

# Radiometric Correction of Anomalous Refraction

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**Abstract:** We propose a scheme to measure and correct anomalous refraction effects for millimeter wave antennas. A radiometer tuned to the 183 GHz atmospheric water vapor line scans the atmosphere above the aperture of an antenna to determine the spatial derivative of the refractive index. In an array of antennas, such as the MMA, it would be used simultaneously to measure the phase delay correction for the antenna. The radiometer feed illuminates an area half the aperture diameter and is scanned by a rotating prism. As well as being cheaper than using three or four radiometers, a single radiometer avoids many of the problems due to systematic differences that would arise with separate receivers. On the Chajnantor site, where anomalous refraction conditions are not correlated with high opacity significant amounts of time could benefit from such correction.

## 1. Introduction

With modern technology extremely accurate antennas are possible for millimeter and sub-millimeter wavelength observations. Receivers are approaching quantum-limited sensitivities, and at high sites the atmospheric absorption is low. The major remaining limitation on sensitivity and imaging quality is the non-uniform tropospheric water vapor distribution which perturbs the wavefront from the astronomical source. The primary effect is to introduce differential path delays between antennas, but a second is to introduce random pointing errors at each antenna (“anomalous refraction”).

At the Chajnantor site proposed for the MMA, anomalous refraction is worst in the afternoon and for a 10-m antenna can be equivalent to a pointing error of 3 arcsec or greater for as much as a third of the day according to the estimates of Holdaway and Woody [1]. Since the mean opacity exhibits virtually no diurnal variation [2] there are significant amounts of time when the atmospheric transmission is very good but the anomalous refraction is seriously detrimental to observations.

Several proposals are being examined for reducing the differential atmospheric phase error between antennas [4] including rapid switching to a nearby calibrator, and using radiometric brightness measurements of the water vapor to derive pathlength corrections. We look at how the radiometric correction may be extended to measure anomalous refraction. An obvious possibility is to use a set of four radiometers at the periphery of the aperture. Although this can in principle be reduced to three radiometers, it still appears too complex and expensive to apply to a large number of antennas. An alternative solution is to use a single radiometer whose beam is scanned round the aperture of the antenna. This gives information about the phase and the phase gradients across the aperture, from which the path delay and pointing correction information may be derived.

## 2. Description

### 2.1 Principle

The scanning radiometer centered on a water emission line, has a beam about half the size of the observing beam in the near-field. This beam rotates around the main beam axis, scanning a cross-section of the water vapor above the antenna. By detecting the output power of the receiver synchronously with the rotation

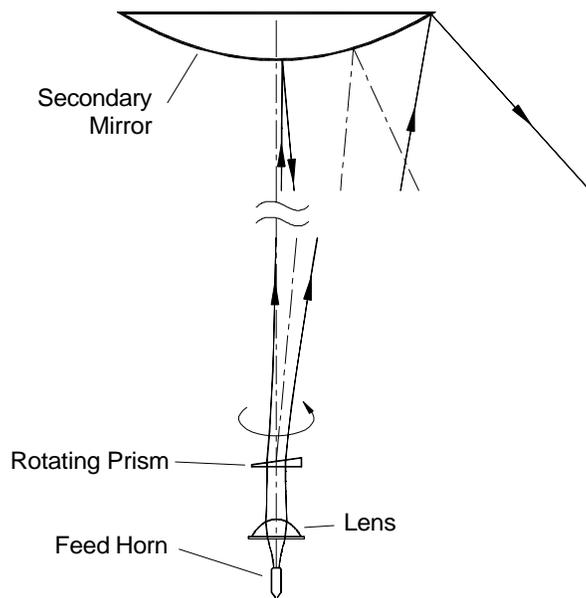


Figure 1: Optical arrangement for the scanning radiometer. The prism rotates around the receiver axis, scanning its illumination pattern round the antenna aperture.

of the primary formed by a dielectric lens, as shown in Fig. 1. The horn diameter is about half the size or slightly less than the primary image. The feed axis points towards the center of the secondary mirror but a prism above the lens near the secondary focus deflects the beam to illuminate a quadrant of the aperture. As the prism rotates about the feed axis the beam sweeps round the aperture and the atmosphere above the antenna (Fig. 2).

At the Cassegrain focus the beam diameter is roughly  $3N\lambda$ , where  $N$  is the numerical aperture at the secondary focus. The lens diameter required is therefore  $\sim 6N\lambda$  or about 60 mm for an  $f/8$  telescope. This occupies about the same focal plane area as a 73-mm receiver. The apex angle of the prism is  $1.8^\circ$  if it is made from a typical low-loss plastic such as polyethylene, so it need have a thickness of only a few millimeters. The plane surfaces and single frequency of operation mean that extremely low reflection layers may be made with bi-periodic grooves cut on the surfaces.

The 5 m wide beam is collimated to a distance of  $\sim 15$  km, well above most of the water vapor at zenith. If separate radiometers were used at the edge of the dish they would need to have collecting apertures of  $\sim 2$  m diameter to have a near-field/far-field transition distance of 2 km, which is roughly the scale height for water. At lower elevations there will be a reduction in the effective gain since there will be some section of atmosphere near the optical axis which is continuously in the beam as it rotates. For a 2 km high layer the overlap will be substantial for elevations of about  $10^\circ$  or less, while for a 4 km layer it will be important at elevations less than about  $16^\circ$ .

Since the beams are not co-axial with the observing receiver, the atmosphere sampled will be different. If the WVR beam is three 73 mm beamwidths ( $3.8$  arcmin) away from the observing channel beam they will overlap up to about 9 km along the line of sight. That will introduce some errors depending on elevation, and the height and scale size of the turbulence.

(using sine and cosine demodulation waveforms) information is obtained about the spatial derivatives of the water emission which are related to the gradients in refractive index. By feeding these back into the antenna servo system the effects of anomalous refraction may be greatly reduced. The average signal may still be used as an indicator of the total pathlength above the antenna for use in the radiometric inter-antenna phase correction.

## 2.2 Optics

It is assumed that the radiometric phase correction scheme will use a dedicated water vapor radiometer (WVR) centered on the water vapor line at 183 GHz. At this wavelength the dimensions of the optics are quite compact whereas at 22 GHz the order of magnitude increase in size makes the scheme too cumbersome for 10-m antennas. The radiometer feed horn is placed at an image

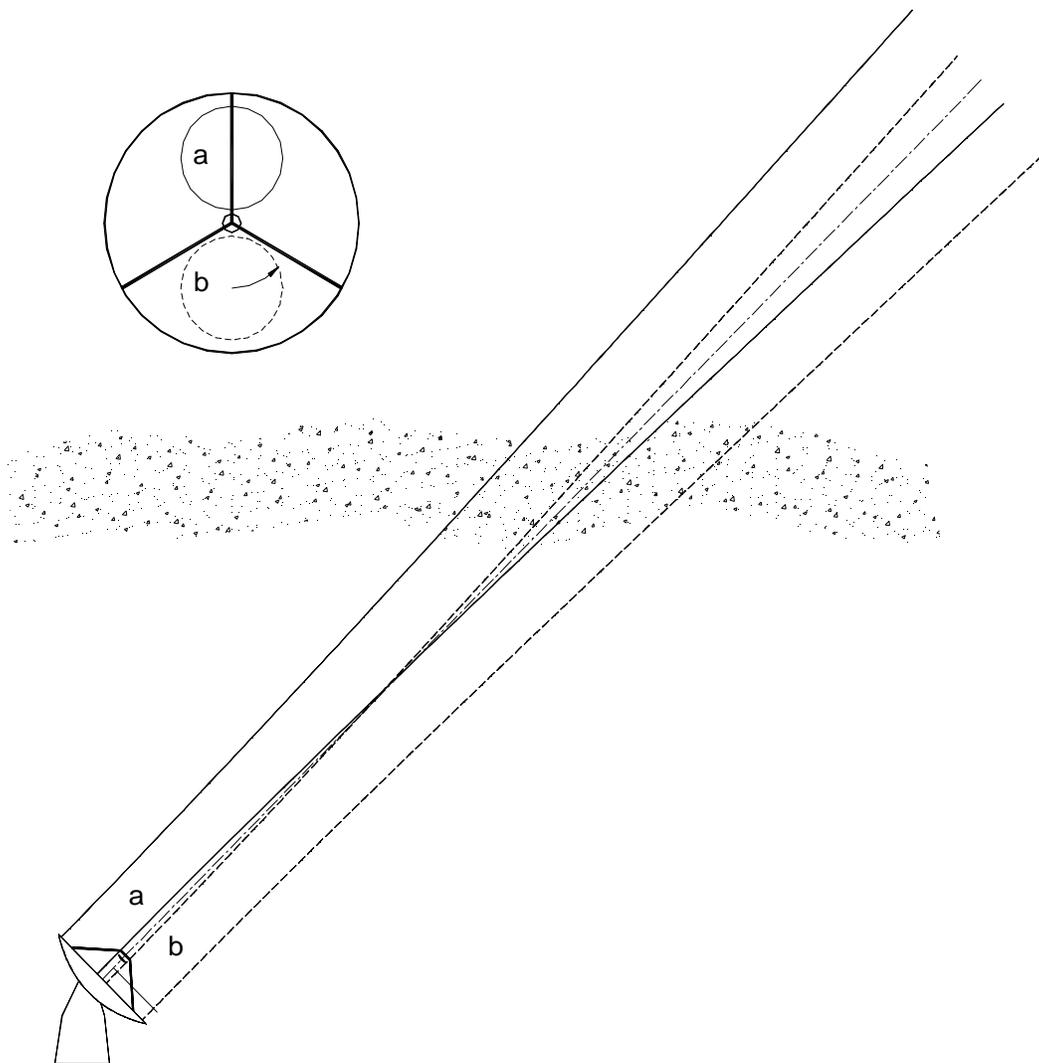


Figure 2: Illustration of two parts of the rotation cycle of the radiometer beam. The elevation gradient would be derived from the difference of these two. Some overlap of the beams in the turbulent layer reduces the difference relative to the true difference, depending on the elevation. The inset in the upper left shows the beam pattern in the antenna aperture.

### 2.3 Sensitivity

At the Chajnator site the worst anomalous refraction is predicted to occur in the afternoon [1], but there is no significant diurnal variation in the opacity [3]. There are therefore likely to be occasions where observing conditions are excellent apart from the atmospheric refraction effects on pointing. Lay has evaluated various sources of error arising in an estimation of phase from atmospheric emission at the 183 GHz [5]. The errors are most severe under conditions of high atmospheric opacity (>5 mm precipitable water vapor (PWV)).

To make a significant improvement in the pointing under conditions of appreciable anomalous refraction we would like the correction to be better than 0.5 arcsec. Since the centroid of the beam follows a

circle of 5 m diameter, the accuracy required of the phase determination is  $\sim 12 \mu\text{m}$  which is more stringent than the requirements for phase stability in the array. Despite this the goal is not unrealistic since the measurement does not rely on having extremely good matching between different radiometers.

The aperture crossing time for a feature in the atmosphere is about 1 s for a  $10 \text{ m s}^{-1}$  wind. Some of the refraction is caused by atmospheric structure with scale sizes larger than the antenna diameter and will contribute variations on longer time scales. Church and Hills found typical time scales of  $\sim 10$  s on Mauna Kea [6]. It is probably safe to assume, therefore, that the shortest integration time will be of order 1 s.

A fundamental limitation to the measurement is the radiometer noise. For a system temperature of 300 K, bandwidth of 0.5 GHz, and integration time of 1 s, the minimum detectable temperature is about  $\Delta T = 13 \text{ mK}$ . In the scanning radiometer we assume that the beam spends about a quarter of the time in each quadrant of the aperture which increases  $\Delta T$  by a factor of two. Taking the difference between two quadrants introduces a further factor of  $\sqrt{2}$  so that the effective  $\Delta T$  is  $\sim 38 \text{ mK}$ . From the plots of brightness increment for a  $50 \mu$  pathlength change given by Lay [5] for various conditions it is evident that this sensitivity is sufficient for PWV of 4 mm or less with some moderate degradation up to 8 mm. To cover the full range we may require dynamic selection of the measurement frequency, but a useful fraction of the time can probably be covered with a single frequency. Some improvement may be made with wider bandwidths and DSB operation to sum both wings of the water line.

There are some differences between the measurement for phase correction and pointing correction. Most notable is the effect of systematic errors. An error in the phase determination common to all antennas cancels out, whereas a pointing correction error common to all the antennas will introduce serious degradation of the data. Two strategies could be applied to remove common mode errors. One is to assume that all the common mode errors are slowly varying and high-pass filter the data to remove these. The alternative is to calibrate and subtract the errors. As pointed out by Hills [7] there is a common mode signal due to the normal refraction of moist air which is normally incorporated into the pointing model. This may be difficult to separate from other common mode effects such as spillover. In the simplest scheme all the common mode errors are removed (by averaging over all antennas) and the normal wet refraction retained in the pointing model. In a more refined scheme the common mode (and even antenna-based) errors would be removed and the wet refraction correction determined from the radiometer output. Separating the wet refraction from the other effects may be done using measurements of the radiometer output over all elevations and under different relative humidity conditions, for example.

Spillover due to scattering by the secondary mirror support struts could contribute  $\sim 0.5 \text{ K}$  to the radiometer input [8]. To reduce this to a negligible level a reduction by a factor of 30 or so is needed. For both tripod and quadrupod supports the symmetry gives first order cancellation in the difference measurement, but there may be some residual requiring calibration.

Although common mode errors may be worse for the pointing correction than the phase, the variations of gain, etc. between antennas are not critical for the refraction correction—only one receiver is involved for each antenna so is no requirement to match between different instruments. (Of course, the need for phase correction still constrains gain matching.) Gain fluctuations at rates higher than the rotation frequency of the prism will corrupt the data. With a rotation rate of 10 Hz or so this should not be a problem—it is similar to a beam-switched single-dish receiver and these are capable of theoretical noise performance.

### 3. Discussion

It appears to be feasible to be able to measure and partially correct for the anomalous refraction component of the atmospheric water vapor non-uniformities for the MMA. There are, however, several areas of uncertainty that need to be resolved to demonstrate the real viability. These are associated with such things as the errors arising from lack of knowledge of the height and physical temperatures of the inhomogeneities. If the refraction is a result of variations in water vapor density at a given altitude then the errors should be smaller than the cases where, for example, there are different blobs of vapor at different altitudes or temperatures over the aperture. The effective height of the layer will also affect how much reduction there will be in the measured gradient due to overlapping of sampled regions at low elevations. This will affect the sensitivity and the scaling factor. Some of these issues may be resolved on the site by

using the information from all the WVR's on the array, possibly supplemented by one or two more specialized instruments which can characterize the atmosphere above the site as a whole. Using the frequent astronomical phase calibrations should allow measurements of scaling factors on short timescales.

There are several important implications for the MMA system design. The method does not seem practical if the 22 GHz water line is used because of the large size of the optics. If 22 GHz is the choice for phase correction on the MMA then a 183 GHz radiometer should be included also for the refraction compensation. For the receiver packaging it is important to understand how much misalignment is permissible between the observing beam and the water vapor monitor so that the optics can be designed to match. If this poses some problems it may be necessary to choose which bands are most important so that those receivers, at least, can be close to the WVR beam.

This method could possibly be tested at an existing mid-altitude millimeter wave site, but would probably not be useful as a permanent piece of instrumentation because of the saturation of the 183 GHz water line. It could however be applicable to large centimeter wave telescopes such as the GBT and the LMT using the 22 GHz line. This has already been suggested as an important factor in being able to operate these antennas at their higher frequencies where the beamwidth is at the 10 arcsec level [7,9]

#### 4. References

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