation of most thermal emission lines, continuum observations will likely dominate the Atacama Array.

We can more than recover this lost bandwidth with a clustered array, in which we configure the antennas into several very tight clusters and correlate the phased signals from each cluster, resulting in fewer baselines to be correlated, which can, depending upon correlator design, be translated into increased bandwidth and increased sensitivity. We explore the advantages and disadvantages of this scheme with respect to sensitivity, phase stability, field of view, and imaging.

2 Sensitivity

The improvement in continuum sensitivity is the primary reason for a clustered array configuration. The image noise from one array will be proportional to

$$\sigma_1 \propto \frac{1}{N_1 D_1^2}, \tag{1}$$

where N_1 is the number of antennas in array 1 and D_1 is the dish diameter of array 1's antennas. If two different arrays observe the same object, and the data is added after correlation, then the image noise will be

$$\sigma_{added} \propto \frac{1}{\sqrt{N_1 D_1^2 + N_2 D_2^2}}, \tag{2}$$

but if all the elements from the two arrays are cross-correlated, the image noise will be

$$\sigma_{corr} \propto \frac{1}{N_1 D_1^2 + N_2 D_2^2}. \tag{3}$$

However, if we are correlator limited, we will need to decrease our bandwidth by a factor of

$$\frac{N_1(N_1-1) + N_2(N_2-1)}{(N_1+N_2)(N_1+N_2-1)} \tag{4}$$

For $N_1 = 50$, $N_2 = 40$, and $D_1 = D_2 = 8$m, the bandwidth reduction is $(780+1125)/4005 \simeq 2$, then

$$\begin{aligned} \sigma_1 &= 1.80 \\ \sigma_2 &= 2.25 \\ \sigma_{add} &= 1.40 \\ \sigma_{corr} &= 1.00 \\ \sigma_{corr,\, bw/2} &= 1.41 \end{aligned} \tag{5}$$

(In this memo, we will normalize all quoted sensitivities to a 90 element array with no change in the bandwidth.) However, if $D_1 = 10$m and $D_2 = 8$m, then

$$\sigma_1 = 1.15$$

$$
\begin{aligned}
\sigma_2 &= 2.25 \\
\sigma_{add} &= 1.03 \\
\sigma_{corr} &= 0.76 \\
\sigma_{corr,\,bw/2} &= 1.07
\end{aligned}
\tag{6}
$$

If both arrays have antennas of the same diameter, cross correlating all antennas is quite attractive if we can keep all of the bandwidth. The $D_1 = 10$m, $D_2 = 8$m case still results in a significant sensitivity improvement. In either case, if we must sacrifice half of our bandwidth in order to perform all of the correlations, we gain nothing over adding the visibilities from the two arrays separately.

2.1 Increased Sensitivity Through Increased Bandwidth

The clustered array concept allows us to get the sensitivity of the full correlated array without losing any bandwidth. In fact, we can *gain* an appreciable amount of bandwidth through using a clustered array since the number of baselines being correlated could be smaller than either correlator normally correlates. The full bandwidth increase might not be realizable due to limitations imposed by the atmospheric windows or by the correlators flexibility, but the potential bandwidth increase is given by the factor

$$
\frac{N_1(N_1-1) + N_2(N_2-1)}{N_c(N_c-1)}, \tag{7}
$$

where N_c is the number of antenna clusters. Table 1 shows the potential bandwidth increase for $N_1 = 50$, $N_2 = 40$, and a variety of possible N_c. So, the cases with $N_c = 45$, 30, and 22 look particularly interesting, potentially increasing the bandwidth by almost an order of magnitude.

N_c Number of clusters	N_{pc} Number per cluster	Bandwidth Increase	Noise ($D_1 = D_2$)	Noise ($D_1 = 10$,$D_2 = 8$)
90	1	0.50	1.41	1.07
45	2	2.03	0.70	0.53
30	3	4.61	0.46	0.35
22	4.1	8.68	0.34	0.26
18	5	13.1	0.27	0.21
15	6	19.1	0.23	0.17

Table 1: Potential increase in bandwidth and image noise as a function of number of clusters in the combined 90 element array.

2.2 Sensitivity with Paired Antenna Calibration

A natural observing mode which follows from the clustered array is paired antenna calibration (Holdaway, 1992; Asaki *et al.*, 1996). One of the antennas in each cluster can be used to observe a calibrator source while the other antenna(s) observe the target source, and the calibrator phase is applied to the target source. We do not perform any detailed analysis of paired antenna calibration in a 90 element clustered array, but it works about as well as fast switching, and can be used even if the LMSA or MMA antennas cannot switch quickly. This mode will require the correlation of two subarrays of the same size, or twice as many correlations as the regular clustered array. The optimal division of the bandwidth between the target array and the calibration array is not clear, but if we assume equal bandwidths for the two arrays, the bandwidth increases in Table 1 should be reduced by 2.0 and the noises increased by $\sqrt{2}$.

3 Problems with the Clustered Array Approach

There are many arguments against the clustered array approach, including complications due to the cluster phasing hardware and the ultra-flexible correlator, the stability of the atmosphere over each cluster, limitations of the field of view, and decrease in the maximum potential dynamic range achievable due to fewer baselines. We speak to each of these concerns, except for the hardware complications, which may very well prevent this idea from becoming part of the MMA design.

3.1 Phasing Up the Cluster

It has been argued that the clustered array is potentially risky because it may be difficult to keep each cluster in phase, and if the cluster antennas are out of phase, they will decorrelate and all baselines to the affected cluster will be down by $\sqrt{N_{pc}}$ in amplitude (the effective collecting area of a randomly phased array is the collecting area of a single antenna). There will be electronic, atmospheric, and antenna deformation contributions to the relative antenna phases. We assume the electronics will hold its phase over time scales of at least 10 minutes, and the electronic phasing can be accomplished by observing a bright calibrator for a fraction of a minute. Since the atmospheric phase errors are a function of the baseline length, we will need to investigate cluster sizes.

3.1.1 Cluster Sizes

To eliminate shadowing above elevation angle θ for all azimuths, antennas must be separated by a baseline of $b_{min} = D/\sin(\theta)$, where D is the antenna dish diameter. To observe down to 30 degrees elevation in all directions requires $2D$ minimum antenna separations, down to 20 degrees requires $2.9D$ separations. It is beneficial to place the antennas as close as possible to maximize the field of view and to minimize the phase errors, but we must be able to see a reasonable fraction of the sky. While 30 degrees elevation seems too high for an elevation limit, this limit applies only in certain directions, and we can certainly arrange the cluster antennas so they are not shadowed until a lower elevation when looking in an important direction such as north. Hence, we will adopt $2D$ as the minimum distance between antennas in a cluster for the calculation here.

The number of antennas per cluster N_{pc} and the geometry of the cluster will also affect cluster size. For this calculation, we adopt the cluster geometries given in Figure 1, with $2D$ as the minimum spacing. From these geometries, the maximum cluster size (from tip to tip of the extreme antennas) and the effective cluster size (ie, the decorrelation of the cluster is the same as if all baselines in it were of this length) are given in Table 2. Since these cluster sizes are generally smaller than the effective calibration baseline length for fast switching, whenever the atmosphere prevents the cluster from phasing up, phase calibration of the whole array will probably be impossible.

N_c Number of clusters	N_{pc} Number per cluster	Maximum Cluster Size	Maximum Cluster Size [m]	Effective Cluster Size [m]
90	1	D	8	8
45	2	$b_{min} + D$	24	16
30	3	$b_{min} + D$	24	16
22	4	$\sqrt{2}b_{min} + D$	31	18
18	5	$2b_{min} + D$	40	22
15	6	$\sim 2b_{min} + D$	40	23

Table 2: The maximum size of clusters from dish tip to dish tip (assuming $D = 8$ m and $b_{min} = 2D$), and the weighted cluster size.

3.1.2 Decorrelation in a Clustered Array

The decorrelation in the voltage produced by summing the signals in a given cluster subject to antenna based phase errors ϕ_A will be $e^{-\sigma^2_{\phi_A}/2}$, and the decorrelation in the visibilities formed from two decorrelated clusters will be $e^{-\sigma^2_{\phi_A}}$, or $e^{-\sigma^2_{\phi_B}/2}$ for *per baseline* phase errors such as atmospheric phase errors. There are several sources of these phase errors, including atmosphere, path length changes due to thermal or wind induced changes in the shape or position of the antenna, and electronic phase drifts, mainly due to thermal drifts in the electronics. It might seem that the two step decorrelation in a clustered array, which first adds several antenna voltages, and then correlates different cluster voltages with wandering phase, might be different from the decorrelation in a generic interferometer. However, after some thought and numerical simulations of the decorrelation in the clustered array scenario, we are confident that the decorrelation is not fundamentally different from the case of a generic interferometer. The only difference is that decorrelation suffered prior to multiplying the summed cluster signals is irreversible, while a generic interferometer could conceivably recover from this decorrelation if data were taken with very short integration times and self-calibration were possible.

First of all, self-calibration will probably seldom be able to correct for these phase fluctuations on time scales of a few seconds; sources bright enough to permit this will seldom be targets of the Atacama array. Hence, the clustered array is not at a disadvantage with respect to a generic interferometer. Second, we will also show that the decorrelation is not too extreme by examining the magnitude of the various phase errors expected:

- **Phase errors due to antenna deformations** will not only occur slowly enough to permit their calibration by phasing up the array every 10 minutes, but since the antennas in a given cluster will be subject to nearly identical thermal and wind environments, antenna phase errors will be nearly identical for all antennas in a cluster. Antennas in different clusters will have different environments, and hence different phases, and will decorrelate when the final cross-correlation is performed among cluster voltages, but this is the same situation as for a generic interferometer.

- **Electronic phase errors** will be primarilly due to temperature changes in the electronics, part of which will be correlated, and part of which will be uncorrelated between antennas. However, any variations on time scales longer than 10 minutes will be removed as each cluster is phased up every 10 minutes. Our electronic phase stability specifications are 7 microns (best) to 16 microns (median) per antenna, (Woody *et al.*). At 850 GHz, 7 microns will reduce the visibility amplitude to 0.98%, while 16 microns will reduce the visibility amplitude to 0.92%. The prime conditions for high frequency observing with the Atacama array will occur at night, and the relatively constant temperatures may result in somewhat better electronic stability. Hence, we adopt the value of 12 microns rms for the electronic phase specification for high frequency observing in the Atacama array. At lower frequencies, much larger electronic and atmospheric phase variations can be tolerated, and day time observations would not be a problem.

- **Atmospheric phase errors** on the effective baselines quoted in Table 2 will largely be benign on the Chile site. The site test interferometer (Radford *et al.*, 1996) provides the distribution of phase structure functions for the Chajnantor site in Chile (Holdaway *et al.*, 1995), which can be used to determine the atmospheric contribution to the cluster voltage decorrelation. It is possible that the atmospheric phase errors could be reduced by highly effective water vapor monitoring instrumentation on each antenna, but we do not need to rely upon this.

Following the above reasoning, the decorrelation in adding the antenna voltages in each cluster will result in power sensitivity which is lowered by a factor of

$$\eta_{cluster} = e^{-(2\pi\sigma_{elec}/\lambda_{\mu})^2} e^{-(2\pi\sigma_{\phi}(\rho_{eff})/\lambda_{\mu})^2/2}, \tag{8}$$

where σ_{elec} is the rms path length due to fluctuations in the electronics (assumed to be 12 microns), and $\sigma_{\phi}(\rho_{eff})$ is the rms atmospheric path length error predicted by the phase structure function on a baseline of ρ_{eff}, the effective baseline length of the cluster. Now we ask the question: how often will $\eta_{cluster}$ be greater than 0.85 (ie, when will the electronic and atmospheric phase result in an irreversible cluster decorrelation sensitivity loss of less than 15%)? We answer this question using one year of site testing data to evaluate what the phase errors are on the effective cluster baselines for several different observing frequencies. Figure 2 indicates that the atmosphere will often permit the array to phase up with very little sensitivity loss, even at the highest frequencies. In fact, the atmosphere will cause more problems with connecting the phases of the different clusters via fast switching, say, than phasing up the clusters.

3.1.3 Phasing the Electronics of Each Cluster

As addressed above, it may not be possible, nor is it necessary, to track the atmospheric phase across a cluster, as these phase errors will be very fast and usually small. Neither is it necessary to track the antenna deformation phase in a cluster, as each antenna in a cluster will suffer very similar deformations. However, we must be able to track the electronic phase of the antennas, at least on intermediate time scales like 10 minutes. Since we are not concerned with the other phase contributions, we are not constrained to phase up on a source which is very close to the target source. We can easily afford the luxury of phasing up the electronics on a bright 1 Jy calibrator at 90 GHz which might typically be 15 degrees away (15 s round trip slew time). Then, even for the worst case of 850 GHz observations which requires a very accurate phase determination at 90 GHz, a mini-correlator with 2 GHz bandwidth could phase a three element cluster to 1% decorrelation in about 20 s. The entire phasing operation could take place in about 40 s. If phasing were performed every 10 minutes, the lost time amounts to a 3% increase in noise. This is a very small penalty in time and sensitivity.

3.2 Limitations in the Field of View

When we phase up each cluster and add the signals, we are synthesizing an aperture the size of the cluster, and this aperture will have a smaller primary beam on the sky. Another way to look at the problem is that we are taking data which belongs in several different (u, v) cells and placing the data in the same cell. Hence, it does no good to have a cell size finer than the cluster size, again limiting our field of view in the image plane. In order to recover the full field of view, we must correlate all antennas, which eliminates the bandwidth advantage of the clustered array, or separately observe each sub-field phasing the clusters to point to that sub-field, which results in an even larger loss in sensitivity if the full primary beam must be observed.

Figure 3 illustrates how the field of view decreases as the number of antennas per cluster increases. However the key targets of the Atacama Array will be very small objects such as protostars. It is quite possible that we could live with a limited field of view.

3.3 Limitations in the Maximum Potential Dynamic Range

In investigating the limitations in image quality due to the reduced number of baselines in the clustered array, it is tempting to make arguments about the number of filled cells in the Fourier plane. Since the Fourier plane cell size increases as the cluster size increases, but the maximum baseline remains constant, the number of cells to fill decreases even as the number of baselines decreases. However, such arguments are artificial: we could always get a much higher number of filled cells from an unclustered array simply by restricting the field of view (ie, using the same large cells in the Fourier plane) and taking advantage of the superior number of baselines. Also, the fraction of filled cells in the Fourier plane may be an attractive relative measure to help compare different arrays, but we cannot at this time translate the fraction of filled cells into an image quality. Hence, we must resort to image simulations.

In the absence of a better working model image, we have used the same old MMA simulation model source, generated from an Hα image of an HII region in M31. This source is much more complicated than most potential targets of the Atacama array. The resolution of the uniformly weighted 10 km Atacama Array is about 14 mas at 300 GHz. We have chosen to use a model cell size of 7 mas, and have tapered to a resolution of 18 mas to reduce the size of the inner sidelobes and improve image quality. The model source is about 630 mas across, or about 35 resolution elements in one dimension.

We have simulated 2 hour observations with 60 s integrations for arrays of between 15 and 90 clusters. This object is large enough that it requires some short spacing information, which we add in the form of 10 minutes of simulated observations from a 40 element 3 km array. No noise or other errors were added to the data as we would like to determine the dynamic range limitations caused by the incomplete Fourier plane coverage. The arrays were made by placing the clusters psuedo-randomly along a ring, but no effort was made to optimize the arrays for good Fourier plane coverage or low sidelobes. The uniformly weighted data (appropriate weighting for high dynamic range objects) were gridded and FFT'ed, and the resulting dirty maps were Clark cleaned with a reasonably tight clean box. Atacama Arrays with 15 and 18 clusters required 3E+4 clean components before the image quality saturated, arrays with 22 and 30 clusters required 1E+5 clean components before the image quality saturated, and arrays with 45 and 90 clusters required about 2E+5 clean components. When the simulated maps generated from a configuration with fewer antenna clusters were cleaned to the same number of clean components as required by simulated maps made with a configuration with more clusters, image quality began to decrease due to over-cleaning, so we have carefully monitored the cleaning processes and stopped the clean when the dynamic range saturates.

The quality of the resulting images is judged by the image fidelity (which gauges the level of errors on source) and the dynamic range (which gauges the level of errors scattered off source). These quantities are plotted against the number of clusters on log-log plots in Figure 4.

Now, each astronomical object will be imaged differently by a given array, so our simulations are only a rough benchmark. However, our simulation object is much more complex than most potential targets of the Atacama Array. At this point we need to estimate the maximum signal to noise we expect to get on our targets based on thermal noise. If we can only obtain 200:1

dynamic range on our brightest target due to thermal noise limitations, it certainly doesn't make sense to keep an array that is capable of a deconvolution limited dynamic range of 50,000:1. Even if we opted for the 22 cluster array, we appear to be able to make 10,000:1 dynamic range images of very complex sources, more image quality than we could ever take advantage of.

4 Discussion

We have explored some of the advantages and disadvantages of the clustered array concept for the Atacama Array. This array design stands to make a very large gain in continuum sensitivity over using the MMA or the LMSA in a 10 km configuration. The atmosphere will not prevent the phasing of the clusters unless it is so bad that fast switching phase calibration will not work anyway. In the event that a radiometric phase correction technique works for phase calibration under these conditions, it will also work to keep the clusters in phase. In fact, the phasing of the clusters appears to be little enough of a problem that we should consider placing the antennas in each cluster further apart to allow observations to lower elevation angles and to prevent solar shadowing and wind blockage in the cluser, which would promote a more homogeneous thermal environment for the cluster antennas. The limited field of view cannot be overcome, but we expect that this will not limit most Atacama Array observations. Very high dynamic range images can be achieved with modest numbers of antenna clusters (22 clusters results in 10,000:1 dynamic range on a very complex source).

A serious discussion of a clustered Atacama Array design would require more scientific input on the field of view limitation and the maximum required dynamic range of a clustered array. Furthermore, is the assumption that the 10 km Atacama Array will be dominated by continuum observations correct? If the Atacama Array ends up performing mostly spectral line observations, there is little utility and much expense in the clustered array concept. The correlator designers must also give some serious thought to the cluster phasing hardware and the added complications of the correlator before we can say that the clustered array concept is feasible. Also, the systems engineers will probably let us know how the maximum transmittable bandwidth will limit our clustered array bandwidth, and hence our sensitivity.

References

Asaki, it et al., 1996, "Phase Compensation Experiments with the Paired Antenna Method", submitted to Radio Science.

Holdaway, 1992, "Paired Antenna Phase Calibration: Residual Phase Errors and Configuration Study", MMA Memo 88

Holdaway, Radford, Owen, and Foster, 1995, "Data Processing for Site Test Interferometers", MMA Memo 129

Radford, Reiland, and Shillue, 1996, "Site Test Interferometer", PASP **108**, 441-445.

Woody, David, *et al.*, "MDC Phase Calibration Working Group Report", MMA Memo 144.

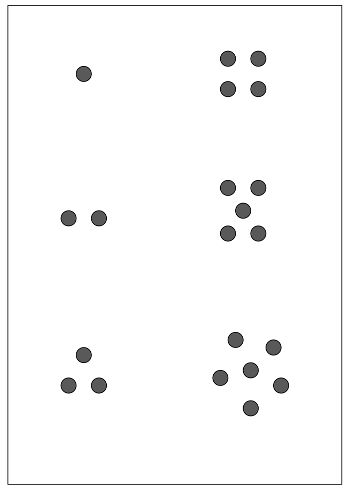

Figure 1: Assumed geometry of the antenna clusters. The minimum separation between antenna centers is $2D$.

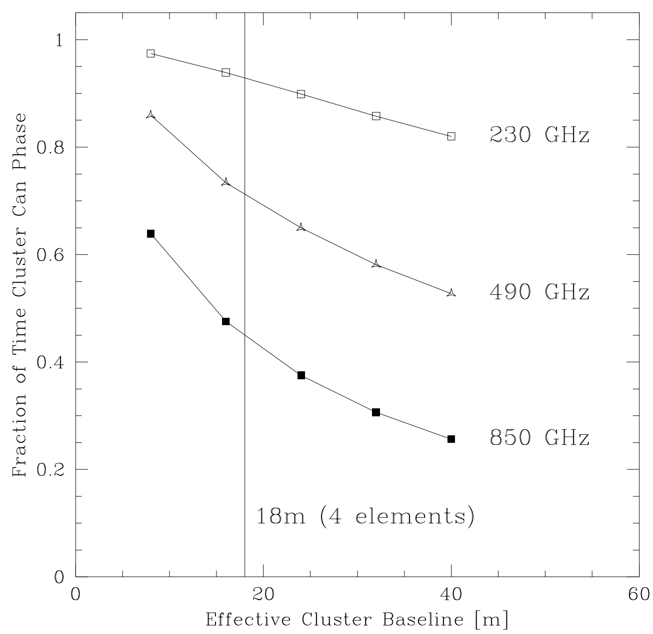

Figure 2: What fraction of the time will the atmosphere permit clusters of various sizes to phase up at various frequencies?

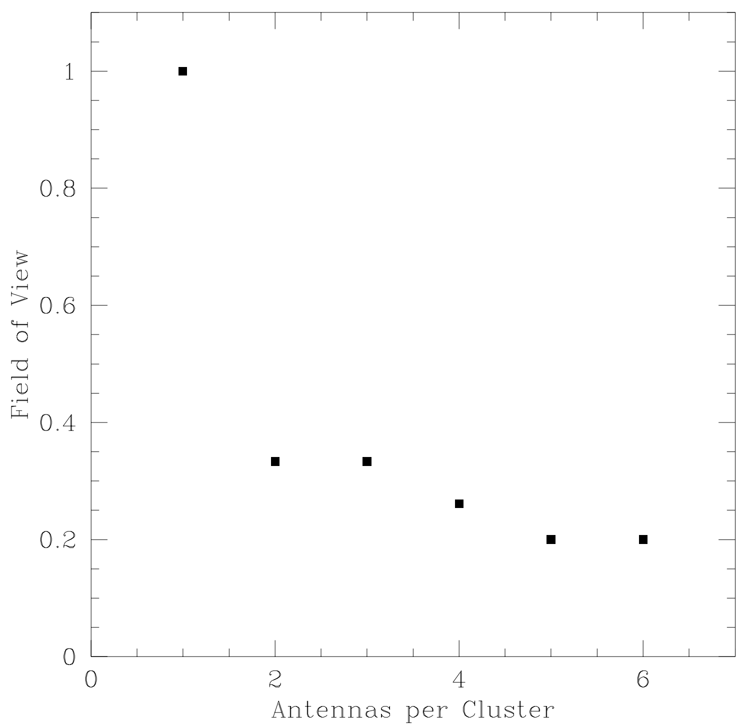

Figure 3: Linear size of the field of view as a fraction of the primary beam size, as a function of the number of antennas in each cluster.

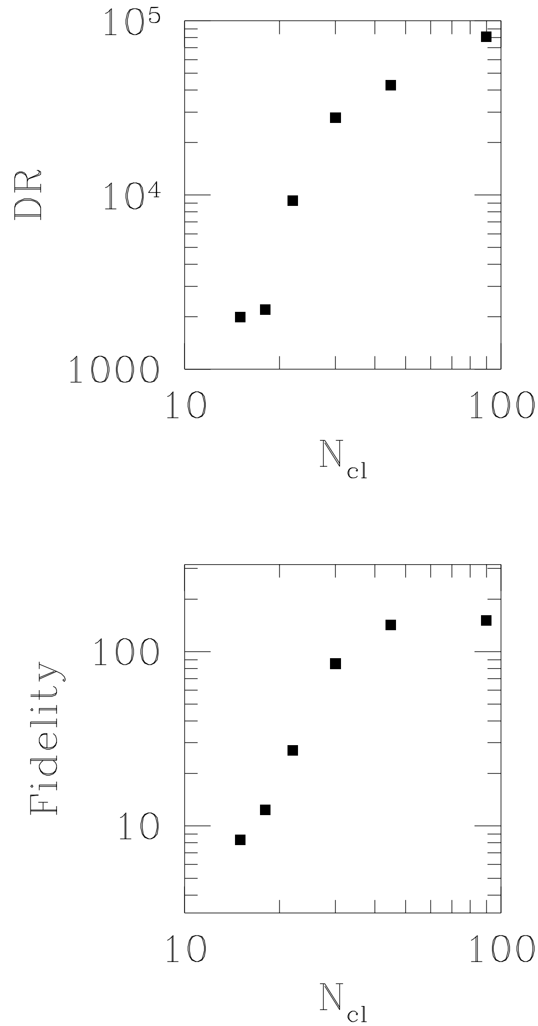

Figure 4: Dynamic range and image fidelity as a function of the number of clusters in the Atacama array for our simulations.