MMA Memo 174: How Many Fast Switching Cycles Will the MMA Make in its Lifetime?

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Abstract

Estimating the MMA's frequency and configuration usage, we calculate the typical fast switching cycle times, calibration source parameters, and atmospheric characteristics for the MMA for an entire year. From these calculations, we are able to estimate during a 30 year life time of the MMA the antennas will perform about 30-50 million fast switching cycles. If radiometric phase correction works well, this may be reduced to about 3 million switching cycles. The uncertainties in these numbers are quite large, at least a factor of 2. The estimated number of switching cycles is required for a fatigue analysis of the MMA antenna members.

Several interesting results arise in conjunction with these calculations:

- At 40 GHz, calibrators are typically 20 mJy and about 0.4 deg away from the target sources, while at 650 GHz, calibrators are typically 120 mJy and about 1.1 deg away. As the path length requirement becomes more severe at high frequencies, more time must be spent integrating on brighter calibrator sources which are typically farther away.
- To achieve low residual phase errors, very short switching cycles are required, reducing the switching efficiency and increasing the noise. If large residual phase errors are permitted, the switching efficiency is high, but decorrelation losses are large, also increasing the noise. The optimal residual phase errors seems to be about 30 deg with an 80% efficiency with respect to decorrelation and time lost to calibration, but some future refinement can be done on this issue.
- By matching the observing frequency to the atmospheric conditions, the D array is *always* phase stable at the observing frequency appropriate to the atmospheric conditions (as defined by 20 deg rms phase errors on the longest baselines) and the C array is *often* phase stable.

1 Introduction

Fast switching phase calibration will remove most of the effects of phase errors on MMA observations (Holdaway, 1992; Holdaway and Owen, 1995; Holdaway, Radford, Owen, and Foster, 1995b; Carilli and Holdaway, 1997). Past analysis has focussed on single frequency observations. In this work, we attempt to make a global view of fast switching at all observing frequencies and all atmospheric conditions. One outcome is an estimate of the total number of switching cycles the MMA antennas will undergo in their lifetimes, which is required for a fatigue analysis of the MMA antenna members.

2 Calculation Strategy

The residual phase errors after fast switching phase calibration are given by

$$\sigma_\phi \simeq \sqrt{D_\phi(\mathbf{vt/2+d})}, \tag{1}$$

where D_ϕ is the phase structure function, $\mathbf{v}$ is the velocity aloft, t is the switching cycle time, and $\mathbf{d}$ is representative of the distance between the lines of site to the calibrator and the target source at the elevation of the turbulent layer. Armed with this expression, source counts of flat spectrum quasars at 90 GHz (Holdaway, Owen, and Rupen, 1994), and a distribution of D_ϕ as obtained from the site testing interferometer (Holdaway, Radford, Owen, and Foster, 1995a), it is possible to estimate the distribution of residual phase errors (Holdaway, Radford, Owen, and Foster, 1995b), or, in this work, the cycle times at various observing frequencies during certain atmospheric conditions.

To approximate the global problem (all frequencies and all observing conditions), one must have a straw man estimate of the frequency and configuration usage and a straw man estimate of the time allocation algorithm to determine what observing frequencies are used during which atmospheric conditions. The frequency distribution is needed because the stability requirements are more demanding at high frequencies. The configuration distribution is needed because some configurations will be phase stable in some atmospheric conditions. The estimate of frequency and configuration usage is roughly based upon Brown 1993. We assume the 40 GHz band is used 10% of the time, and the 90, 140, 230, 345, and 650 GHz bands are each used 18% of the time. We assume single dish observing is performed 10% of the time, while D (70m), C (200m), and B (800m) arrays are each used 20% of the time, and the A (3km) and "Atacama Array" (10km) are together used 30% of the time. We also assume the frequency and configuration distributions are independent. We explore two simple time allocation algorithms: phase stability matched, in which the best phase stability conditions is allocated to the highest frequency observations; and opacity matched, in which the best opacity conditions are allocated to the highest frequency observations. (Since there is only a rough correlation between the phase stability and opacity, these two algorithms can produce very different results.)

Now, to correctly solve this problem, we need to fold in the distributions of the calibrator sources, phase structure functions, velocity aloft, opacity, observing frequencies, and observing elevation angles (and probably several other things I don't even want to think about). Rather than performing a complete treatment, we do the Monte Carlo simulations as in Holdaway and Owen (1995) to determine, for each observing frequency, the distributions of the optimal calibrator brightness and distance (which imply distributions of calibrator integration time, slewing time, and distance in the atmosphere), and then form the medians of the relevant quantities. For each frequency, we also form the medians of the atmospheric parameters such as phase structure function and opacity at 225 GHz. With these medians, for each frequency we can back-solve for the cycle time in Equation 1 which is required to achieve some required rms phase error.

We assumed the observations were at 50 deg elevation angle (see Holdaway *et al.*, 1996) for a model distribution of observed elevation angles). The opacity will scale as the airmass, 1.30 in this case, and the phase stability will scale like the square root of the airmass (Holdaway and Ishiguro, 1995), 1.14 in this case.

In all cases we assume phase calibration is performed at 90 GHz, and the phase solutions are scaled to the target source frequency.

2.1 Atmospheric Conditions

The median atmospheric conditions and the median characteristics of the optimal calibrators for each frequency bin are presented in Table 1. The data in this Table are directly derived the site testing data between May 1995 and April 1996, and are reported at the measured frequencies. How do these parameters scale to the MMA's observing frequencies? The rms phase increases almost linearly with frequency up until about 300 GHz, but there is some deviation from linearity (ie, dispersion) in the submillimeter. This is problematic for fast switching, as electronic delays are non-dispersive while the water vapor results in some dispersion. We do not propose a solution at the present time, but water vapor radiometry may be required. The opacity varies in a complicated way with frequency. We address the opacity variation with frequency below.

Table 1: Median atmospheric conditions and median calibrator properties for each of the frequency bins.

Freq	phase matched		opacity matched				
[GHz]	tau at 225 GHz	rms ϕ, 11 GHz,300m	tau at 225 GHz	rms ϕ, 11 GHz,300m	Scal [Jy]	Dist [deg]	min $vt/2+d$ [m]
40	0.092	10.1	0.152	5.55	0.022	0.393	29.2
90	0.093	6.08	0.108	5.11	0.037	0.508	34.6
140	0.078	3.72	0.072	3.50	0.049	0.622	40.9
230	0.061	2.40	0.053	2.73	0.066	0.817	46.3
345	0.046	1.48	0.039	1.85	0.079	0.937	51.4
650	0.035	0.79	0.028	1.24	0.119	1.113	62.8

2.2 Switching Calculation Results

The results of the fast switching calculations are presented in Table 2 for phase matched time allocation and Table 3 for opacity matched time allocation, both covering 20 deg, 30 deg, and 45 deg rms residual phase errors. It is important to consider sensitivity. It has always been appreciated that fast switching results in inefficiency due to the time spent off-source, and that residual phase errors of 30 deg will lead to a modest 13% decorrelation which must be corrected. These two factors trade off of each other: if we work harder at fast switching in order to reduce the residual phase errors to decrease the decorrelation loss, we will spend less time on source. Another similar tradeoff exists between the opacity matched and phase matched time allocation algorithms. To optimize sensitivity, one may be led to schedule the highest frequency observations during the low opacity times. However, since the opacity and atmospheric phase errors are not totally correlated, the best opacity conditions will not always be the best phase conditions, so more time will be spent on fast switching, quite possibly resulting in a loss in sensitivity. This point is strikingly made in the 650 Ghz frequency bin in the opacity matched case where, in order to achieve 20 deg residual phase errors for the median atmospheric parameters, one must switch faster than one can detect the typical calibration sources, resulting in no time (or sensitivity) on the target source.

We can quantify these sensitivity arguments for our current set of computations by defining the *relative sensitivity* as

$$\frac{\epsilon_a e^{-\sigma_\phi^2/2}\sqrt{t_{frac}}}{T_{sys}}, \tag{2}$$

where ϵ_a is the aperture efficiency, t_{frac} is the fraction of time spent integrating on source, and T_{sys} is the system temperature given in Brown's "Uniform Assumptions for the Atacama Array Workshop" by

$$T_{sys} = T_{rx}e^{a\tau} + T_{atm}(e^{a\tau} - \epsilon_{sp}) + T_{bg} \tag{3}$$

where T_{rx} is receiver temperature, generally taken as $2h\nu/k$, a is the airmass, taken as 1.3 for these calculations (airmass at 50 deg elevation), τ is the opacity at the *target source frequency*, T_{atm} is the atmospheric temperature, $\epsilon_{sp} = 0.96$ is the warm spill over efficiency, and T_{bg} is the 2.7 K back ground radiation. Now, in a departure from Brown, we submit that the submillimeter opacity may be significantly higher than previous NRAO memos have assumed. Earlier NRAO work is based on Liebe's 1989 MPM code. Pardo and Cernicharo's 1997 ATM code, based on a more recent version of Liebe's transmission code, agrees very well with the MPM results for frequencies below about 400 GHz when the PWV is normalized to give the same opacities at 225 GHz, but predicts opacities higher by a factor of about 2 in the 650 and 850 GHz windows. For the 650 GHz window, the ATM model predicts much higher opacity, which translates into much higher system temperatures and much lower sensitivity. At this point, we cannot tell which transmission model is correct, but we report results from both models here in Tables 2 and 3.

Comparing the relative sensitivities in the two time allocation algorithms, the MPM transmission model indicates the phase matched algorithm results in superior sensitivities (time lost to calibration dom-

Table 2: For each frequency bin, we report the median cycle time, the median fraction of time on source, the median τ_{225} and σ_ϕ on a 300m baseline at 11.2 GHz, the relative sensitivity assuming the 1989 MPM transmission code, the relative sensitivity assuming the 1997 ATM transmission code, and the normalized sensitivity. The relative sensitivity is defined in the text, and the normalized sensitivity is a measure of how much sensitivity is lost due to decorrelation from the residual phase errors and the time lost from on-source integration during the calibration cycle. These quantities are reported for the phase matched time allocation algorithms with 20, 30, and 45 deg rms residual phase errors.

ν [GHz]	Median cycle time [s]	Fraction on source	Sens. with MPM	Sens. with ATM	Norm. Sens.
phase matched, 20 deg residual phase					
40	18.4	0.903	0.0175	0.0182	0.89
90	10.4	0.809	0.0182	0.0171	0.84
140	10.7	0.798	0.0191	0.0188	0.84
230	8.40	0.701	0.0109	0.0109	0.78
345	8.81	0.681	0.0053	0.0058	0.77
650	6.47	0.496	0.0016	0.0008	0.66
phase matched, 30 deg residual phase					
40	35.9	0.950	0.0167	0.0173	0.84
90	21.4	0.906	0.0178	0.0168	0.82
140	22.5	0.903	0.0188	0.0185	0.82
230	19.1	0.869	0.0113	0.0113	0.81
345	21.0	0.866	0.0055	0.0060	0.81
650	18.5	0.823	0.0019	0.0010	0.79
phase matched, 45 deg residual phase					
40	69.1	0.974	0.0142	0.0148	0.72
90	42.0	0.952	0.0154	0.0145	0.71
140	45.1	0.951	0.0163	0.0160	0.71
230	40.0	0.937	0.0098	0.0098	0.71
345	45.3	0.938	0.0048	0.0053	0.71
650	44.0	0.926	0.0017	0.0009	0.70

Table 3: As with Table 2, but for the opacity matched time allocation algorithms with 20, 30, and 45 deg rms residual phase errors.

ν [GHz]	Median cycle time [s]	Fraction on source	Sens. with MPM	Sens. with ATM	Norm. Sens.
opacity matched, 20 deg residual phase					
40	49.2	0.964	0.0180	0.0185	0.92
90	13.8	0.855	0.0184	0.0173	0.87
140	11.5	0.811	0.0197	0.0194	0.84
230	5.81	0.568	0.0104	0.0104	0.70
345	4.58	0.387	0.0045	0.0049	0.58
650	3.25	0.000	0.0000	0.0000	0.00
opacity matched, 30 deg residual phase					
40	95.5	0.981	0.0169	0.0173	0.86
90	28.2	0.929	0.0177	0.0167	0.84
140	24.6	0.911	0.0193	0.0190	0.83
230	14.4	0.826	0.0117	0.0117	0.79
345	12.8	0.782	0.0059	0.0064	0.77
650	5.42	0.399	0.0016	0.0010	0.55
opacity matched, 45 deg residual phase					
40	184.	0.990	0.0143	0.0147	0.73
90	55.9	0.964	0.0152	0.0143	0.72
140	49.9	0.956	0.0167	0.0164	0.71
230	31.5	0.920	0.0104	0.0104	0.70
345	29.6	0.905	0.0054	0.0058	0.69
650	16.4	0.801	0.0020	0.0012	0.65

Table 4: Median cycle times in seconds for the various frequencies and single dish (SD), D (70m), C (200m), B (800m), and A (3000m) arrays, for the phase matched 30 deg residual phase case. If the array is phase stable, we assume that switching will still occur every 300 s for calibration of the electrical path length of the antenna and the electronics.

Array	Cycle Time [s]					
	40G	90GHz	140GHz	230GHz	345GHz	650GHz
SD	300	300	300	300	300	300
D	300	300	300	300	300	300
C	300	21.4	22.6	19.2	21.0	18.5
B	36.0	21.4	22.6	19.2	21.0	18.5
A	36.0	21.4	22.6	19.2	21.0	18.5

inates). However, if the ATM atmospheric model is correct, with its higher opacities in the submillimeter windows, then the two picking algorithms result in very similar sensitivities.

In addition, we also report a normalized sensitivity, which is the former sensitivity divided by the perfectly phase calibrated sensitivity (ie, $t_{frac} = 1$ and $\sigma_\phi = 0$, as would result if there were no phase errors, or if phase calibration could be achieved by magic). This normalized sensitivity is a realistic expression of the cost of fast switching phase calibration on the scientific output of the array, and they can be used effectively to gage the relative merits of various fast switching strategies. For example, in the phase matched allocation algorithm, it is not really advantageous to attempt to achieve residual phase errors of 20 deg or less for 650 GHz observations: the normalized sensitivity is 0.66 for the 20 deg case, and 0.79 for the 30 deg case. However, at 45 deg residual phase errors, the effects of decorrelation dominate any sensitivity improvement due to the more leisurely cycle times. The normalized sensitivity in the phase matched Table cannot be compared to the normalized sensitivity in the opacity matched Table because the median opacities in the two different time allocation algorithms are different.

2.3 Phase Stable Arrays

If the switching time is comparable to the crossing time of the atmosphere above the array, the array will be phase stable and will not require fast switching. (A more precise criteria can be derived directly from the structure function.) In fact, all frequency bins are phase stable in the D (70m) array. This does not imply that D array is always stable; rather, during the good conditions in which the high frequency observations would be made, the D array is phase stable at all planned frequencies, and during the poorer conditions when the low frequency observations would be made, the D array is phase stable at these low frequencies. Several of the frequency bins are phase stable in the C array. In the best condition bin, the array would be phase stable on 3000 m baselines at 115 GHz, but these most excellent conditions are required for the 650 Ghz observations, even with fast switching. Anyway, when the array is phase stable, we assume that calibration will still be performed every 300 s. Table 4 shows the cycle times for the different frequencies and arrays for the phase matched 30 deg residual phase error case.

2.4 Total Number of Switching Cycles

From Table 4 which lists median cycle times for eacg frequency and array configuration (and the analogous tables for the other cases), together with the weights given to each frequency and array configuration, we may calculate the total number of switching cycles the MMA would perform in an assumed 30 year life time (see Table 5).

The 45 deg residual phase error case should not usually be considered, as it has already lost 27% of its sensitivity to decorrelation. (There are some cases though, such as very high frequency observations in non-optimal phase conditions, when such high residual phase errors may be required.) Similarly, in

Table 5: Numbers of fast switching cycles required in the 30 year life of the MMA for different residual phase errors and the phase matched and opacity matched time allocation algorithms.

Residual Phase Error [deg]	Millions of Cycles, Phase matched:	Millions of Cycles, Opacity Matched:
20	73	114
30	31	49
45	12	18

order to achieve 20 deg residual phase errors, the array must switch an awful lot. The 30 deg residual phase errors are a happy medium, resulting in only a 13% decorrelation sensitivity loss, but not quite so many cycles. Hence, it looks like the MMA will make about 30-50 million fast switching cycles.

3 Additional Factors Not Considered

There are a few factors which would further augment these estimates of the number of switching cycles in the MMA's life time. If radiometric phase correction worked well, we would only need to calibrate the electronic phase, with some calibration observations to support the radiometric phase correction. If calibration were performed once every 300 s, we are down to 3 million switching cycles.

The above calculations were not performed with the full distributions, but rather on the medians of distributions. If the full blown calculations were to be performed, the tails of the distributions would increase the calculated number of cycles in the life time of the array.

The phase matched and opacity matched time allocation algorithms are not optimal. Presumably a more complicated algorithm which considered rms phase, opacity, and winds aloft could do a better job at matching a project with the atmospheric conditions. On the other hand, the atmospheric conditions will change over the course of the observations. If the atmospheric conditions change too much, the observations will be halted and replaced by observations more appropriate to the conditions. This will result in some inefficiency, with observations being done in somewhat better or somewhat worse atmospheric conditions than required. Also, some of the very good atmospheric conditions will be allocated to some very demanding lower frequency observations, which would force some high frequency observations to use less than optimal conditions.

Two observing modes that have not been considered in the analysis of the number of switching cycles are mosaicing and total power observing. While neither observing technique will require very much in the way of fast switching phase calibration, both techniques have the potential to significantly increase the number of telescope movements. Mosaics with hundreds or even thousands of pointings will not be uncommon. These large mosaics may require very short integration times, such as 10-20 s per pointing, making mosaiced observations similar to fast switching in the number of switching cycles it requires per unit time of observing. If shorter integrations are required, then On-The-Fly mosaicing can be used (Holdaway and Foster, 1994).

Total power continuum observations of large sources may be much worse. Holdaway, Owen, and Emerson (1995) suggested that nutating subreflectors may not be required on the MMA as position switching and On-The-Fly mapping may perform about as well or better than the traditional beam switching of Emerson, Klein, and Haslam(1979). This issue will be revisited in an upcoming MMA Memo, and the expected result is that beam switching will still be required, but that very fast and rapidly repeating On-The-Fly observing (cycle times of a few seconds) will still be a useful observing technique for mapping large regions of the sky. This sort of observing technique would increase the total telescope movement if it were utilized very often.

If "bright" continuum sources were observed, self-calibration could remove the atmospheric phase

Table 6: What are the weakest sources we can self-calibrate on (assuming the phase matched time allocation algorithm)? Calculations use the MPM transmission model except where stated.

ν [GHz]	max T_{int} [s]	τ_{225}	τ at ν	S_{min} mJy
phase matched, 20 deg residual phase				
40	11.57	0.0920	0.036	3.17
90	7.20	0.0931	0.021	3.67
140	7.56	0.0776	0.038	3.40
230	6.46	0.0613	0.080	6.03
345	6.87	0.0463	0.21	12.0
650	5.66	0.0345	0.43	36.9
650	5.66	0.0345	0.81[a]	70.9[a]
phase matched, 30 deg residual phase				
40	21.80	0.0920	0.036	1.54
90	13.57	0.0931	0.021	1.78
140	14.38	0.0776	0.038	1.64
230	12.55	0.0613	0.080	2.88
345	13.66	0.0463	0.21	5.65
650	11.99	0.0345	0.43	16.9
650	11.99	0.0345	0.81[a]	32.5[a]
phase matched, 45 deg residual phase				
40	41.10	0.0920	0.036	0.75
90	25.58	0.0931	0.021	0.86
140	27.38	0.0776	0.038	0.79
230	24.40	0.0613	0.080	1.38
345	27.18	0.0463	0.21	2.67
650	25.42	0.0345	0.43	7.73
650	25.42	0.0345	0.81[a]	14.9[a]

[a]The second of each set of 650 GHz calculation uses the ATM transmission model, which produces significantly higher submillimeter opacities than the old MPM transmission model.

fluctuations, and fast switching phase calibration would not be required.

3.1 Self Calibration

For bright sources, self-calibration permits the solution for the antenna phases from the target source detection. Now, for the MMA, "bright" is frequency dependent, but the source strength required for self-calibration in our frequency binned atmospheres will range from about 1 mJy at 40 GHz to about 17 mJy (if one assumes Liebe's 1989 MPM atmospheric transmission code) or 32 mJy (if one assumes Pardo and Cernicharo's 1997 ATM atmospheric transmission code). The maximum self-calibration solution interval and the corresponding minimum point source strength is listed for the different frequency bins, time allocation algorithms, and required residual phase error are listed in Table 6. These numbers were calculated assuming that thermal noise in the gain solutions and changes in the atmospheric phase over the integration time make equal contributions to the residual phase.

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