Reference pointing of LSA/MMA antennas

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November 3, 1997

Abstract

We investigate the accuracy of pointing measurements for proposed versions of the LSA/MMA array. We show that in interferometric mode, the sensitivity of pointing measurements is governed, expressed as a fraction of the beam width, by the product of the surface of one antenna by the square root of the number of antennas, and thus does not scale like the total collecting power. Frequent pointing calibration to the accuracy needed for mosaics should be possible, relaxing the need for pointing stability on time scales longer than 30 min. In that respect we have checked the pointing performance of the Plateau de Bure antennas; the pointing accuracy in September weather is nearly always limited by atmospheric seeing (independently of operating frequency).

1 Introduction

Table 1: Pointing specifications for the MMA/LSA antennas

Diameter	Beam	Pointing	
	(300 GHz)	map	mosaic
8m	32″	3.2″	1.1″
10m	25″	2.5″	0.8″
12m	21″	2.1″	0.7″
15m	17″	1.7″	0.6″

The pointing needs for the antennas considered for the proposed LSA/MMA collaboration are summarized in Table 1. The requirement to point at a tenth of a beam width seems sufficient for good quality imaging in a field smaller than the primary beam width. However there is a lot of concern for being able to image fields larger than one beam width by mosaicing. From the work of Holdaway ([1997]) it appears that mosaics with high dynamic range (~ 500) require pointing to 1/30th of a beam width, hence the last column in Table 1. Note however that anomalous refraction (Holdaway [1997]) limits pointing accuracy to about $0.5''$ for a large fraction of the available time.

In this memo I try to address the following topics: (i) Will sensitivity allow the pointing be calibrated to such an accuracy, and how frequently? (ii) What is our experience with pointing calibration of the 15m Plateau de Bure antennas?

2 Sensitivity of pointing calibration measurements

The most efficient way to measure the pointing offsets is to move the antennas to a continuum source and perform a five point map. From the measured baseline amplitudes one has to determine the antenna amplitude gains as a function of antenna positions. Then, for each antenna, the gain variation as a function of antenna offsets is used to find the pointing offsets.

2.1 Sensitivity of antenna amplitude gain determination

Assume we have an interferometer with N antennas. We want to determine the power gain g_i of antenna i, by observing a point source of flux density S.

The quantities we measure are the baseline amplitudes: Let us note b_{ij} the amplitude of the correlation product of the outputs of antennas i and j.

One has:

$$b_{ij} = S_\nu \sqrt{g_i g_j}$$

which may be written, since the power gains g_i are positive:

$$\beta_{ij} = \gamma_i + \gamma_j$$

where $\beta_{ij} = \log b_i j/S$, and $\gamma_i = 1/2 \log g_i$. We thus have $N(N-1)$ linear equations to solve for the N unknowns γ_i. Such a system is usually solved using the method of least squares. One minimizes:

$$\sum_{j \neq i} (\gamma_i + \gamma_j - \beta_{ij})^2$$

for which the N conditions are:

$$\sum_{j \neq i} (\gamma_i + \gamma_j - \beta_{ij}) = 0 \qquad [i = 1, \ldots, N]$$

which may be rewritten as:

$$(N-2)\gamma_i + \sum_{j=1,N} \gamma_j - \sum_{j \neq i} \beta_{ij} = 0 \qquad [i = 1, \ldots, N]$$

Adding these equations one obtains:

$$(N-1) \sum_{j=1,N} \gamma_j = \sum_{j=1,N} \sum_{k>j} \beta_{jk}$$

It is then straightforward to substitute this back and get:

$$\gamma_i = \frac{1}{N-2} \sum_{j \neq i} \beta_{ij} - \frac{1}{(N-1)(N-2)} \sum_{j=1,N} \sum_{k>j} \beta_{jk}$$

Here the second term contains *all* the baseline amplitudes. This formula is derived in a much more elegant way (and in French) by E. Anterrieu ([1992]). Let's rewrite it in a slightly different way:

$$\gamma_i = \frac{1}{N-1}\sum_{j\neq i}\beta_{ij} - \frac{1}{(N-1)(N-2)}\sum_{j\neq i}\sum_{k\neq i,>j}\beta_{jk}$$

Now the first term contains all baselines connected to antenna i, the second one contains all the other baselines; for instance for 3 antennas, one obtains the well-known formula:

$$\gamma_1 = \frac{1}{2}(\beta_{12}+\beta_{13}) - \frac{1}{2}\beta_{23}; \qquad g_1 = b_{12}b_{23}/b_{23}$$

Now all the β_{ij} contain noise terms which are uncorrelated. Then for the corresponding r.m.s. fluctuations we get:

$$\overline{\delta\gamma_i^2} = \left(\frac{1}{N-1}\right)^2 \sum_{j\neq i}\overline{\delta\beta_{ij}^2} + \left(\frac{1}{(N-1)(N-2)}\right)^2 \sum_{j\neq i}\sum_{k\neq i,>j}\overline{\delta\beta_{jk}^2} \tag{1}$$

In the large signal-to-noise limit: $\delta\gamma_i = 1/2\delta g_i/g_i$, $\delta\beta_{ij} = \delta b_{ij}/b_{ij}$. Let us assume further that all antennas have the same gain $g_i = G$ and sensitivity: $\delta b_{ij} = \sigma$:

$$\overline{\delta G^2} = 4\frac{\sigma^2}{S_\nu^2(N-1)^2}\left(N-1+\frac{(N-1)(N-2)}{2(N-2)^2}\right)$$

$$\sigma_G = 2\frac{\sigma}{S_\nu}\sqrt{\frac{2N-3}{2(N-1)(N-2)}}$$

The rms of the power gain is thus behaving like $2\sigma/(S_\nu\sqrt{N})$ in the large N limit. This is because in Eq. 1 the first $(N-1)$ terms are going to dominate the summation when N is large, since the other $(N-1)(N-2)/2$ are multiplied by a $1/(N-2)^2$ factor. The rms gain also diverges for $N<3$: it is well-known that it is not possible to measure the gain of a single antenna in a two-element interferometer. This formula slightly differs from that of Cornwell and Fomalont ([1989]); the asymptotic behaviour is the same ($\sigma/(S_\nu\sqrt{N})$ for the *amplitude* gains) but their result diverges for $N=3$.

In the case of heterogeneous arrays the previous analysis has to be refined; We do this in Appendix A. The result for a large number of antennas is simply:

$$\sigma_{G_1} = 2 * \frac{\sigma 1}{S_\nu}\sqrt{\frac{\pi D_1^2}{A}}$$

where G_1 is the gain of kind 1 , $\sigma 1$ the rms in one baseline connecting two antennas of kind 1, and A is the total collecting area of the array.

2.2 Sensitivity of pointing offset determination

The 1σ error Δx on the position measurement using the five point method, assuming a Gaussian beam of half-power size $\theta_{\rm B}$, and using offsets of $\frac{\theta_{\rm B}}{\sqrt{8\log 2}}$, is given by:

$$\Delta x = \frac{\sigma_G \theta_B \sqrt{e}}{4\sqrt{\log 2}} \sim 0.495 \Delta g_i \theta_B$$

or

$$\Delta x = 0.99 \frac{\sigma \theta_B}{S_\nu \sqrt{N}}$$

But it is more efficient to move the antennas so that one out of five is always pointed. Then each pointed antenna is worth 4 displaced ones, in terms of final signal to noise ratio. Thus

$$\Delta x \sim 0.99 \frac{\sigma \theta_B}{S_\nu \sqrt{8N/5}} \sim 0.80 \frac{\sigma \theta_B}{S_\nu \sqrt{N}}$$

Let us now estimate σ for N antennas of diameter D, at frequency ν. Assuming a band with of 8 GHz and an integration time of τ seconds:

$$\sigma = 24 \left(\frac{15}{D}\right)^2 \frac{T_{\rm SYS}}{100} \tau^{-0.5} \quad \text{mJy}$$

then using $\theta_B = 51(100/\nu)(15/D)''$:

$$\Delta x = \frac{190}{S_\nu} \left(\frac{15}{D}\right)^3 \frac{T_{\rm SYS}}{\nu} \left(\frac{40}{N\tau}\right)^{0.5}$$

Note that the dependence on frequency is to first order only through the source flux S_ν, since one may assume $T_{\rm SYS}/\nu \sim 0.5$ at all frequencies.

The actual duration of the pointing measurement can be estimated as $\tau_p = 5\tau + 10 + 2 * \alpha_p$ where α_p is the angular distance (in degrees) to find a suitable pointing source. Here I assume a settling time of 2 seconds between two consecutive points in the five-point map, and a slewing rate of 1 degree per second.

If we want to be able to point frequently (say every 15 minutes), and are willing to spend less than 10% of the time on pointing, then a practical upper limit on τ is 15 seconds; we do not wish to go further than $\alpha_p \sim 5$ degrees, since a higher value will put severe constraints on the accuracy on the pointing model.

Then the minimum usable flux is set by:

$$S_\nu = \frac{25}{\Delta x} \left(\frac{15}{D}\right)^3 \left(\frac{40}{N}\right)^{0.5}$$

Values for different array options are given in Table 2. The pointing goal there is to obtain a rms pointing error of $\theta_B/30$ at 300 GHz. Thus we set Δx to half of this value so that the measurement error does not contribute significantly to the rms pointing error. This sets the minimum flux; the density of sources above that minimum flux is estimated from Holdaway et al. ([1994]). It appears that the 40×8m may have serious difficulties to find enough pointing sources, while the proposed large homogeneous arrays should have enough sensitivity for frequent, high accuracy pointing measurements, thus relaxing the requirements on pointing stability on time scales longer than about 30 minutes.

For heterogeneous arrays with (N_1, N_2) antennas of sizes (D_1, D_2), the minimum flux becomes (see Appendix A):

$$S_\nu = \frac{25}{\Delta x} \left(\frac{15}{D}\right)^2 \left(\frac{40 \times 15^2}{N_1 D_1^2 + N_2 D_2^2}\right)^{0.5}$$

The results in Table 2 are then unchanged if part of the arrays are replaced by antennas of different sizes, provided the total collecting area is conserved. Note that here the need to correlate the smaller antennas with the larger ones is essential.

We have assumed here that the pointing could be calibrated at any frequency. It is of course desirable to calibrate the pointing at the observing frequency. Different aperture illuminations at widely different frequencies combined with time-dependent structural deformations might cause a time variable pointing difference between the two beams. This should be more closely investigated.

Table 2:
Weakest usable pointing calibrator (integration τ = 15s). The minimum flux does not depend on frequency (assuming $T_{\rm SYS}/\nu$ = 1K/GHz), while the source counts apply only at 90 GHz. The last column is the angular radius of the cone in which one pointing source is to be found on average.

N	D (m)	Δx ($''$)	S_ν (mJy)	$n(S_\nu)$ (sr^{-1})	α_p (deg.)
128	8	0.53	173	166	2.5
90	10	0.43	132	249	2.0
64	12	0.35	109	333	1.8
40	15	0.28	88	457	1.5

3 Performance of Plateau de Bure antennas

3.1 Pointing measurements

We have tested the performance of the Plateau de Bure antennas by doing repeated pointing measurements on strong sources. The measurement technique was either cross scans of about 30 seconds duration, or five point maps with 10 second integrations on each of the five points. After each measurement the pointing corrections were updated. Focusing was also done on the same sources.

The sources were 2230+114, 3C454.3, 0923+392; their fluxes were in the range 4 to 6 Jy. The 1σ measurement errors were $0.12-0.20''$ for the five point method and $0.25-0.50''$ for the cross scan method. The results are summarised in Table 3. Only four antennas were available. The measured pointing errors show a slow drift (a few seconds per hour), and short term fluctuations. The results are shown in Figs. 1 to 5. For each day I have computed the residual r.m.s. deviation after removing a polynomial baseline (linear for the short experiments, of degree 2-4 for the long ones).

3.2 Effect of Wind

It is clear from Table 3 and the Fig 6 that the mechanical effect of wind on the antennas is not the dominant cause of pointing fluctuations. The night with the strongest wind (October $1^{\rm st}$) had actually one of the lowest r.m.s pointing fluctuations. On that night the elevation pointing of antennas 4 and 5 (and to some extent antenna 1) showed a drift by about $5''$ in about half an hour while the wind increased from 6 to 8 m/s (fig. 5).

3.3 Seeing

Anomalous refraction was first observed with the 30-m telescope (Altenhoff et al. [1987]). It was observed to occasionally move the source images by a large fraction of the beam width. It is well recognized that it is due to the same random fluctuations of the atmospheric water content, constantly observed by millimeter interferometers, which would limit the angular resolution of our synthesis maps to about one arc second on average nights, without the help of the radiometric phase correction. In that case the term 'seeing' seems more appropriate, in relation with optical astronomy. These random fluctuations persist on the scale of a single dish; a linear variation in the water content across the telescope aperture causes a linear phase gradient and thus a deviation of the beam.

The atmospheric rms phase fluctuation σ_ϕ in an interferometric observation is generally related to the baseline length b by a relation of the form $\sigma_\phi(b) \propto b^\beta$ (Olmi and Downes [1992]). The power law exponent β is in the range $0.4-0.8$. If one extrapolates this relation to scales smaller than the antenna size, one may predict the amplitude of the random pointing deviations (seeing) caused by these phase fluctuations:

$$\sigma_\alpha = \frac{\sigma_\phi(D)}{2\pi}\frac{\lambda}{D}$$

Table 3: Summary of pointing measurements

19 Sep.	Wind: 2.1 m/s Elevation ~ 42.0							
night	Seeing(Az) .31 Seeing(El) .40							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.38	.70	.36	.48	.48	.41	.71	.94
Tracking rms	.57	.37	.33	.76	.35	.23	.27	.18
Measurement rms	.52	.44	.20	.26	.38	.23	.23	.24
20 Sep.	Wind: 4.5 m/s Elevation ~ 35.0							
noon	Seeing(Az) 2.90 Seeing(El) 3.56							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	2.13	1.43	2.42	1.64	2.71	3.35	3.89	3.17
Tracking rms	.36	.25	.25	.46	.35	.29	.31	.25
Measurement rms	1.18	1.01	1.02	.98	.98	.95	.94	.98
21 Sep.	Wind: .0 m/s Elevation ~ 75.0							
sunrise	Seeing(Az) .69 Seeing(El) .71							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.83	.60	.85	.40	1.06	.96	2.27	.93
Tracking rms	.38	.27	.27	.39	.39	.28	.30	.29
Measurement rms	.53	.46	.23	.22	.36	.30	.29	.42
21 Sep.	Wind: .0 m/s Elevation ~ 40.0							
day	Seeing(Az) 3.21 Seeing(El) 4.23							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	2.61	2.12	2.76	2.46	3.11	2.94	2.80	3.14
Tracking rms	.38	.27	.27	.39	.39	.28	.30	.29
Measurement rms	.73	.70	.54	.58	.61	.52	.55	.60
25 Sep.	Wind: 2.0 m/s Elevation ~ 32.0							
day	Seeing(Az) .69 Seeing(El) 1.01							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.51	.65	.94	.34	1.40	1.00	1.29	1.05
Tracking rms	.47	.58	.26	.67	.20	.13	.23	.18
Measurement rms	.12	.12	.12	.12	.12	.12	.13	.12

Table 4: Summary of pointing measurements (continued)

28 Sep. night	Wind: 2.5 m/s Elevation ∼ 50.0 Seeing(Az) .59 Seeing(El) .69							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.84	.65	.74	.37	.83	.58	.82	.79
Tracking rms	2.67	.49	.29	.79	.35	.22	.55	.45
Measurement rms	.12	.12	.12	.12	.12	.12	.13	.12
29 Sep. noon	Wind: 2.5 m/s Elevation ∼ 65.0 Seeing(Az) 1.54 Seeing(El) 1.57							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.63	.85	.62	.74	1.15	.77	1.10	.92
Tracking rms	.54	.52	.29	.60	.28	.26	.21	.19
Measurement rms	.14	.13	.14	.14	.14	.14	.15	.14
29 Sep. afternoon	Wind: 2.0 m/s Elevation ∼ 65.0 Seeing(Az) 3.19 Seeing(El) 3.42							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	1.19	1.66	2.02	1.89	1.90	2.42	2.27	1.47
Tracking rms	1.25	.42	.40	.60	.38	.32	.39	.30
Measurement rms	.14	.15	.16	.15	.15	.15	.15	.15
01 Oct. night	Wind: 7.0 m/s Elevation ∼ 60.0 Seeing(Az) .50 Seeing(El) .54							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.52	.57	.36	.47	.83	.72	1.33	.84
Tracking rms	1.12	.60	.34	.83	1.02	.26	.31	.22
Measurement rms	.18	.18	.19	.19	.18	.18	.19	.18
01 Oct. night	Wind: 8.5 m/s Elevation ∼ 50.0 Seeing(Az) .66 Seeing(El) .76							
Axis	Azimuth				Elevation			
Antenna	1	2	4	5	1	2	4	5
Pointing rms	.59	.56	.84	.71	.77	.71	1.08	.71
Tracking rms	2.11	.40	.39	1.03	.95	.31	.55	.33
Measurement rms	.18	.18	.19	.19	.18	.18	.19	.18

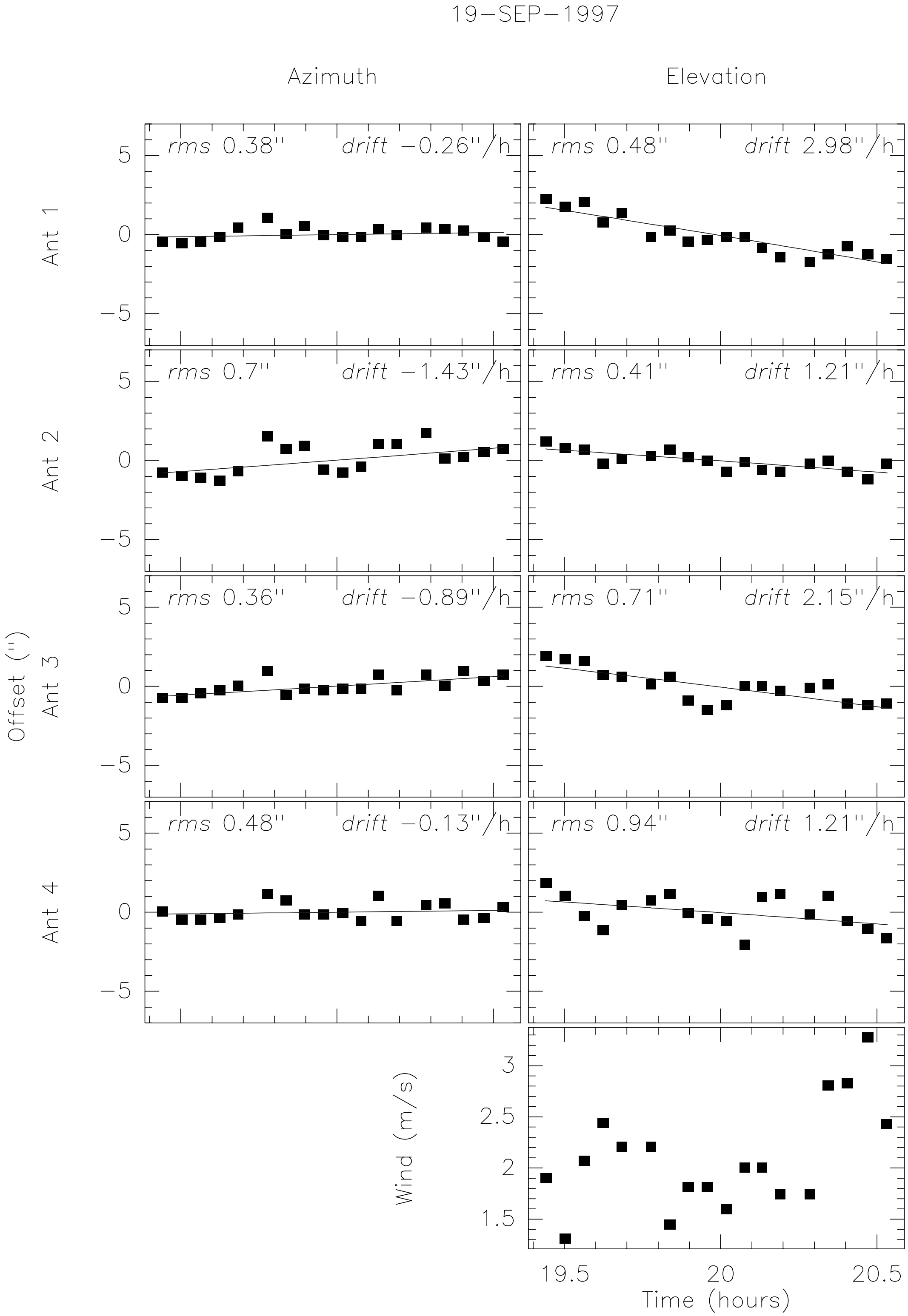

Figure 1: September 19th data

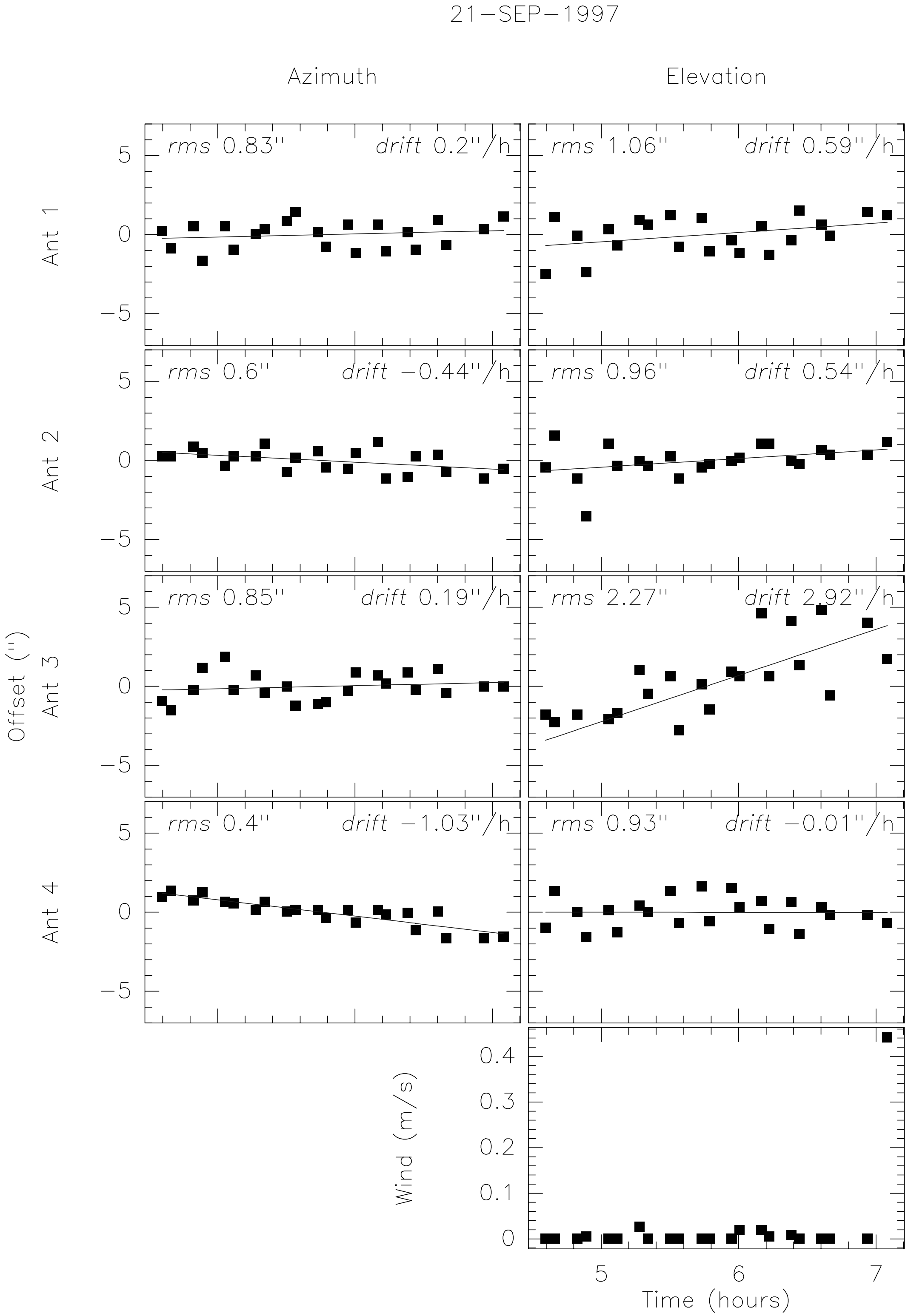

Figure 2: September 21st data

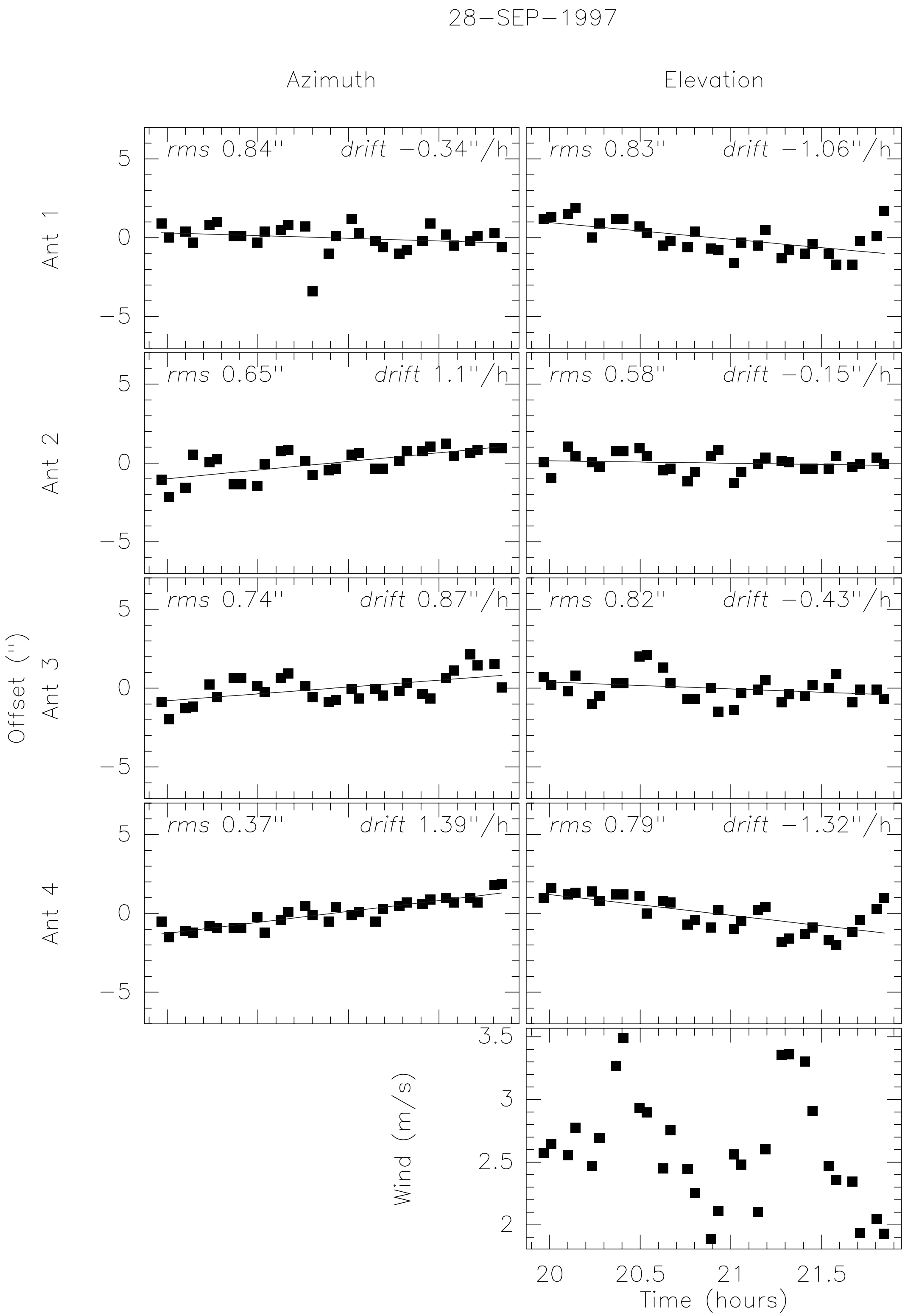

Figure 3: September 28th data

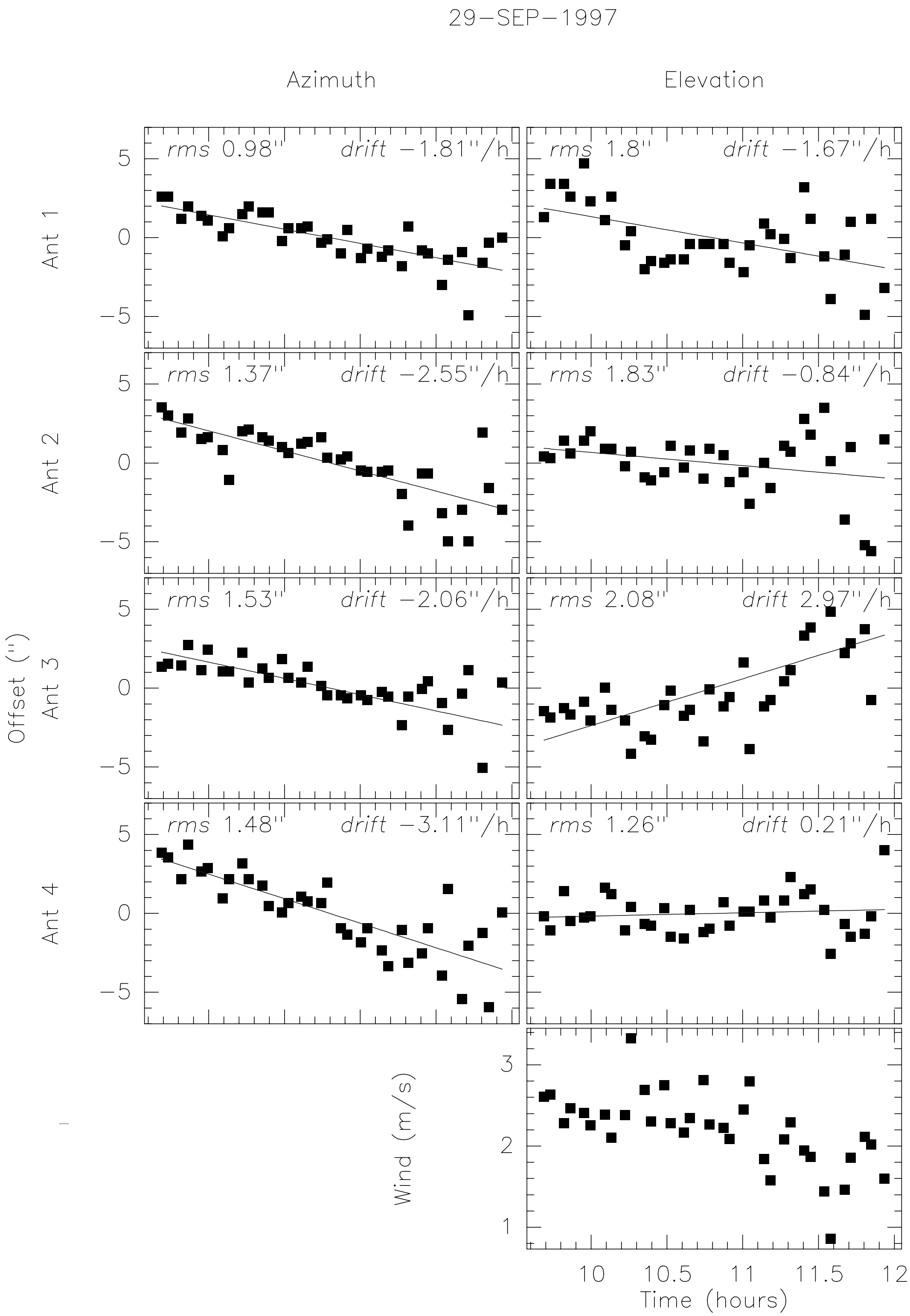

Figure 4: September 29th data

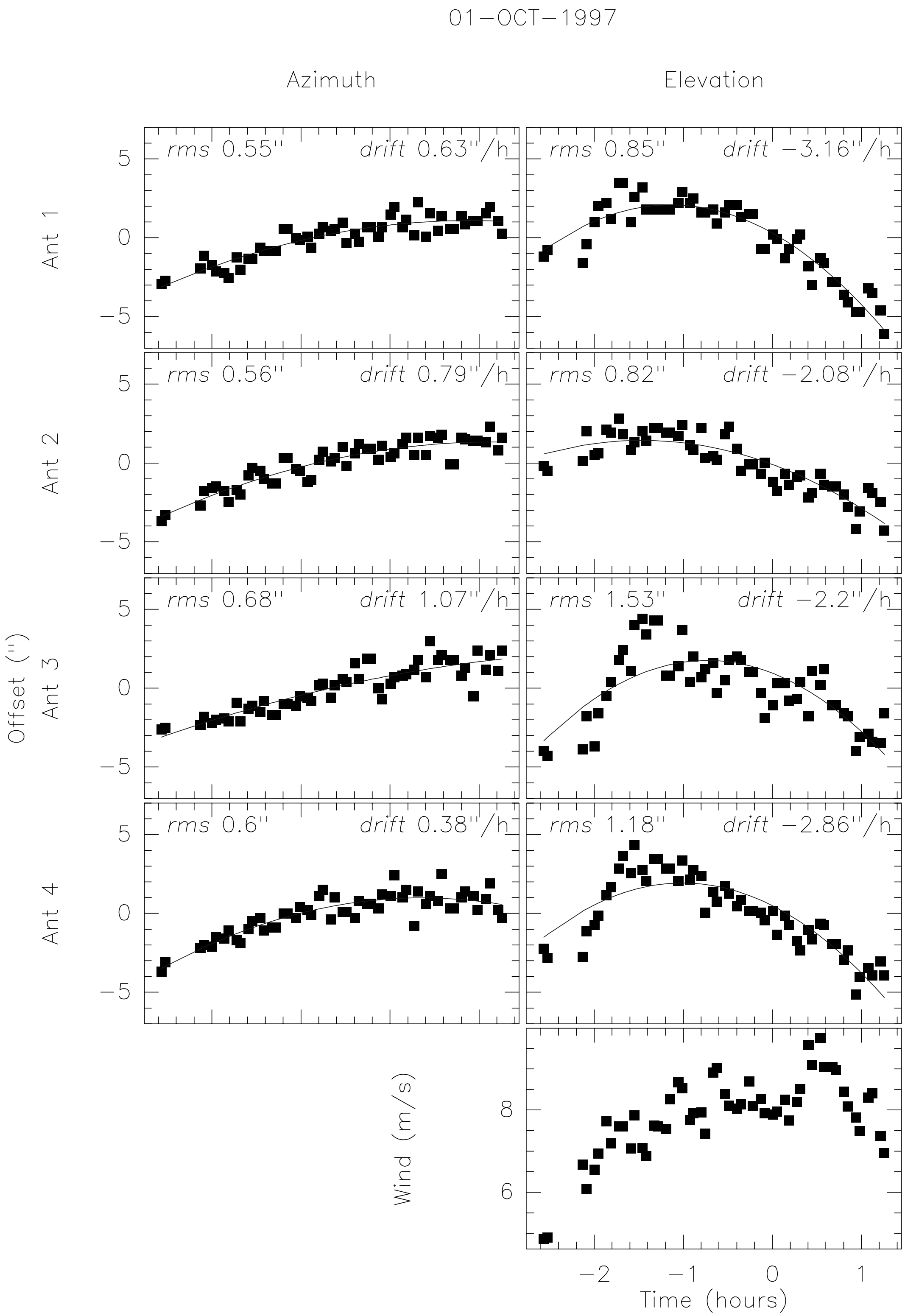

Figure 5: October 1st data. The wind direction was almost exactly North; The antennas were pointed to the South-West at 40-60 degrees elevation.

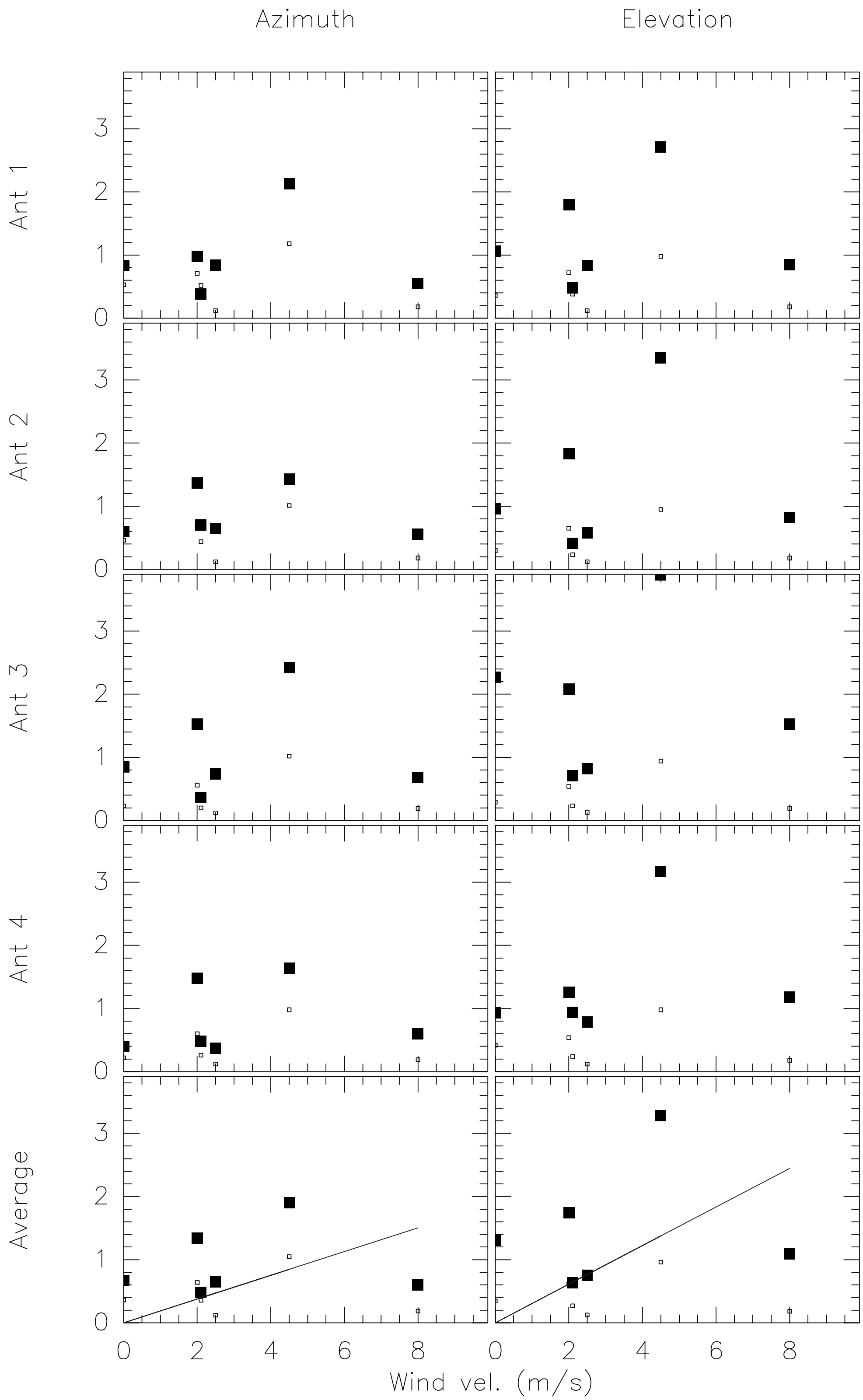

Figure 6: Attempt to correlate observed pointing errors with wind velocity.

This formula should actually only be valid for azimuth deviations; for elevation a more appropriate formula at elevation θ should be

$$\sigma_\theta = \frac{\sigma_\phi(D/\sin\theta)}{2\pi}\frac{\lambda}{D}$$

since the antenna beam intercepts an ellipse of axes D and $D/\sin\theta$ in each layer of the atmosphere.

For the above pointing data the information on phase fluctuations is available on six baselines in the range 24 to 64 m. I have assumed an index $\beta = 0.6$ and extrapolated the phase data down to 15-m scale to compute the seeing parameters σ_α and σ_θ. The observed pointing errors are plotted as a function of these parameters in Fig. 7. The pointing errors averaged on all antennas are also shown in Fig. 8.

It is clear that the observed pointing errors are well correlated with atmospheric seeing; however, as can be shown by averaging the data of the three points with highest fluctuations, the above method must be overestimating the seeing by about 50%. This is most probably due to different sampling times in the pointing measurements and the phase measurements. The individual pointing scans were separated by longer time intervals than the typical integration time of phase measurements (actually estimated from the pointing scans themselves). One should ideally do more frequent pointing scans, and interrupt them from time to time to perform longer on-source integrations, suitable to sample the temporal structure function of the phase fluctuations. The seeing parameter could have also been overestimated if the exponent β was systematically higher for scales lower than 24m.

On the other hand, the points with lowest fluctuations in Fig. 8 do lie above the straight line; this could be due to overestimation of the pointing error r.m.s. (at this level the measurement error may contribute to the statistics). However measured tracking errors are in the $0.2 - 0.5''$ range and must contribute to the observed r.m.s.

In a recent memo Holdaway ([1997]) computed the expected pointing degradation due to seeing on the Chajnantor site. He found that r.m.s. deviations of $0.5''$ should be expected about half of the time.

4 Conclusions

- The absolute precision to which the pointing of array elements can be measured scales like the inverse cube of their diameter and the inverse square root of their number. Therefore larger antennas are favored.

- Frequent pointing calibration should be possible with the LSA/MMA antennas, in order to reach a pointing accuracy suitable for high dynamic range mosaics ($0.3-0.5''$, depending on the antenna size), provided the tracking is able to reach that accuracy. This could relax the need for pointing stability on time scales longer that 30 min.

- Measured on a 1 hour time scale, the pointing stability of the Plateau de Bure antennas is about $\sim 1''$/hour r.m.s. in low wind conditions ($< 5m/s$)

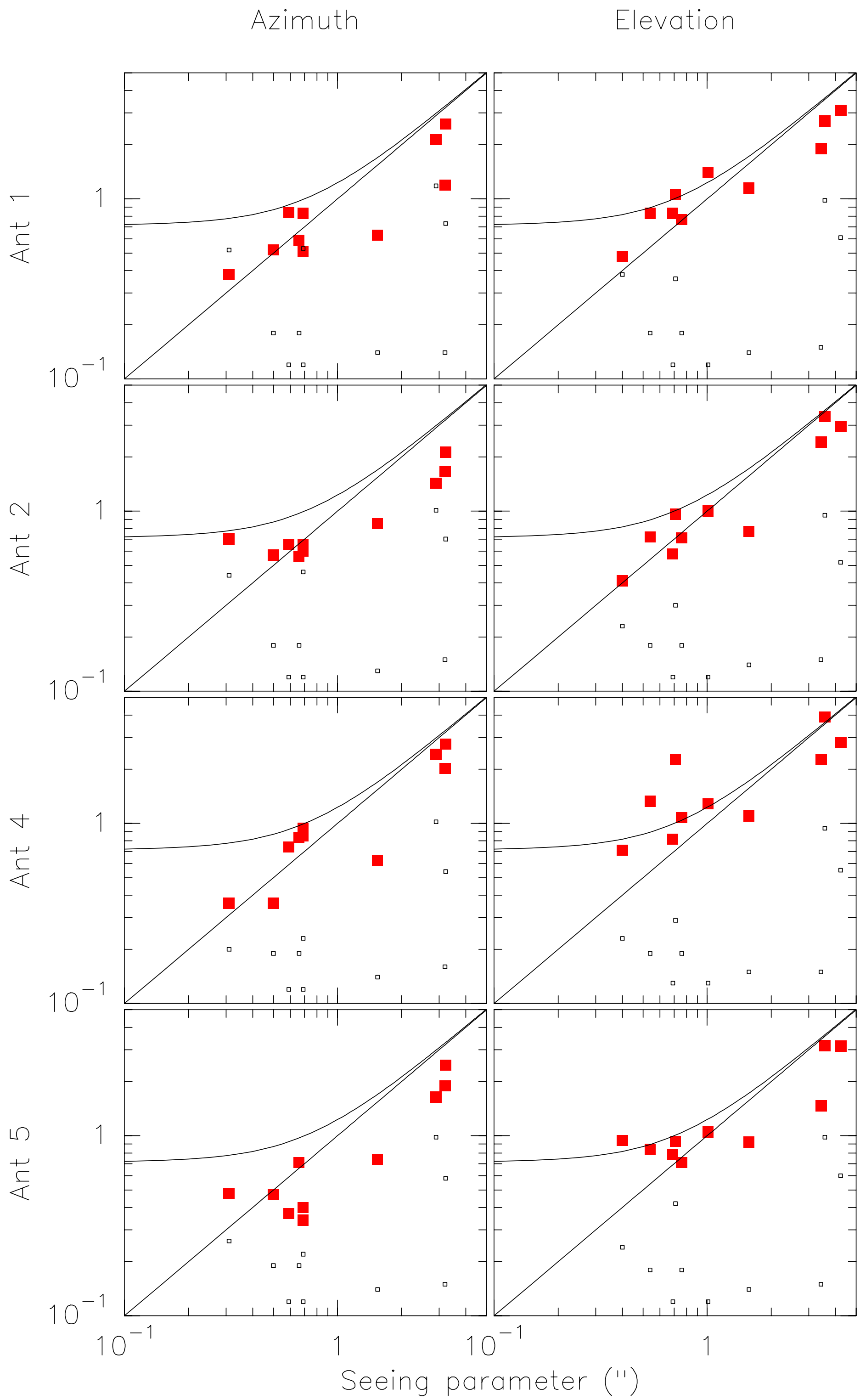

Figure 7: Correlation of observed pointing errors with the seeing parameter. The big squares are the observed pointing r.m.s. deviations in arc seconds; the small squares indicate the measurement r.m.s. errors. The curved lines represents $\sqrt{s^2+0.7^2}$ where s is the seeing parameter

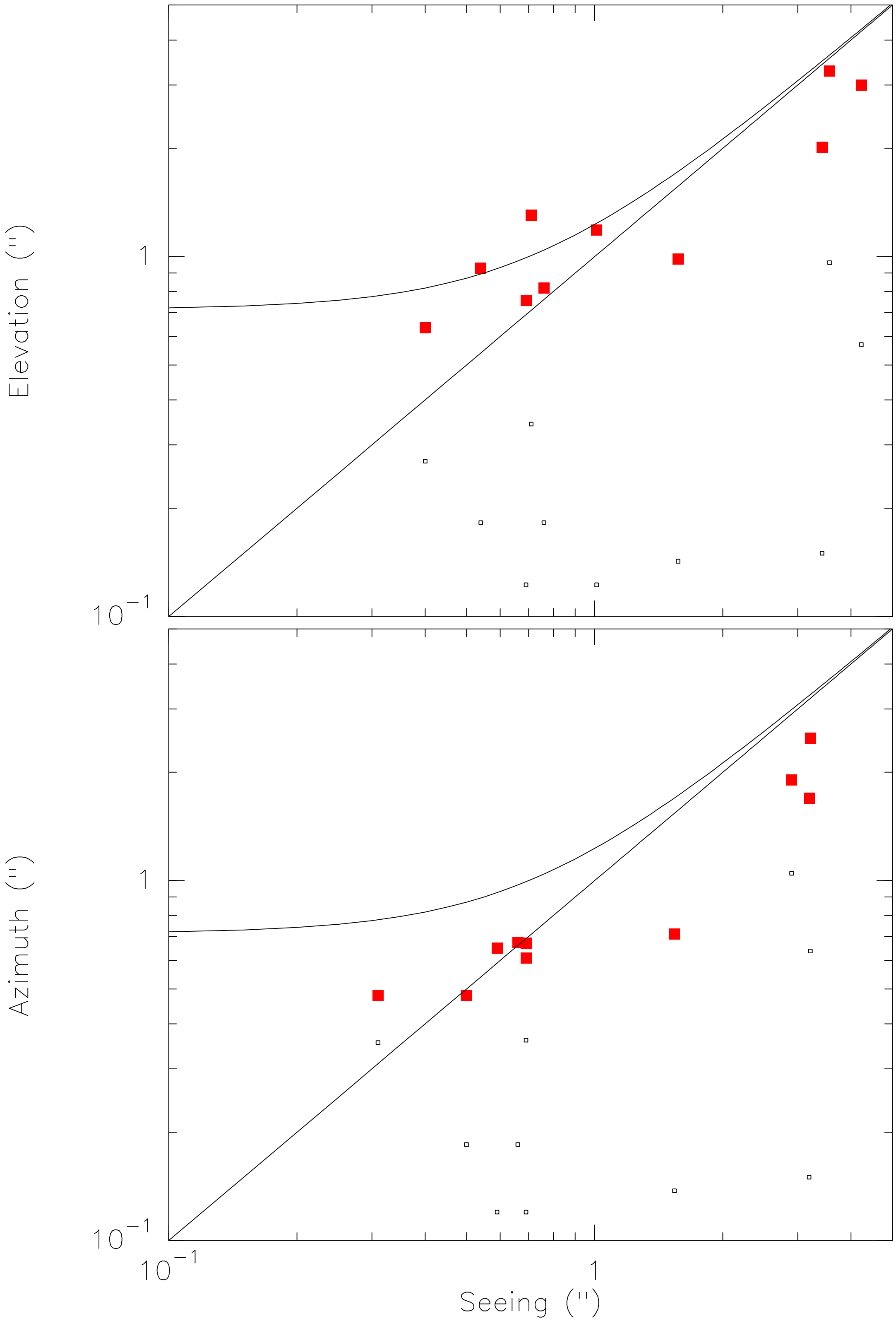

Figure 8: Correlation of observed pointing errors with the seeing parameter: same as Fig. 7, for the average of the four available antennas

- On shorter time scales the specifications ($0.7''$ rms on each axis) seem well fulfilled; the accuracy is actually $\sim 0.5''$ in azimuth.

- More measurements are needed in good seeing conditions ($< 0.5''$) with strong sources to investigate the limiting performance of the tracking.

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A Gain determination in heterogeneous arrays

Assume we have a heterogeneous array with N antennas, of diameters D_i. The r.m.s. visibility rms on baseline ij is:

$$\overline{\delta b_{ij}^2} \propto \frac{T_{\mathrm{SYS}i} T_{\mathrm{SYS}j}}{D_i^2 D_j^2}$$

In the least square analysis one should then weigh each baseline by $w_i w_j$ where $w_i \propto D_i^2$ (assuming the system temperatures equal). One may normalize the weights by

$$\sum_{i=1,N} w_i = 1$$

Now the least squares analysis gives:

$$\sum_{j \neq i} w_j (\gamma_i + \gamma_j - \beta_{ij}) = 0$$

which may be rewritten:

$$\gamma_i(\sum_{j\neq i} w_j) + \sum_{j\neq i} w_j\gamma_j = \sum_{j\neq i} w_j\beta_{ij}$$

We define

$$\overline{\gamma} = \sum_{i=1,N} w_i\gamma_i$$

Then

$$\gamma_i = \frac{1}{1-2w_i}(\sum_{j=1,N} w_j\beta_{ij} - \overline{\gamma})$$

after multiplying by w_i and summation over i one may compute :

$$\overline{\gamma} = \sum_{j=1,N}\sum_{k>j} c_{jk}\beta_{jk}$$

where

$$c_{jk} = w_j w_k \frac{\frac{1}{1-2w_j} + \frac{1}{1-2w_k}}{1 + \sum_{j=1,N}\frac{w_j}{1-2w_j}}$$

and obtain a closed form for γ_i:

$$\gamma_i = \frac{1}{1-2w_i}\left(\sum_{j\neq i}(w_j - c_{ij})\beta_{ij} - \sum_{j\neq i}\sum_{k>j,\neq i} c_{jk}\beta_{jk}\right)$$

For the r.m.s. fluctuations we get:

$$\overline{\delta\gamma_i^2} = \frac{1}{(1-2w_i)^2}\left(\sum_{j\neq i}(w_j - c_{ij})^2\overline{\delta\beta_{ij}^2} - \sum_{j\neq i}\sum_{k>j,\neq i} c_{jk}^2\overline{\delta\beta_{jk}^2}\right)$$

However if we note σ_i the flux sensitivity of a baseline connecting antenna i with an *identical* antenna:

$$\overline{\delta\beta_{ij}^2} = \frac{D_i^2}{D_j^2}\frac{\sigma_i^2}{S_\nu} = \frac{w_i}{w_j}\frac{\sigma_i^2}{b_{ij}^2}$$

$$\overline{\delta\beta_{jk}^2} = \frac{D_i^2}{D_j^2}\frac{D_i^2}{D_k^2}\sigma_i^2 b_{jk}^2 = \frac{w_i^2}{w_j w_k}\frac{\sigma_i^2}{b_{jk}^2}$$

Then

$$\overline{\delta g_i^2} = 4\frac{\sigma_i^2}{S_\nu^2(1-2w_i)^2}\left(w_i\sum_{j\neq i} w_j(1-\frac{c_{ij}}{w_j})^2 + w_i^2\sum_{j\neq i}\sum_{k>j,\neq i}\frac{c_{jk}^2}{w_j w_k}\right)$$

Now if N is large enough, $w_j \sim 1/N$ and $c_{jk} \sim 1/N^2$. Neglecting all terms in $1/N$:

$$\overline{\delta g_i^2} = 4\frac{\sigma_i^2}{S_\nu^2}w_i$$

$$\sigma_{g_i} = 2\frac{\sigma_i}{S_\nu}\sqrt{\frac{\pi D_i^2}{A}}$$

which shows than when the number of elements N is large, the precision of the gain measurement for one element is only determined by the size of this element and the total collecting area.