MMA Memo 177: Sensitivity Comparisons of the Various LSA/MMA Collaboration Options

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Abstract

For the purpose of helping to evaluate the various options for collaboration between the LSA and the MMA, we review the optimum point source sensitivity and optimum imaging sensitivity criteria for array design, and also develop a new criteria based on optimizing surface brightness sensitivity in wide field imaging.

We calculate the noises under the various criteria for the various possible LSA/MMA joint arrays. If the array has antennas of two different diameters, cross-correlating all antennas is required to maximize the array's sensitivity. Processing of visibilities from antennas of different diameter is feasible with our current software, although with an array of 8 m and 15 m dishes, full cross-correlation mosaicing will increase the cpu requirements by an order of magnitude. Given that all cross correlations are formed, there is not a great deal of difference between the various sensitivities of the homogeneous array options (50-60 x 12 m antennas) and the heterogeneous array options (40 x 8 m plus 25-35 x 15 m antennas).

1 Introduction

On June 25 and 26 of 1997, European representatives of the LSA project met with representatives of the MMA project to discuss the possibility of building a single collaborative array. In addition to a resolution which stated the goal of working towards a single array, we also agreed to study two different array design concepts: a homogeneous array with 50-60 12 m antennas, and a heterogeneous array consisting of both 40 8 m antennas and 25-35 15 m antennas. (The range in the numbers of antennas is due to the uncertainty in the cost of the 12 m and 15 m antennas. We consider both the poor and rich ends of these options.) Here, we investigate the various sensitivities of these different array options and compare them with each other and with the "default" arrays of 40 8 m antennas and 35 15 m antennas.

2 Analytical Arguments

For a heterogeneous array, simple analytical expressions exist for the point sensitivity and the imaging sensitivity. We review these expressions, as well as develop an expression for the imaging surface brightness sensitivity of an array.

2.1 Optimizing Point Source Sensitivity: nD^2

The noise σ_{int} in a naturally weighted image from interferometric observation is equal to

$$\sigma_{int} = \frac{\sqrt{2}kT_{sys}}{\eta A\sqrt{\Delta t\Delta\nu N_{bl}}}, \tag{1}$$

where T_{sys} is the system temperature, A is the collecting area of each dish, $\pi D^2/4$, Δt is the integration time, $\Delta\nu$ is the band width, and N_{bl} is the number of baselines $n(n-1)/2$, n being the number of antennas. Taking out the items we are not interested in,

$$\sigma_{int} \propto \frac{2}{A\sqrt{n(n-1)}}. \tag{2}$$

The analogous proportionality for a single dish of area A_{sd}, all other factors such as efficiency, T_{sys}, integration time, and band width being the same, is

$$\sigma_{sd} \propto \frac{2}{A_{sd}}. \tag{3}$$

Since $\sqrt{n(n-1)}$ is very close to n, the noise in an interferometer image is very nearly inversely proportional to the total collecting area, and very nearly the same noise as for a large single dish of the same collecting area. If autocorrelations are included, then $N_{bl} = n(n+1)/2$, and the interferometer noise is actually a bit less than for a single dish of the same collecting area. *To optimize the point source sensitivity of an instrument, we then seek to maximize the total collecting area, for either an interferometer or a single dish.* Hence, we arrive at the consideration of maximizing nD^2

The interferometer offers some advantages: its potential for higher resolution reduces confusion, its ability to image multiple resolution elements simultaneously in the primary beam results in a speed up of its imaging with respect to a single dish of the same collecting area, and its multiple small elements often represent a cost savings, as compared to a single dish of the same collecting area. The single dish also offers one main advantage: it puts a given amount of point source sensitivity (set by the collecting area) into the largest possible beam on the sky, hence optimizing surface brightness sensitivity, at least for single pointing observations.

2.2 Optimizing Large Field Imaging Sensitivity: nD

If point source sensitivity were all that mattered, we would get as much collecting area as we could, as cheaply as we could. However, the universe is not made up of point sources, in spite of the Clean algorithm. Especially at millimeter wavelengths where the primary beams become smaller, there are many sources which will require multiple pointings to image. Since larger dishes result in smaller primary beams, part of the sensitivity gained by increasing the dish size to increase the collecting area is lost as more pointings must be spent to cover a large source and each pointing will receive less integration time, as compared to a smaller dish.

The imaging sensitivity, a measure of the sensitivity in imaging two dimensional objects which are much large than the primary beam, is proportional to the point source sensitivity multiplied by the square root of the amount of time which the array is able to spend on each pointing, given a fixed amount of time to cover the entire region of interest. The integration time which each pointing receives will be inversely proportional to the number of pointings required to cover the region of interest; the number of pointings is inversely proportional to the area of the primary beam; so the integration time which each pointing receives is proportional to the area covered by the antenna's primary beam B, which is inversely proportional to D^2. Hence, the imaging sensitivity is given by

$$S \propto nD^2 t^{1/2} \quad (4)$$

$$S \propto nD^2 B^{1/2} \quad (5)$$
$$S \propto nD^2 (D^{-2})^{1/2} \quad (6)$$
$$S \propto nD. \quad (7)$$

For one dimensional objects, the integration time each pointing receives will be proportional to D^{-1}, so the imaging sensitivity will go like $nD^{1.5}$. For zero dimensional objects (ie, point sources, or sources small compared to the beam), the integration time the pointing receives is independent of the dish size, and nD^2 is recovered.

2.3 Optimizing Wide Field, Low Resolution Brightness Sensitivity

The nD^2 and nD criteria are well known in array design. We wish to consider another criteria for the number of elements and the dish size which has not been considered by others: how does the surface brightness sensitivity of a packed configuration vary in the case of wide field imaging?

A naive approach would reason that the brightness sensitivity of an array is independent of n and D, but only depends upon the filling factor f of the array (ie, the fraction of the array area which is filled up with antenna collecting area), which is ultimately limited by the antenna design and how close it allows the antennas to be placed. For the time being, we will assume that all antenna diameters can have designs which permit the same maximum filling factor, which is something like 0.5. Now, as in imaging sensitivity, the number of pointings required to span the two dimensional object will be proportional to D^2, which will augment the sensitivity by D^{-1}. This suggests that the wide field surface brightness sensitivity criteria is f/D, or just $1/D$ for equally filled arrays. How can this be? In the limit, it seems we should have an unspecified number of vanishingly small antennas!

In order to compare the wide field surface brightness sensitivity of different arrays, we really need to hold the resolution constant. A 50% filled array with n large dishes will have higher resolution than a 50% filled array with n small dishes. To compare the two arrays, we must taper the array of larger dishes to the natural resolution of the array of smaller dishes. Some amount of point source sensitivity is lost from the array of large dishes, but the point source sensitivity will still be better than for the array of small dishes. To address how much point source sensitivity is lost to tapering, we have generated arrays with a range of antenna size and number, simulated data sets from each array with noise per visibility proportional to D^{-2}, and tapered the simulated visibilities to produce the same image resolution for each array. The resulting image noise is proportional to the single pointing surface brightness sensitivity at the chosen resolution. A power law expression $n^\alpha D^\beta$ was fit to the simulated image noise for the different arrays of various n and D, and the best fit to the point source noise at constant resolution is $n^{-0.6} D^{-0.9}$, though there is considerable scatter in the relationship, presumably due to details of the simulated array configurations. The wide field surface brightness sensitivity at this constant resolution will be one over the point source noise, divided by D to account for the number of pointings and time per pointing, or

$$n^{0.6} D^{-0.1} \simeq n^{1/2}. \tag{8}$$

Hence, for the case of optimally filled arrays, the wide field brightness sensitivity at constant resolution is approximately independent of the dish diameter and only depends upon the number of elements, subject to the implicit constraint that the array is packed (ie, ∼50% filled) at a size corresponding to the desired resolution.

2.4 Significance of nD^2, nD, and $n^{1/2}$ Criteria

Following the work of R. Brown, as presented at the June 25 LSA/MMA meeting, we can optimize the quantities nD^2, nD, or $n^{1/2}$ subject to the constraints given by the cost equation and the total amount of money we think we have to spend. Optimizing nD^2 will favor larger antennas (as per the LSA's 15 m antennas), optimizing nD will favor smaller antenna (as per the MMA's 8 m antennas), and optimizing $n^{1/2}$ will favor small antennas.

3 Computed Sensitivities for Mixed Arrays

While the analytical quantities in the previous sections are approximately correct for homogeneous arrays, computation of the imaging and low resolution surface brightness sensitivities for a heterogeneous array in which all baselines are correlated, such as what we are considering for the joint MMA/LSA array, must be performed in a somewhat more detailed manner. Only the point source sensitivity remains a simply computed quantity.

We consider the various sensitivities of six different array options: 50 12 m antennas, 60 12 m antennas, 40 8 m antennas plus 25 15 m antennas, 40 8 m antennas plus 35 15 m antennas, 40 8 m antennas alone, and 35 15 m antennas alone. We assume here that the 8 m antennas will have 1 arcsecond pointing and 25 micron surfaces, the 12 m antennas will have 1 arcsecond pointing and 25 micron surfaces, and the 15 m antennas will have 1.5 arcsecond pointing and 30 micron surfaces. In addition to the differences in diameter and pointing and surface specifications, there will be other differences in the performance of the antennas. Since the feed legs do not go to the edge of the dish in the LSA design, there will be more blockage and more scatter of stray radiation, so the low frequency antenna efficiency will not be as high as the MMA design, and increased warm spill over will slightly increase the system temperature. Since the efficiency and spill over effects are small, we have not included them in these computations.

3.1 Mosaicing with a Heterogeneous Array

In order to maximize the sensitivity of a heterogeneous array, signals from all antenna pairs must be correlated, and all antennas must have a common pointing center at each time. In the case of the heterogeneous choice for the MMA/LSA collaboration in which 40 8 m dishes and between 25 and 35 15 m dishes have been proposed, we will have three different primary beams: the 8 m primary beam, the 15 m primary beam, and the 8x15 m primary beam. If an object is much smaller than the smallest primary beam (as in the case of MERLIN or in *ad hoc* VLBI arrays), then the different primary beam sizes do not matter. However, if the object is as large or larger than the primary beams, we must correctly treat the effects of the primary beam via a mosaicing algorithm.

Consider an object which is much larger than our primary beams which we wish to image with our heterogeneous array. In order to maintain uniform sensitivity in the mosaic image, the common pointings must be set by the Nyquist sampling rate of the smallest primary beam, in this case, the 15 m. This results in the 8 m data being oversampled by a factor of 15/8, or 3.5 times as many pointings required to sample the same two dimensional region of sky as for an array of only 8 m dishes. However, the oversampling does not result in any loss of speed, as the extra sensitivity from adjacent pointings permits 1/3.5 times less integration time per pointing to obtain the same final noise level in the mosaic.

The non-linear mosaic algorithm (Cornwell, 1988; Cornwell, Holdaway, and Uson, 1993) is already flexible enough to deal with mosaic data from a heterogeneous array. To demonstrate this, we have made a toy simulation with an array of 27 8 m dishes and 13 15 m dishes, with 121 pointings sampled on the sky by $\lambda/2D$ for $D = 15m$. Baselines correlating 8x8, 15x15, and 8x15 dishes were treated as different telescopes, each with their own primary beam. Hence, a total of 363 logical pointings (ie, 121 actual pointings times 3 different primary beam types) were processed in the mosaic algorithm, as opposed to the $\sim$ 30 pointings which would have been required if only 8 m dishes had been used. With 10 times the number of pointings to process, the cpu time required to image a mosaic of a given size will increase by a factor of 10. The toy simulation which demonstrates heterogeneous mosaicing works produced an image with only about 1000:1 dynamic range. Error-free mosaic simulations with 40 elements typically produce much higher dynamic range images. We attribute the lower than expected dynamic range to the fact that the point spread functions for each logical pointings (ie, one with $13 \cdot 12/2$ baselines, one with $27 \cdot 26/2$ baselines, and one with $27 \cdot 13$ cross baselines) had higher side lobes than 40 elements in a homogeneous array ($40 \cdot 39/2$ baselines). A heterogeneous mosaic with 40 8 m antennas and 25-35 15 m antennas would have higher dynamic range, but not as high as the same number of elements in a homogeneous array. However, we expect that pointing errors will be a more serious limitation to the image quality than the higher side lobes in each logical pointing, especially for the 15 m dishes which will have large pointing errors and a smaller beam (a future MMA Memo will address heterogeneous array mosaicing with pointing errors).

While there is still much work that needs to be done in the subject of heterogeneous mosaicing (ie, examination of pointing errors and total power considerations, optimization, etc), it is reasonable to assume at this time that there are no fundamental software limitations which will prevent us from making heterogeneous mosaic in the future.

The imaging sensitivity of a heterogeneous array mosaic will be somewhat more complicated than nD: it will be proportional to the quantity S defined as:

$$S_i = \sqrt{N_{BL_i}} D_i^2 \sqrt{B_i} \tag{9}$$

$$S = \sqrt{\sum_i S_i^2}, \tag{10}$$

where S_i is the imaging sensitivity of the i^{th} primary beam type, N_{BL_i} is the number of baselines that have the i^{th} primary beam type, D_i is the effective dish diameter (the geometric mean in the case of the cross baselines), and B_i is the area on the sky covered by the i^{th} primary beam. The individual sensitivities S_i for each different primary beam type (ie, 8x8, 15x15, and 8x15) add in quadrature as they are essentially different arrays observing the same object.

3.2 Frequency Dependent Sensitivity Losses

Because the proposed 8 m, 12 m, and 15 m antennas have different pointing and surface specifications, a proper comparison should look at the array sensitivities over a range of frequencies. To properly do this, we must consider the following frequency dependent effects:

- The system temperature will increase with frequency as the receiver temperatures and the atmospheric transmission degrade. Assuming $T_{rec} = 4h\nu/k$ and otherwise the same analysis as R. Brown's "Uniform Assumptions for the Atacama Array Workshop" analysis, we adopt the following numbers:

Freq [GHz]	T_{sys} [K]
230	65
350	110
650	280
850	330

- Ruze losses due to wide angle scattering off of the surface irregularities will result in a decreased efficiency of $e^{-(4\pi\sigma/\lambda)^2}$. The Ruze formula is actually too pessimistic as σ gets large, due to the fact that when surface errors cause wave front deformations of more than π radians the wave front may actually be more in phase, and a correction has been made to account for this effect (Levy, 1996).

- For wide field imaging, pointing errors will result in poor imaging, as emission is scattered through the image. This effect cannot be quantified as a sensitivity loss, but will be addressed in an upcoming MMA Memo. For single pointing imaging, extreme pointing errors cause antenna-based amplitude fluctuations, resulting in a loss of sensitivity. We quantify this loss of sensitivity via pointing error simulations and find:

Freq [GHz]	D=8m, PE=1$''$	D=12m, PE=1$''$	D=15m, PE=1$''$.5
230	1.00	1.00	0.98
350	1.00	0.99	0.96
650	0.99	0.96	0.87
850	0.97	0.94	0.81

With a 13% sensitivity loss at 650 GHz and a 19% loss at 850 GHz, the 15m dish will sometimes experience extreme pointing deviations during which the visibility amplitude will drop substantially. It will be difficult to correct for the effect of these amplitude fluctuations on imaging, and they will help set the upper frequency at which the 15 m dish will work.

3.3 Point Source Sensitivities

For the six array options mentioned above, we calculate the rms image noise for single pointing observations of 1 minute, 8 Ghz bandwidth, and two polarizations, for frequencies ranging from 230 GHz to 850 GHz. For the two configuration options with both 8 m and 15 m antennas, we assume all possible correlations are performed. Ruze losses and pointing inefficiency are included, but no time losses due to phase calibration or pointing calibration are included. The point source sensitivities are shown in Table 1.

3.4 Wide Field or Imaging Sensitivities

For the six array options mentioned above, we calculate the rms image noise for observations of a source which is many pointings across in both directions, spending 60 s of time per square arcminute of source, with all other parameters being the same as in the point source sensitivity case. For the two configuration options with both 8 m and 15 m antennas, we assume all possible correlations are performed. The wide field imaging sensitivities are shown in Table 2.

In reviewing the entries in Table 2 for 40 8 m antennas plus 25 or 35 15 m antennas, remember that the 1.5 arcsecond pointing errors and the smaller beam of the 15 m will drastically limit its usefulness in mosaicing at high frequencies. The 15 m dishes will probably not be used at all in the 650 and 850 GHz bands. At 350 GHz, the 15 m dish's pointing errors will set a limit to the image quality which can be achieved. Bright sources which could possibly exceed this image quality limit would be mosaiced with only the 8 m antennas, but weak sources which would be limited by thermal noise could employ both 8 m and 15 m dishes for mosaicing. The image quality of the 12 m mosaics will also degrade more quickly than the 8 m, but not as drastically as the 15 m. However, the 12 m option cannot rely upon smaller dishes for better mosaic image quality at the highest frequencies.

3.5 Wide Field Brightness Sensitivities at Constant Resolution

Using model arrays with approximately 50% filling factor, we have calculated the increase in point source noise which results from tapering to a $3''\!.5 \cdot 230$ GHz/ν and a $7''\!.0 \cdot 230$ GHz/ν beam, applied this noise increase to the values in Table 2, and converted the noise values into brightness sensitivities. These values for the wide field surface brightness sensitivities of the various array options are presented in Tables 3 and 4

4 Discussion

From Tables 1 through 4 we see that the homogeneous and heterogeneous arrays actually have very similar sensitivities. The heterogeneous options are a bit better than the homogeneous options (compare the "poor" homogeneous array to the "poor" heterogeneous array, and the "rich" homogeneous array to the "rich" heterogeneous array). For the heterogeneous arrays, the addition of the 40 8 m antennas adds a small improvement to the point source sensitivity. The cross-correlations between dishes of different diameter are not so important to point source sensitivity either. However, the 8 m antennas, with their wider beams, make a strong contribution to the imaging and imaging surface brightness sensitivities (as do the inter-dish cross-correlations).

So, it appears that both the homogeneous and the heterogeneous options are acceptable from a sensitivity point of view. A decision between the collaboration options must be made on other issues such as image quality (ie, high frequency mosaicing), scientific flexibility of the instruments, engineering considerations for a single or multiple antenna designs, software considerations, and political factors.

References

Cornwell, 1988, "Radio-interferometric imaging of very large objects", A&A 143, 77.
Cornwell, Holdaway, and Uson, 1993, "Radio-interferometric imaging of very large objects: implications for array design", A&A 271, 697-713.
Levy, Roy, 1996, *Structural Engineering of Microwave Antennas*, IEEE Press, New York, p. 288.

Table 1: Reciprocal point source sensitivity: noise in mJy for a single pointing for the six studied array options for a 60 s continuum observation with 8 GHz bandwidth and two polarizations.

Freq [GHz]	50x12m	60x12m	40x8m plus 25x15m	40x8m plus 35x15m	40x8m	35x15m
230	0.043	0.036	0.038	0.030	0.120	0.040
350	0.083	0.069	0.073	0.057	0.219	0.078
650	0.352	0.293	0.301	0.241	0.785	0.352
850	0.678	0.564	0.563	0.459	1.303	0.718

Table 2: Reciprocal imaging sensitivity: noise in mJy for imaging a very large source in continuum, spending 60 s of time per square arcminute of source.

Freq [GHz]	50x12m	60x12m	40x8m plus 25x15m	40x8m plus 35x15m	40x8m	35x15m
230	0.16	0.13	0.14	0.12	0.29	0.18
350	0.46	0.38	0.40	0.34	0.81	0.54
650	3.60	2.99	3.01	2.54	5.35	4.50
850	9.06	7.54	7.12	6.13	11.62	12.01

Table 3: Reciprocal imaging brightness sensitivity in mK for 50% filled arrays tapered to a resolution of $3\farcs5 \cdot 230$ GHz/ν, spending 60 s of time per square arcminute.

Freq [GHz]	50x12m	60x12m	40x8m plus 25x15m	40x8m plus 35x15m	40x8m	35x15m
230	0.38	0.32	0.33	0.28	0.55	0.54
350	1.12	0.95	0.93	0.81	1.52	1.57
650	8.73	7.41	6.88	6.10	10.09	13.03
850	21.97	18.66	16.28	14.73	21.91	34.75

Table 4: Reciprocal imaging brightness sensitivity in mK for 50% filled arrays tapered to a resolution of $7\farcs0 \cdot 230$ GHz/ν, spending 60 s of time per square arcminute.

Freq [GHz]	50x12m	60x12m	40x8m plus 25x15m	40x8m plus 35x15m	40x8m	35x15m
230	0.17	0.14	0.15	0.12	0.21	0.22
350	0.50	0.40	0.43	0.35	0.57	0.64
650	3.88	3.15	3.17	2.66	3.79	5.28
850	9.77	7.94	7.51	6.42	8.24	14.09