MMA Memo 199: Cost-Benefit Analysis for the Number of MMA Configurations

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Abstract

We perform a new sort of cost/scientific-benefit analysis to determine the optimal number of configurations the MMA should have. We trade the costs of building extra configurations, moving antennas among them, and lost observing time, against the loss of sensitivity which results from tapering when a specific image resolution different from the natural resolution of the array is required. With our assumptions, we find that the optimal number of configurations is between five and eight, depending upon what fraction of the time tapering is required. However, four configurations is fairly close to the optimum.

1 Introduction

Up until now, we have assumed that the MMA, like the VLA, will have four different major configurations, with minor modifications of these configurations to enhance observing to the far north or far south. However, no justification has been given for four configurations, rather than three or five or six. With between 32 and 100 antennas, it would be possible to do many observations with a single configuration; however, to achieve high brightness sensitivity at low resolution, one would have to taper the array rather dramatically. Obviously it is more efficient to build multiple arrays which produced the desired resolutions. But what resolutions do the astronomers want? There will clearly be a wide distribution of resolutions, and we probably cannot settle on just a few required resolutions. If we build a wide variety of array configurations to cater to everyone's desired resolution, we also suffer from inefficiencies, as we will spend an inordinate amount of money to build the vast number of antenna pads and cabling, and will waste an unfortunate amount of time moving the antennas among the configurations.

This memo seeks to find that compromise between the few configurations (which require much tapering) and the many configurations (which require much construction and moving) which will optimize the integrated sensitivity, and hence the scientific output, of the MMA.

2 Cost-Benefit Approach

The cost-benefit approach we take in addressing the issue of the number of configurations required for the MMA assumes that we can trade off the extra costs associated with configurations (ie, pads, cables, antenna moving costs) for scientific benefit in the form of usable sensitivity (ie, instead of buying configuration, we buy more antennas).

In order to proceed, we will need to know a very lot about the Millimeter Array and how it will be used. Table 1 lays out the symbols for each quantity, what the quantity is, and our current estimate for that quantity.

3 Calculating the Observing Efficiency ϵ_o

3.1 Array Design

We assume that the array configurations are bounded by a most compact array with approximately filling factor f_{min} of 40%, and by a most extended array with maximum baselines b_{max} of 10^4 m. Further, we assume that all configurations except the most compact are basically ring arrays, and that the resolution of adjacent arrays are related by a resolution scale factor S. The maximum baseline of the compact array is given by

$$b_{compact} = D_a \sqrt{N_a / f_{min}}. \tag{1}$$

However, because the average baseline is shorter in this array than in a ring array, this array will have the same resolution as a ring array about 70% as large. Then, S is related to the number of configurations N_c and b_{max} and $b_{compact}$ by

$$S = R^{1/(N_c - 1)}, \tag{2}$$

or

$$N_c = \frac{\ln R}{\ln S} + 1, \tag{3}$$

where R is defined to be the ratio of the largest array and smallest array effective baselines, $b_{max}/(0.7 b_{compact})$.

3.2 Assumptions About Array Use

We need to assume the density of required resolutions for the MMA. To first order, we assume that the required resolutions will be constant in logrithmic bins. Since the array configurations have been designed with a constant resolution scaling factor S, this implies that each configuration will have the same proposal pressure. The VLA, which also has constant S configurations, finds very similar observing pressure on each of its four configurations, though the lowest resolution array has been slightly more oversubscribed than the other configurations for the last several years. While there are more sources which can be detected from the compact arrays, it will take longer to detect the fewer sources in the large arrays, so array use sort of balances out.

Many observers will simply take the natural resolution of the array configuration they observed in. An extreme case of this is the observation of a point source, which can be

Symbol	What it Means	Guess the Numbers
Array Design		
N_a	Number of Antennas	36
D_a	Diameter of Antennas	10m
N_c	Number of Configurations	2-12
S	Resolution scale factor between adjacent configurations	2-100
$N_\circ$	Number of Ants that don't move between adjacent configurations	3
b_{max}	Size of the largest array	3000 m
f_{min}	Filling factor of compact array	0.4
N_t	Number of Transporters	3
Costs		
C_a	cost of a fully outfitted antenna	\$3.55M
C_p	cost of an antenna pad	\$40k
C_c	cost of cabling to a pad	\$10k
C_t	cost of one transporter	\$1M
n_w	number of workers to move antenna	4
C_w	cost of worker for 1 hour	\$20
C_{rt}	cost to run and maintain the transporter per move	\$200
C_m	cost to move one antenna	$n_w \, t_m C_w + C_{rt}$
Operational Assumptions		
t_m	Time required to move one antenna	2 hour
t_w	Time each day available for outdoor work	10 hour
f_r	Fraction of total reconfiguration which is not science-usable	0.5
θ	Angular resolution	
$\rho(\theta)$	Density of angular resolutions from array observing pressure	(see below)
$f_\circ$	fraction of time configuration requires no tapering	0.0 or 0.5
$\psi(\theta)$	distribution of array sensitivity considering tapering	(see below)
ϵ_o	observing efficiency: how much sensitivity is lost to tapering?	
ϵ_r	reconfiguring efficiency: how much sensitivity is lost to reconfiguring?	
ϵ_t	total efficiency	$\epsilon_o \epsilon_r$

observed in any configuration large enough to avoid confusion. So, one component of the density of required resolutions is a set of delta functions at the full resolution of each array being considered.

However, sources which are not point sources have a maximum resolution at which they may be profitably observed. The astronomer is playing off resolution and brightness sensitivity. The resolution of one array may not be high enough to see what the astronomer needs to see, but the brightness sensitivity of the next larger array may not be high enough to permit the astronomer to see anything. Hence, in addition to the set of delta functions, there is also a continuous component of the density of required resolutions. Most observers who take an array's natural resolution (ie, the delta function crowd) are actually part of this continuous distribution, but allow themselves to be lumped into the delta function out of convenience. They might actually benefit from slightly higher resolution or slightly higher brightness sensitivity (and lower resolution), but there is no array configuration which can provide this, so they take the nearest configuration.

And finally, there are people who absolutely need images at non-standard resolutions. One fundamental analysis method used in millimeter astronomy is comparing lines of different molecular species, or different transitions of the same molecular species. In general, these will be at different frequencies and hence different resolutions. While the VLA was designed to be scalable for multi-frequency comparisons (ie, spectral index maps between 15 GHz in C array, 5 GHz in B array and 1.4 GHz in A array), the number of specific frequencies and resolutions which are important to the MMA is too large to optimize the configurations for just a few. Rather, we need to accept that astronomers will be making images of arbitrary resolution, and will have to taper sometimes to get the resolution they require.

We assume that some fraction of observations, $f_\circ$, will require no tapering (the delta function crowd), and the remaining observations for the i^{th} configuration will have a required resolution distribution of

$$\rho_i(\theta) = ((N_c - 1)\ln S)^{-1}\theta^{-1}. \tag{4}$$

The factor $((N_c - 1)\ln S)^{-1}$ normalizes $\rho_i(\theta)$ for the coordinate convention that $\theta = 1$ is the natural resolution of the array and S is the resolution scaling factor between arrays, or the most one would ever taper. This expression considers all resolutions greater than the resolution of the most compact configuration. The most compact configuration will also require tapering, but since the configuration is constrained by the close packing, there are no options open to consideration, and it should not be included in this optimization attempt.

One complication arises in the case of extreme tapering (ie, by more than a factor of $S^{0.5}$). Imagine we are comparing two different line maps, and we want to make the resolutions identical. Further imagine that to get the same resolution in the second line as we have in the first, we need to observe in the B array and taper almost all the way back to the C array. From a sensitivity point of view, it is actually much more advantageous to observe the second line in the C array and just mildly taper the first line map. Of course, the case becomes more complicated when more than two lines are included in the astrophysical analysis and an extreme tapering event (ETE) cannot always be avoided. However, for calculational purposes, lets assume that most extreme tapering events can be avoided, and that we only need to consider tapering up to S^p where p is 0.5. Then we only consider $\rho_i(\theta)$ between normalized θ of 1 and S^p for each configuration, and the new normalized form of $\rho_i(\theta)$ is given by

$$\rho_i(\theta) = ((N_c - 1)p\ln(S))^{-1}\theta^{-1}. \quad (5)$$

3.3 Sensitivity after Tapering

I argued in MMA Memo 156 (Holdaway, 1996) that since tapering would be so important for the MMA, we should design each configuration such that it performed optimally with respect to tapering, losing a minimum of sensitivity. Filled arrays meet this requirement, while ring-like arrays, with their more uniform Fourier plane coverage, lose more sensitivity when tapered to a given resolution. Since then, imaging simulations (Holdaway, *unpublished*; Morita, *in preparation*) indicate that ring arrays provide superior imaging quality in spite of their large sidelobes. However, the superior imaging quality is not due to the ring array's "uniform" Fourier plane coverage, but due to the fact that the ring array has much shorter shortest baselines than the filled array (and quite a lot of them, too), and the image quality in the simulations is being dominated by the very short baseline distribution. At this point, I am ready to move ahead with ring-like arrays, though there are investigators who still favor filled arrays (Kogan, 1997). Recently, Kogan (private communication) has produced arrays which are fat rings, or donut arrays. They produce a partially tapered Fourier plane distribution, and so will lose less sensitivity upon tapering than the pure ring-like, uniform coverage arrays. The approach taken in this memo is more global: to design the entire set of array configurations to perform optimally with respect to tapering.

For a ring-like array with approximately uniform Fourier plane coverage, increasing the resolution by a factor a will require tapering, leaving a fraction of a^{-2} of the visibilities. Since the sensitivity is proportional to the square root of the number of visibilities, the residual sensitivity after tapering will be proportional to a^{-1}. Hence, we define the sensitivity function, intended for use between a tapered resolution θ between 1 and S:

$$\psi(\theta) = \theta^{-1}. \quad (6)$$

3.4 Observing Sensitivity

We now define the normalized observing sensitivity, integrated over all resolutions between the natural resolution of the largest and the smallest configuration, based on the above considerations as

$$\begin{aligned}
\epsilon_o &= f_\circ + (1 - f_\circ) \sum_{i=1}^{N_c-1} \int_1^{S^p} \rho(\theta)\psi(\theta)d\theta & (7)\\
&= f_\circ + (1 - f_\circ) \sum_{i=1}^{N_c-1} ((N_c - 1)p\ln(S))^{-1} \int_1^{S^p} \theta^{-2}d\theta & (8)\\
&= f_\circ + (1 - f_\circ)(p\ln(S))^{-1}(1 - (S)^{-p}) & (9)\\
&= f_\circ + (1 - f_\circ)\left(\frac{N_c - 1}{p\ln R}\right)\left(1 - R^{-p/(N_c-1)}\right) & (10)
\end{aligned}$$

If everyone were happy with the natural resolution of the array configuration, $f_\circ$ would be 1, and ϵ_o would be 1. Table 1 considers the case were $R = 45$ (ie, 3000 m/(.7 $\cdot$ 95 m)) $f_\circ = 0$

N_c	ϵ_o
12	0.92
10	0.90
8	0.88
6	0.83
5	0.80
4	0.74
3	0.64
2	0.45

Table 1: Observing efficiency ϵ_o as a function of number of configurations N_c assuming $R = 45$ and $f_\circ = 0$ (nobody doesn't taper).

(nobody doesn't taper), but with tapers only out to $S^{0.5}$ (ie, $p = 0.5$), for a variety of N_c. Hence, to get as much as 0.80 of the sensitivity of the MMA when observers always tapered with a distribution like $\rho(\theta) = \theta^{-1}$, you would need 6 different array configurations. Or, with the 4 proposed MMA configurations, you end up with 0.70 of the sensitivity. We remind here that the observing efficiency depends strongly on $f_\circ$.

4 Reconfiguration Efficiency and Configuration Costs

4.1 Time Lost to Reconfiguration

For reconfiguring the array, we assume:

- t_m, the time to move one antenna, is 1 hr (see MMA Memo 147, Holdaway and Owen 1996).
- t_w, the time available each day for outdoor work, is 10 hr.
- we will be in each of the N_c configurations twice a year.
- some of the time spent reconfiguring will permit useful science. However, sensitivity will be lost while antennas sit during the day, after they have been moved and before pointing and baseline determinations have been made at night. Also, we assume that the scientific demand for the oddly configured hybrid array may not be 100%. Lumping all these factors together, we assume that a fraction f_r of the time spent during reconfiguration will *not* be useful for scientific purposes.

Then the time lost to the array, in days per year, will be about

$$2N_c(N_a - N_\circ)t_m f_r/(N_t t_w), \tag{11}$$

and the normalized reconfiguration efficiency ϵ_r is given by

$$\epsilon_r = (1 - N_c(N_a - N_\circ)t_m f_r/(365 N_t t_w))^{0.5} \tag{12}$$

4.2 Costs of Reconfiguration

In the cost-benefit analysis, we sum the monetary costs of reconfiguring and trade the money for antennas. We can then ask if it is better to have a few more antennas and fewer configurations, or more configurations and somewhat fewer antennas.

The monetary cost to move one antenna is estimated to be

$$C_m = n_w t_m C_w + C_{rt}, \tag{13}$$

so the cost to move $N_a - N_\circ$ antennas through N_c configurations in a year will be

$$N_c(N_a - N_\circ)(n_w t_m C_w + C_{rt}). \tag{14}$$

Meanwhile, the cost of making the configurations will be

$$N_c(N_a - N_\circ)(C_p + C_c). \tag{15}$$

Now, since operating expenses and capital costs will come from different sources for the MMA, we can't really trade one off against the other. But for the cost-benefit analysis, lets add up the move costs for a 20 year period. Then the total cost of the configurations plus moves will be

$$N_c(N_a - N_\circ)\left(20(n_w t_m C_w + C_{rt}) + (C_p + C_c)\right) \tag{16}$$

5 Results

We have calculated the various efficiencies subject to both tapering and reconfiguration for numbers of configurations N_c ranging from 2 to 12, for 36 10 m antennas, assuming $f_\circ = 0.5$ (see Table 2). Even for a very large number of configurations, the efficiency lost to reconfiguring the array is negligibly small, and it would seem that the choice would be to make many configurations

We have also calculated the additional costs that extra configurations impact upon the array. Under our assumptions, the extra costs are linear with the number of configurations, and are equivalent to 1.57 antennas per configuration. In order to compare $N_c = 6$ on an equal footing with $N_c = 4$, we must keep the total cost of the two options equal; in other words, we must take the \$10M which was spent on the two extra configurations out of the antenna budget, implying we building 3 fewer antennas and our sensitivity is down. To reflect this, we correct the total efficiency of the 6 configuration option by $(N_a - 3)/N_a$. This corrected total efficiency is reported as ϵ_t^* in Table 2.

We plot both the total efficiency and the corrected total efficiency for the $f_\circ = 0.5$ case in Figure 1, and for the $f_\circ = 0$ case in Figure 2. Most remarkably, the results do not come out too differently from the presumed $N_c = 4$ option. In the $f_\circ = 0.5$ case, the optimal N_c is about 5, but 4 and 6 are also extremely close to the optimal ϵ_t^*. If a larger fraction of the observations will require tapering, ie, if $f_\circ = 0$, the optimal number of configurations will shift upwards to about $N_c = 8$, but even in this case, the $N_c = 4$ case is still just about 7% below the optimal ϵ_t^*.

References

N_c	ϵ_o	ϵ_r	ϵ_t	Config Cost [M$]	Moving Cost [M$]	Total Cost [M$]	Lost Antennas	ϵ_t^*
2	0.72	0.99	0.72	3.3	1.0	4.3	-1.2	0.74
3	0.82	0.99	0.81	5.0	1.4	6.4	-0.6	0.83
4	0.87	0.99	0.86	6.6	1.9	8.5	0.0	0.86
5	0.90	0.98	0.88	8.2	2.4	10.6	0.6	0.87
6	0.92	0.98	0.90	9.9	2.9	12.8	1.2	0.87
7	0.93	0.98	0.91	11.6	3.3	14.9	1.8	0.86
8	0.94	0.98	0.91	13.2	3.8	17.0	2.4	0.85
9	0.94	0.97	0.92	14.8	4.3	19.1	3.0	0.84
10	0.95	0.97	0.92	16.5	4.8	21.3	3.6	0.83
11	0.96	0.97	0.92	18.1	5.2	23.4	4.2	0.82
12	0.96	0.96	0.92	19.8	5.7	25.5	4.8	0.80

Table 2: Efficiencies and costs for various numbers of array configurations assuming $f_\circ = 0.5$.

N_c	ϵ_o	ϵ_r	ϵ_t	Config Cost [M$]	Moving Cost [M$]	Total Cost [M$]	Lost Antennas	ϵ_t^*
2	0.45	0.99	0.44	3.3	1.0	4.3	-1.2	0.46
3	0.64	0.99	0.64	5.0	1.4	6.4	-0.6	0.65
4	0.74	0.99	0.73	6.6	1.9	8.5	0.0	0.73
5	0.80	0.98	0.78	8.2	2.4	10.6	0.6	0.77
6	0.83	0.98	0.82	9.9	2.9	12.8	1.2	0.79
7	0.86	0.98	0.84	11.6	3.3	14.9	1.8	0.80
8	0.88	0.98	0.85	13.2	3.8	17.0	2.4	0.80
9	0.89	0.97	0.87	14.8	4.3	19.1	3.0	0.79
10	0.90	0.97	0.87	16.5	4.8	21.3	3.6	0.79
11	0.91	0.97	0.88	18.1	5.2	23.4	4.2	0.78
12	0.92	0.96	0.88	19.8	5.7	25.5	4.8	0.77

Table 3: Efficiencies and costs for various numbers of array configurations assuming $f_\circ = 0.0$.

Holdaway, M.A., 1996, "What Fourier Plane Coverage is Right for the MMA?", MMA Memo 156.

Holdaway, M.A., and Owen, F.N., 1996, "How Quickly Can the MMA Reconfigure?", MMA Memo 147.

Kogan, L., 1997, "Optimization of an array configuration minimizing side lobes", MMA Memo 171.

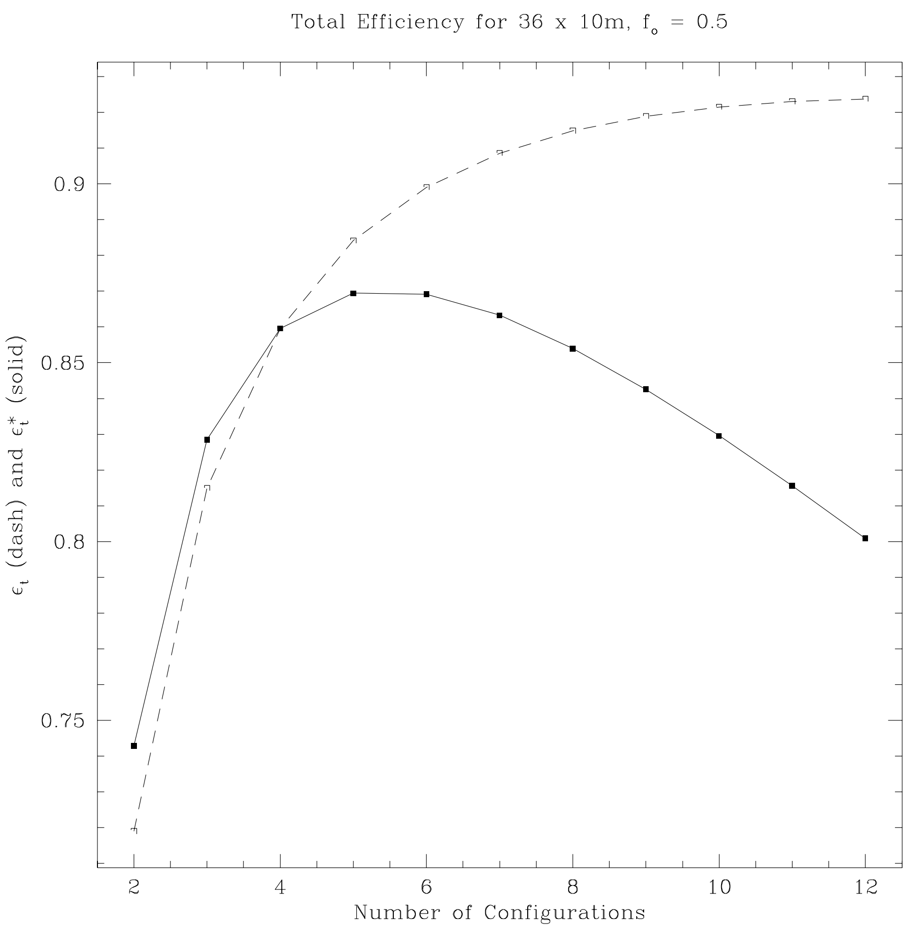

Figure 1: Total efficiency (dash) and corrected total efficiency (solid) for the $f_\circ = 0.5$ case.

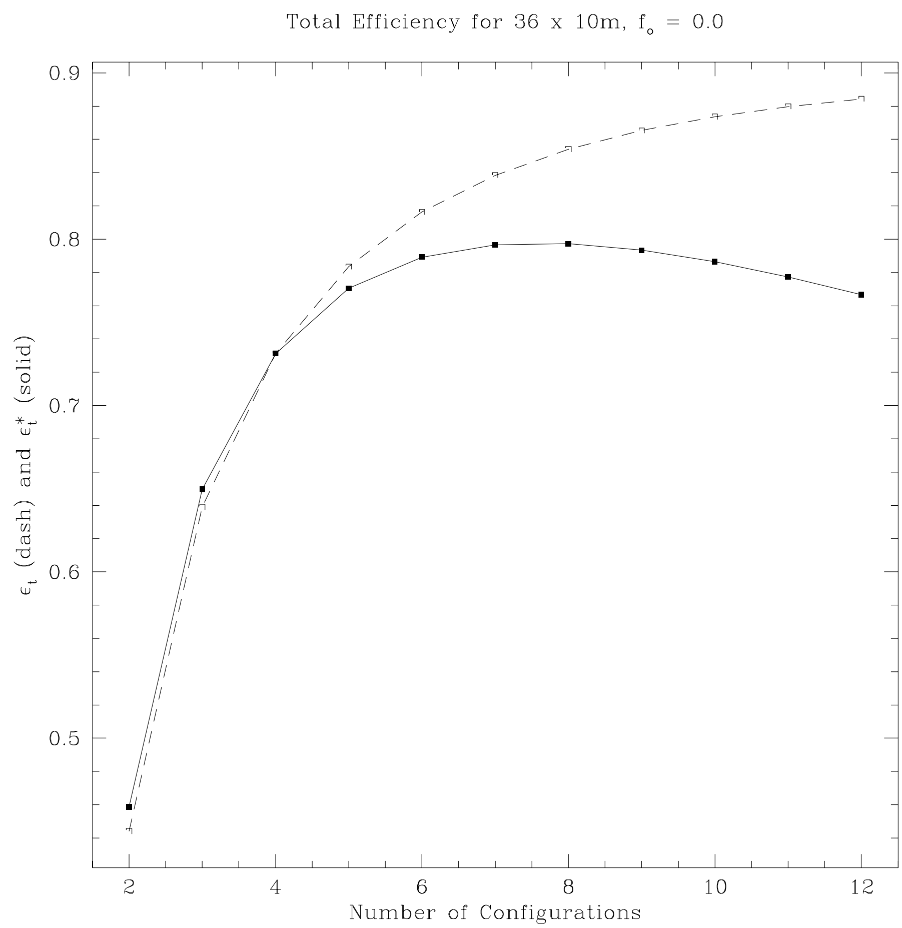

Figure 2: Total efficiency (dash) and corrected total efficiency (solid) for the $f_\circ = 0$ case.