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On the Noise Properties of Balanced Amplifiers

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Abstract

The balanced amplifier is used in applications requiring a better input match than is possible with a single-ended amplifier. While the impedance matching property of the balanced amplifier is well known, its noise behavior appears not to be widely understood. It is shown that the outgoing noise waves at the input and output of a balanced amplifier are uncorrelated even though they originate in the same components. Hence, a sliding short-circuit at the input produces no variation in the output noise of the amplifier. The properties of a balanced amplifier are similar to those of an amplifier preceded by an isolator, although the noise wave emerging from inputs of the two circuits originates in different elements. The noise theory of the balanced amplifier applies also to balanced mixers based on quadrature hybrids.

Introduction

The balanced amplifier was proposed by R. S. Engelbrecht of Bell Labs as a way of providing a good input match when an amplifier was tuned for optimum noise performance. It was first reported by Engelbrecht and Kurokawa [1] in 1965, and a subsequent paper by Kurokawa [2] explored the theory of the balanced amplifier in more detail. Kurokawa's noise analysis considered the effects of mismatches at the terminated ports of the input and output quadrature hybrids, and of a mismatched output load, but did not explore the effect of a mismatched source on the noise output of the balanced amplifier. The purpose of this paper is to show that the noise properties of a balanced amplifier are in many ways similar to those of an amplifier with an isolator at its input [3]. In particular, it is shown that the outgoing noise waves at the input and output of a balanced amplifier are uncorrelated. This implies that the output noise is independent of the phase of the source reflection coefficient, so moving a sliding short-circuit at the input produces no variation in the output noise. This is not obvious on first consideration because, although there is no correlation between the noise of the individual component amplifiers within the balanced amplifier, the noise emerging from the input and output ports of each of the component amplifiers is, in general, correlated. Noise emerging from the input of the balanced amplifier, reflected by a mismatched source, re-enters the amplifier where it combines with correlated output noise components.

The need for an amplifier with a well-matched input arises when a poorly matched source must be connected to the amplifier through an electrically long transmission line. If the amplifier and source are both poorly matched to the transmission line, multiple reflections between them result in a variation of the overall gain with frequency. Because the source impedance seen by the amplifier is frequency dependent, the output noise also varies with frequency. If an isolator is placed between the source and amplifier, the gain and noise become independent of frequency. Then the output noise includes a component from the internal termination of the isolator, whose thermal noise is incident on the (mismatched) source and is partially reflected into the amplifier via the isolator. This is shown in Fig. 1 where b_1 and b_2 are the outgoing noise waves at the input and output of the

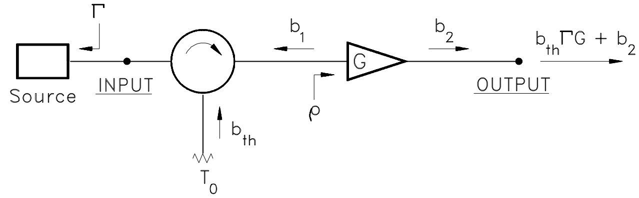

Fig. 1. An amplifier preceded by an isolator.

amplifier, and b_{th} is the outgoing thermal noise wave from the isolator's termination. With an ideal isolator, moving a sliding short circuit at the input of the isolator produces no change in the output noise of the amplifier. However, changing the magnitude of the reflection coefficient of a noiseless source *does* change the output noise of the amplifier because of the change in the noise power from the isolator's termination reflected into the amplifier.

In the case of the balanced amplifier, depicted in Fig. 2, the noise behavior is less apparent. The noise of each of the two component amplifiers is characterized by outgoing noise waves b_1 and b_2 which are, in general, correlated to some degree. If the amplifiers are identical, their noise waves are equal in magnitude, although the noise of amplifier A is not correlated with that of B. The noise waves (b_1^A, b_1^B) from the input ports of the two amplifiers propagate towards the input quadrature hybrid which divides the noise power between the input port and the terminated fourth port. A source reflection coefficient $\Gamma(f)$ reflects part of the noise from each amplifier back into the circuit. The reflected noise propagates through the balanced amplifier, as any other input signal, and appears at the output port. The output noise waves (b_2^A, b_2^B) of the component amplifiers are also coupled to the output port through the output quadrature hybrid. Since b_1^A and b_2^A are correlated, and also b_1^B and b_2^B, it has often been assumed that the output noise of the balanced amplifier depends on the magnitude and phase of the source reflection coefficient. It will be shown that, given ideal input and output quadrature hybrids and identical component amplifiers, the output noise depends on the magnitude of the source reflection coefficient but not on its phase.

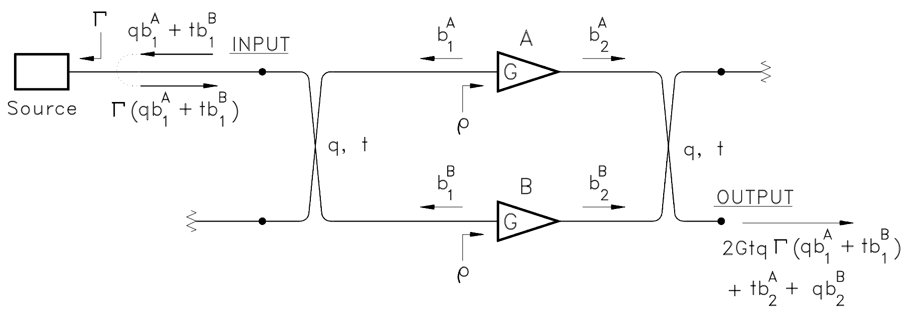

Fig. 2. A balanced amplifier, showing the noise components which originate in the component amplifiers.

There are two additional components of output noise in a balanced amplifier and these are from the resistive terminations, assumed matched, on the fourth ports of the input and output hybrids. First consider the noise from the fourth port termination on the input hybrid, as depicted in Fig. 3. The thermal noise wave a_0 from this termination is reflected by the reflection coefficient ρ at the input ports of the component amplifiers, and then coupled to the input of the complete balanced amplifier. There, it is reflected by the source reflection coefficient Γ, and propagates forward through the balanced amplifier to the output port. It is shown here that the output noise power originating at this fourth port termination depends on the magnitude of the source reflection coefficient but not on its phase. Noise from the termination on the fourth port of the *output* hybrid also contributes an output

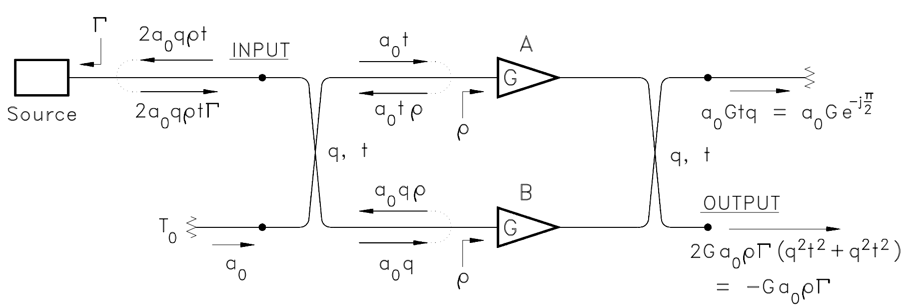

Fig. 3. A balanced amplifier, showing the noise components which originate in the termination on the fourth port of the input hybrid.

component if the outputs of the component amplifiers are not well matched, but this is usually negligible provided the gain of the component amplifiers is sufficiently large. In the present work it will be neglected for simplicity.

The *noise waves* used in this work are as described by Meys [4]. They have the dimension of $(\text{power})^{0.5}$ and are normalized to an arbitrary real characteristic impedance, normally that of the surrounding transmission line or waveguide elements. This is somewhat different from the earlier use by Penfield [5] of noise waves to characterize amplifier noise, in which case the normalizing impedance was complex and equal to the noise-optimum source impedance of the amplifier. The noise power per unit bandwidth carried by a noise wave a_n is given by the ensemble average $<|a_n|^2>$. The noise temperature associated with the noise wave is $<|a_n|^2> / k_B$, where k_B is Boltzmann's constant. Meys shows that the noise of an amplifier is completely characterized by two noise waves at the input port, one ingoing and the other outgoing. It is easily shown that an equivalent representation has two outgoing noise waves, one at the input port of the amplifier, and the other at the output port [6].

Analysis

Consider the balanced amplifier as depicted in Fig. 2. Assume the quadrature hybrids at the input and output have gains (*i.e.*, S-parameters) q and t to their quadrature and in-phase ports, respectively. For an ideal hybrid $q = \frac{1}{\sqrt{2}} e^{j(\theta - \frac{\pi}{2})}$ and $t = \frac{1}{\sqrt{2}} e^{j\theta}$, where for simplicity the factor $e^{j\theta}$ will be omitted without loss of generality in the following calculations. Assume the component amplifiers A and B are identical and have gain G, input reflection coefficient ρ, and are characterized by noise waves b_1 and b_2 at their input and output, as shown.

<u>Sinusoidal Input Signal</u>

First, consider a signal of unit amplitude incident at the input. The corresponding output from balanced amplifier consists of components from each branch of the amplifier; from the upper branch qGt, and from the lower branch tGq, resulting in a net output $2qtG = G e^{-j\frac{\pi}{2}}$. At the terminated port of the output hybrid the signal produced by the unit input likewise consists of the sum of two components, qGq and tGt; that is, $G(q^2 + t^2)$, which is equal to zero. Some of the unit input signal is reflected at the inputs of the component amplifiers and emerges as an outgoing wave from the input port of the balanced amplifier with amplitude $q\rho q + t\rho t = 0$; that is, the balanced amplifier is matched at the input, as expected.

<u>Noise from the Component Amplifiers</u>

The outgoing noise from the *input* ports of the component amplifiers is coupled through the input quadrature hybrid to produce an outgoing noise wave $b_1^A q + b_1^B t$ emerging from the input of the balanced amplifier, as shown in Fig. 2. The outgoing noise waves, b_2^A and b_2^B, from the *output* ports the component amplifiers are coupled to the output of the balanced amplifier giving $t b_2^A + q b_2^B$.

Consider the effect of a mismatched source with reflection coefficient Γ. Then, all the noise components which emerge from the input port of the balanced amplifier appear at the output multiplied by the factor $\Gamma G e^{-j\frac{\pi}{2}}$. As shown in Fig. 2, the noise of the component amplifiers A and B results in a net output

$$b_{out} = \left[2G\Gamma t q^2 b_1^A + t b_2^A\right] + \left[2G\Gamma t^2 q b_1^B + q b_2^B\right]. \quad (1)$$

Assume the noise waves b_1 and b_2 of each amplifier are fully correlated, so $b_2^A = k\, b_1^A$ and $b_2^B = k\, b_1^B$. Then:

$$b_{out} = b_1^A\, t \left[2G\Gamma q^2 + k\right] + b_1^B\, q \left[2G\Gamma t^2 + k\right] . \tag{2}$$

Since b_1^A and b_1^B are uncorrelated, and $|t| = |q| = 1/\sqrt{2}$, the average noise output power

$$<|b_{out}|^2> = \frac{1}{2}\left(<|b_1^A|^2> \left|2G\Gamma q^2 + k\right|^2 + <|b_1^B|^2> \left|2G\Gamma t^2 + k\right|^2\right) . \tag{3}$$

Assume that the two component amplifiers A and B are identical, so $|b_1^A| = |b_1^B| = |b_1|$. Let $R_1 = \left[2G\Gamma q^2 + k\right]$ and $R_2 = \left[2G\Gamma t^2 + k\right]$. Then

$$<|b_{out}|^2> = \frac{1}{2}<|b_1|^2> \left(\left|R_1\right|^2 + \left|R_2\right|^2\right) . \tag{4}$$

Since $q^2 = -t^2$, it is simply shown using the vector diagrams in Fig. 4 that $|R_1|^2 + |R_2|^2 = 2(|k|^2 + 4|G\Gamma q^2|^2)$, which is independent of the phase of Γ. It follows that $<|b_{out}|^2>$ depends on the magnitude of Γ but is independent of the phase of Γ. In cases where the noise waves b_1 and b_2 of each amplifier are not fully correlated, the problem can be considered in two parts: the correlated component of the noise, for which the above analysis applies, and the uncorrelated component, whose output contribution is independent of the phase of Γ, anyway.

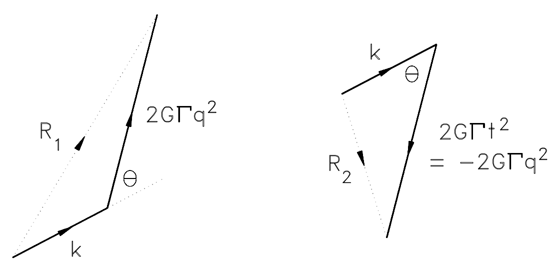

Fig. 4. Vector diagrams representing the quantities R_1 and R_2 in equation (4). It is simply shown trigonometrically that $|R_1|^2 + |R_2|^2$ is independent of theta and therefore of $\arg(\Gamma)$.

Noise from Termination on Input Hybrid

Using the same argument as in the case of a sinusoidal signal, the thermal noise wave a_0 from the termination on the fourth port of the input hybrid is delivered to the terminated port of the output hybrid with amplitude $a_o G e^{-j\frac{\pi}{2}}$ as shown in Fig. 3. In the ideal case, no power is delivered to the output of the balanced amplifier.

Some of the noise a_0 from the termination is reflected at the inputs of the component amplifiers and emerges as an outgoing noise wave from the input port of the balanced amplifier, with amplitude $a_o q\rho t + a_o t\rho q = a_o \rho e^{-j\frac{\pi}{2}}$, as shown in Fig. 3. If this is reflected by a source reflection coefficient Γ, it produces at the output port of the balanced amplifier an amplitude $b_{out} = -a_0\rho\Gamma G$. The corresponding noise output power $<|b_{out}|^2> = <|a_0|^2> |\rho\Gamma G|^2$ is clearly dependent on the magnitude of Γ but independent of the phase of Γ.

Discussion

It has been shown that the output noise of the ideal balanced amplifier depends on the magnitude of the source reflection coefficient Γ, but is independent of the phase of Γ. This implies that the outgoing noise waves at the input and output ports of the balanced amplifier are uncorrelated, for otherwise a change in the phase of Γ would change the noise measured at the output port.

It is interesting to compare the balanced amplifier with an amplifier preceded by an isolator. Both have a matched input, and the noise wave emerging from the input port in each case is not correlated with the noise wave emerging at the output port. If these outgoing noise waves from the input ports are denoted b_{in}^{bal} and b_{in}^{circ}, then $<|b_{in}^{bal}|^2> \; = \; <|b_1|^2> \; + \; k_B T_0 |\rho|^2$ W/Hz, and $<|b_{in}^{circ}|^2> \; = \; k_B T_{circ}$W/Hz, where b_1 is the outgoing noise wave for the component amplifiers (assumed identical) in the balanced amplifier, T_0 is the physical temperature of the termination on the fourth port of the input hybrid, and T_{circ} is the physical temperature of the termination of the isolator. This allows the relative merits of the two types of amplifier in a given application to be compared.

The theory developed here in the context of the balanced amplifier is equally applicable to the balanced mixer of the type based on quadrature hybrids. However, it does not apply to the kind of balanced mixers based on 180° hybrids (transformers).

Acknowledgments

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