MMA Memo 175: Gain Degradation in a Symmetrical Cassegrain Antenna Due to Laterally Offset Feeds

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1 Abstract

The maximimum distance off-axis that a feed can be placed in a symmetrical Cassegrain antenna without significant degradation in efficiency is investigated. The results place limits on the size and position of receivers at the secondary focus. For example, for the proposed MMA geometry, if we use a one percent reduction in gain as a figure-of-merit, then the maximum off-axis distance is about 165 mm or 6.5 in. for a 350 micron wavelength. For longer wavelengths the allowable distance is much greater.

2 Introduction

Early discussions of MMA receiver geometries have suggested the use of one or two large dewars at the secondary focus with feeds clustered around the central axis of the antenna. Aberrations are thereby introduced which degrade antenna performance, and this memo attempts to quantify the degradation. One of the receiver geometries that has been suggested is used as an example.

The loss of antenna efficiency due to lateral feed displacement for a symmetric cassegrain antenna, such as is planned for the MMA, is investigated. The assumption is made that the feed is repointed towards the center of the subreflector so that the field amplitude aperture illumination is symmetric to first-order. This is a necessary condition for many applications where the offset of the feed is so large that the illumination of the subreflector would be vignetted were the feed not repointed.

3 Aberration Terms

Classical optical theory is used and the system is treated as an equivalent prime-focus telescope with a focal length of $f_{eq} = mF$, where m is the Cassegrain magnification parameter and F is the focal length of the primary mirror. The expression for the Seidel aberrations, which are classical optical aberration functions that give wavelength dependent phase-shifts (and which excludes the linear phase-shift which leads to a pure beam squint) is [1]:

$$\Phi(\alpha, r) = -C\alpha^2 r^2 \cos\phi^2 - \frac{1}{2}D\alpha^2 r^2 + E\alpha^3 r \cos\phi + F\alpha r^3 \cos\phi \tag{1}$$

where the coefficients (C-F) are given by:

$$C = -\frac{md}{2f_{eq}d_s} \qquad \text{(Astigmatism)} \tag{2}$$

$$D = -\frac{m^2 d}{2 f_{eq} d_s} \qquad \text{(Curvature)} \tag{3}$$

$$E \simeq 0 \qquad \text{(Distortion)} \tag{4}$$

$$F = \frac{-1}{4 f_{eq}^2} \qquad \text{(Coma)} \tag{5}$$

Eq.(1) is an expression of the path length errors developed on the primary aperture due to the off-axis feed. The variable r is radius on the primary, and $\alpha = h/f_{eq}$ where h is the off-axis distance and f_{eq} is the equivalent focal length of the Cassegrain. The variables d_s is the distance from the primary to the secondary focus. The variable d is the distance from the primary focus to the vertex of the hyperbolic subreflector.

4 Minimizing Aberrations [2]

We are interested in a measure of the gain reduction. In general, quadratic and higher order terms in the phase distribution widen the antenna beamwidth. Severe phase errors also bring up the sidelobe level, including the well-known coma lobe. Since we are interested in limiting the gain reduction to one percent or less, the effect should only be a slight beam-broadening.

It is noted that the loss in gain is proportional to the weighted squared sum of phase deviations. There are two ways to reduce the aberrations on the telescope: repointing the antenna to the beam peak, and refocussing the antenna, usually by moving the subreflector. Re-pointing takes out aberration terms proportional to $r\cos\phi$ and refocussing takes out terms in r^2. This leaves the astigmatism and coma terms as the most significant.

The astigmatism term, when expressed as $r^2\cos^2\phi$, contains a net focus error; removing this focus error minimizes the residual wavefront RMS and results in an astigmatic error of the form $r^2\cos^2\phi - \frac{1}{2}$. The coma term expressed in the form $r^3\cos\phi$ contains a net telescope pointing error. The magnitude of the best fit pointing error depends on the illumination taper. Integrating as $r^3 - \frac{2}{3}\cos\phi$ removes the pointing error for a uniform aperture illumination, and integrating as $r^3 - 0.58\cos\phi$ removes the pointing error for a 12-dB tapered parabolic illumination.

There is another detrimental effect of moving the feed off-axis. The feed illuminates the primary aperture in an asymmetrical way, which increases the spillover on one side of the primary. Lugten and Welch discussed this and estimate that it would add $0.65°K$ to the system temperature [3]. Alternatively, they say, a skirt could be added to the primary to direct the spillover onto the sky.

5 Gain Calculation

The aberration function can be used in the expression for the gain of a circular aperture:

$$G = \frac{4\pi}{\lambda^2} \frac{\left|\int_0^{2\pi}\int_0^1 f(\hat{r}) exp(jk\Phi(\hat{r},\phi))\hat{r}d\hat{r}d\phi\right|^2}{\int_0^{2\pi}\int_0^1 f^2(\hat{r})\hat{r}d\hat{r}d\phi} \tag{6}$$

Here the function $f(\hat{r})$ expresses the aperture illumination function, with $\hat{r}$ the normalized radial parameter. In our calculations we used:

$$f(\hat{r}) = 0.25 + 0.75(1 - \hat{r}^2) \tag{7}$$

This is a parabolic illumination function with a 12-dB edge taper.

Eq (6) can be integrated directly to get the gain degradation due to aperture phase errors. Alternatively, Ruze [4] gives a simplified expression for Eq (6) by expanding the exponential, which is valid for phase errors only on the order of a radian or less. In practice, we are looking for the lateral offset which will give us a one percent gain degradation, which invariably will meet the criterion of a radian or less. The expanded expression is given:

$$\frac{G}{G_0} \simeq 1 - \overline{\delta^2} + \bar{\delta}^2 \tag{8}$$

where:

$$\overline{\delta^2} = \frac{\int_0^{2\pi}\int_0^1 f(\hat{r})k^2\Phi^2(\hat{r},\phi)\hat{r}d\hat{r}d\phi}{\int_0^{2\pi}\int_0^1 f(\hat{r})\hat{r}d\hat{r}d\phi} \tag{9}$$

$$\bar{\delta} = \frac{\int_0^{2\pi}\int_0^1 f(\hat{r})k\Phi(\hat{r},\phi)\hat{r}d\hat{r}d\phi}{\int_0^{2\pi}\int_0^1 f(\hat{r})\hat{r}d\hat{r}d\phi} \tag{10}$$

Eq (8) was calculated for the MMA geometry proposed by Lugten and Welch [3]. The relevant parameters used were:

Antenna Parameters	
$D = 8000$ mm	Diameter of Primary Mirror
$d = 2777.27$ mm	Primary Vertex to Secondary Vertex
$F = 3040$ mm	Primary Focal Length
$d_s = 4562$ mm	Primary to Secondary Focal Distance
$m = 16.35$	Cassegrain Magnification
$f_{eq} = 49680$ mm	Effective focal length of Cassegrain

The gain versus lateral offset distance was calculated using Eq (1) and Eq (8), for cases using the coma term, the astigmatism term, and both terms. Ruze states that the contribution from the curvature term is dominant but can be completely removed by placing the feed on the Petzval surface [5]. This was checked using the ray-tracing procedure and found to be true. Moving the feed axially affects to first-order only the curvature term, and thus can be ignored. Adjusting the subreflector axially should also tune out the curvature term.

6 Ray-Tracing Method

Classical aberration theory is based on paraxial optics, in which it is assumed that all geometric rays are nearly parallel to the optical axis. The theory breaks down when the off-axis angles are too large. Realizing this, we checked the validity of the aberration theory on the proposed MMA geometry, in which receivers will be 10.5 arcsec off of the axis of the equivalent paraboloid (resulting in a beam scan of about 60 beamwidths on the sky at 0.35 mm wavelengths). To check the aberration theory, the method of geometrical ray-tracing was used, in which the path length from the feed to the aperture in the Cassegrain system was explicitly calculated. After subtracting the tilt term in $r\cos\phi$, a curve fit was done to the phase expression of Eq.[1] with the coefficients as unknowns, and the results were compared to the coefficients defined by Eq. [2], [3], and [5].

The results were as follows:

Coefficient	Classical Theory	Ray Tracing	Percent Difference
C (Astigmatism)	-1.06 e-4	-1.11 e-4	4.4
D (Curvature)	-1.74 e-3	-1.67 e-3	4.0
F (Coma)	-1.01 e-10	-1.00 e-10	1.0

From the above table it can be seen that the difference between the classical theory and the ray-tracing is very small, and the ray-tracing is not necessary for a good first-order calculation.

7 Results of Calculation

The proposed MMA geometry of Lugten, Welch [3] calls for the shortest wavelength receiver (0.35 mm) to be 6 inches off-axis. Fig.1 shows the loss in percent for the receiver at that frequency versus off-axis distance. The curve that shows coma and astigmatism together is the relevant result, and the curves for coma and astigmatism alone are incuded as well. The loss at an off-axis distance of 6 in. (152.4 mm), and at a wavelength of 0.35 mm, which is the highest loss condition for the proposed MMA receiver layout, is 0.75 percent. This is very small, especially considering that at the shorter wavelengths the phase efficiency degradation will be dominated by the loss of efficiency due to the surface accuracy of the dish. It should also be noted that the plot is only showing the shortest wavelength, and that the loss at longer wavelengths is negligible.

The calculation of the integrals shown in Eq [9] and Eq [10] were done in Mathematica.

8 Acknowledgements

Discussions with Peter Napier alerted me to the fact that that the classical aberration terms might not be valid for our geometry.

John Lugten corrected my first calculation by pointing out that using the aberration terms from Ref. [1] overstates the loss, and suggested the corrected form that is dicussed in Section 3.

Discussions with James Lamb led me to find an error in my calculation. James had done calculations in parallel to these, with a different formulation but nearly identical result.

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References

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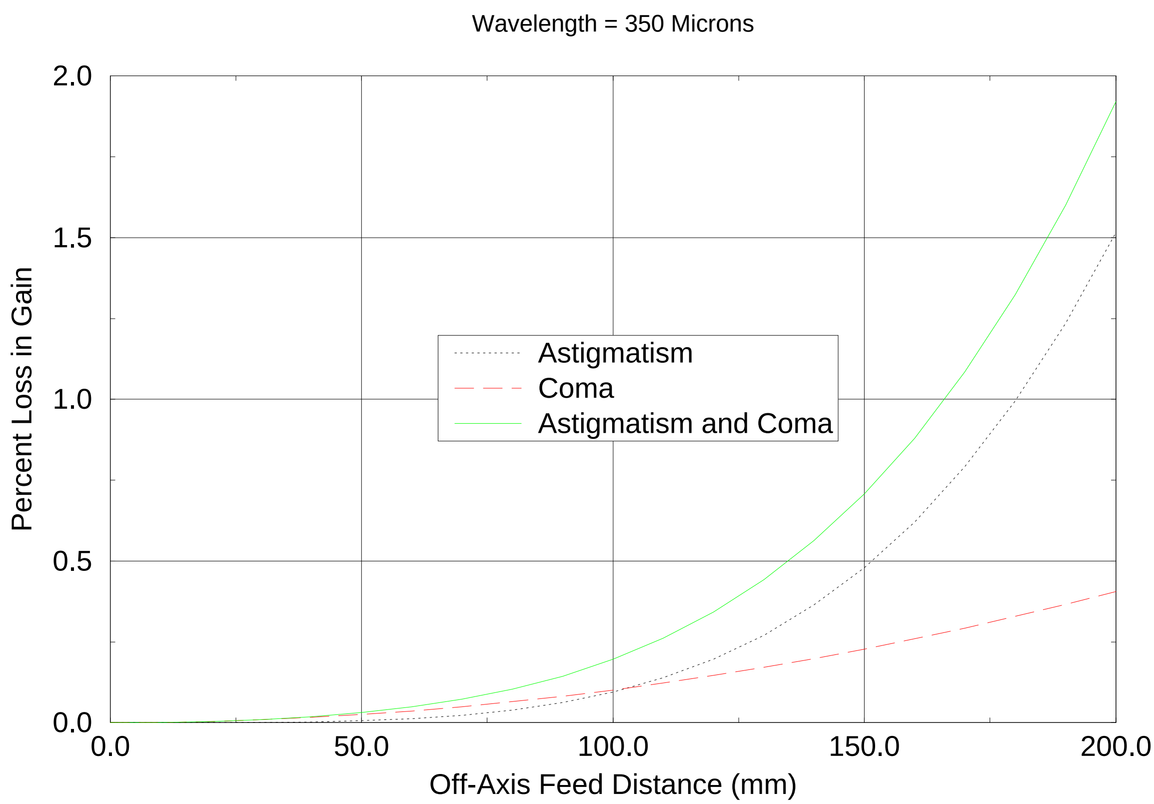

Figure 1: Loss of Gain for Proposed MMA Geometry due to Off-Axis Feed at 350 Micron Wavelength