ALMA Memo 330
The overlap of the astronomical and WVR beams*

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Abstract

The variation of the overlap of the astronomical and water-vapor radiometer beams with height in the atmosphere is investigated for the current BIMA setup, and is compared with similar calculations for OVRO and a proposed system for ALMA. The findings are that at BIMA, the 3-mm and 1.3-cm beams essentially do not overlap above a height of ∼1 km. The linear separation of the beams is ∼35 m at a zenith height of 4 km. The dominant factor in determining the beam overlap is the angular offset between the beams, with the overlap decreasing very rapidly with increasing offset. Under conditions where the dominant source of water-vapor fluctuations has a scale greater than the beam separation, the small beam overlap will generally not pose a serious problem to correcting the phase.

However, we do find that the current 28-arcmin beam offset at BIMA potentially places a limit on the measurement of path length differences achievable with the APHID water vapor radiometer, although this is dependent on the height of the dominant source of fluctuations in the water vapor content. For a number of reasons, the beam separation will not be a problem for an instrument which has recently been proposed for ALMA. Finally, we also suggest that, if possible, future instruments should have beams which are offset in azimuth to minimize the error introduced by looking through atmospheric lines of sight of unequal length.

1 Introduction: Formulating the problem

A great deal of effort is being made to understand the relationship between the measured atmospheric water-vapor content and the astronomical phase. In most current systems, the water line at 22.235 GHz is monitored with a radiometer whose beam is offset on the sky relative to the astronomical beam. The beam offset is determined by the size of the beams, which is in turn determined by the antenna diameter. Larger antennas allow for a smaller beam offset. One aspect which has not yet been examined is how the coupling of the two beams varies with height through the atmosphere and the importance of how well the beams are coupled. A simple analytic examination is presented here and its implications discussed.

The optical setup for most radio telescopes (including the BIMA antennas) produces a beam whose radial power response falls off as a gaussian of the form $\exp(-2r^2/w^2)$. The scale-length for the gaussian, w, is defined as the radius at which the power falls to 1/e^2 of the peak. However, it should be noted that w is not constant: it depends on the distance from the antenna and the wavelength of operation. The overlap of two gaussian beams of radii w_1 and w_2 separated by a distance s is given by

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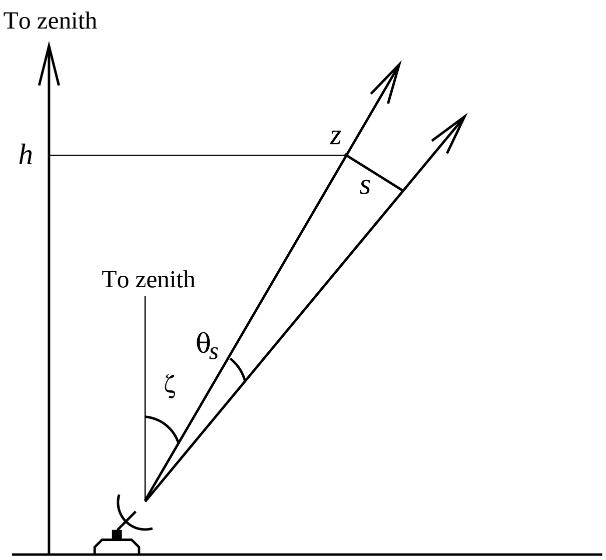

Figure 1: Geometry of the telescope and beam arrangements on the sky. All quantities are as in equation 5. The height in the atmosphere is denoted by h, and the zenith angle is ζ.

$$\eta(s) = \left(\frac{2w_1 w_2}{w_1^2 + w_2^2}\right)^2 \exp\left(\frac{-2s^2}{w_1^2 + w_2^2}\right) . \tag{1}$$

The beam-radius, w, varies with distance (z) from the aperture, and, when the beam is expanding from the location of the beamwaist, is given by

$$w(z) = w_0\sqrt{1 + \hat{z}^2} , \tag{2}$$

where $\hat{z} = \lambda z/(\pi w_0^2)$, z being the distance along the axis of the beam. The beam has its smallest radius (w_0) at the position of the beamwaist. When $\hat{z} \ll 1$ the beam radius is equal to the beamwaist. The regime in which $\hat{z} \gg 1$ corresponds to the far-field limit of the telescope at a wavelength λ.

The size of the beamwaist is determined by the amount by which the beam is tapered, such that

$$w_0 = \sqrt{\frac{5 \log \mathrm{e}}{F}}\, D , \tag{3}$$

where D is the telescope diameter and F is the edge taper in dB. For a typical edge-taper of 10 dB, w_0 is close to the telescope radius ($0.47D$). Note w_0 is independent of wavelength.

The beam separation s is related to the distance along the direction of the beam by $s = z \tan\theta_s$. Equation 1 may then be expressed as

$$\eta(z) = \frac{4w_{01}^2 w_{02}^2 (1 + \hat{z}_1^2)(1 + \hat{z}_2^2)}{[w_{01}^2(1 + \hat{z}_1^2) + w_{02}^2(1 + \hat{z}_2^2)]^2} \exp\left(\frac{-2z^2 \tan^2\theta_s}{w_{01}^2(1 + \hat{z}_1^2) + w_{02}^2(1 + \hat{z}_2^2)}\right) . \tag{4}$$

When the edge-tapers are equal ($w_{01} = w_{02} = w_0$) and θ_s is small (so that $\tan\theta_s \to \theta_s$) this equation becomes

$$\eta(z) = \frac{4(A_\mathrm{w}^2 + \lambda_1^2 z^2)(A_\mathrm{w}^2 + \lambda_2^2 z^2)}{(2A_\mathrm{w}^2 + (\lambda_1^2 + \lambda_2^2)z^2)^2} \exp\left(\frac{-2\pi A_\mathrm{w}\theta_s^2 z^2}{2A_\mathrm{w}^2 + (\lambda_1^2 + \lambda_2^2)z^2}\right) , \tag{5}$$

where $A_\mathrm{w} = \pi w_0^2$ represents the cross-sectional area of the beam at the waist, and θ_s is in radians. This equation also assumes the telescope is pointed towards the zenith if z is interpreted as altitude within the atmosphere. To calculate the overlap as a function of height in the atmosphere for the general case where the telescope is pointed at a source of zenith angle ζ, z should be replaced by $h/\cos\zeta$ where h is the vertical height in the atmosphere. Figure 1 shows the assumed geometry.

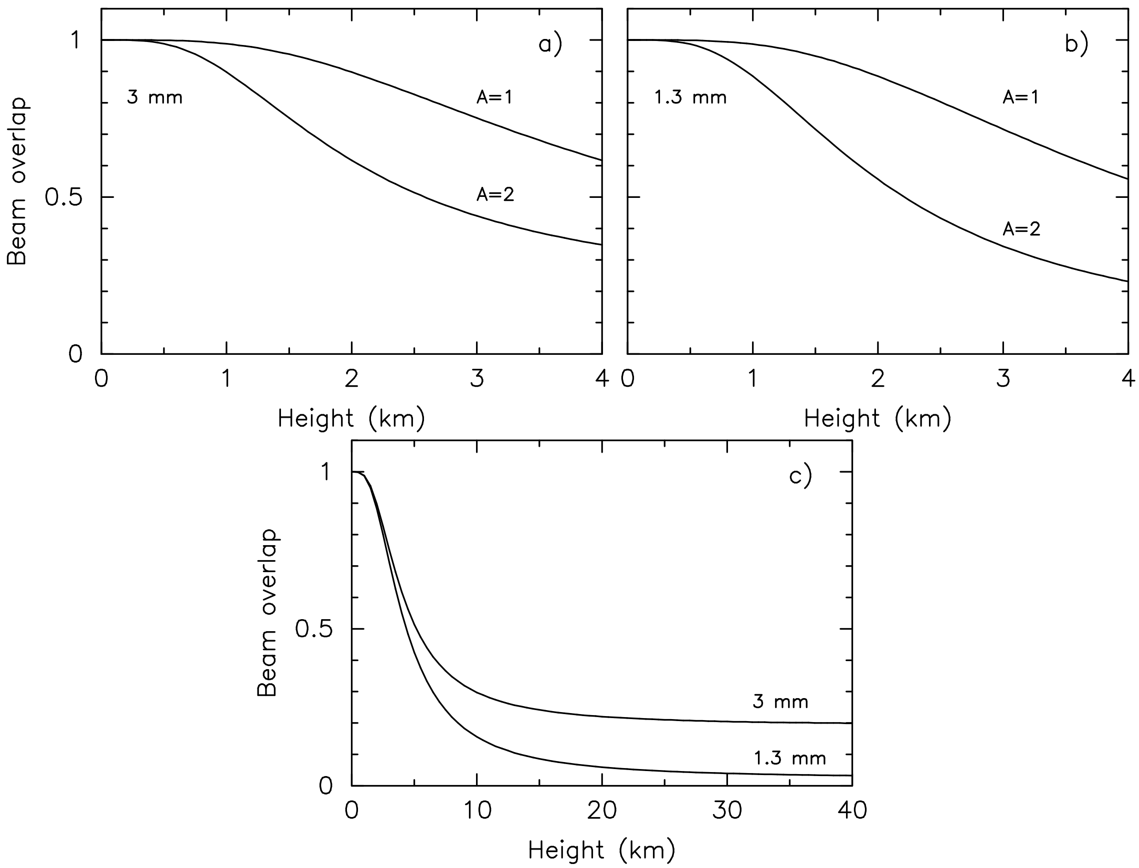

Figure 2: Overlap of the astronomical and WVR beams as a function of height in the atmosphere for the current BIMA setup. (a) The case of no offset between the beams. (b) As (a) but for an observing wavelength of 1 mm. Note the overlap falls off slightly more quickly with height. In both (a) and (b), results are plotted for an airmass A of 1 and 2. (c) The asymptotic behavior of equation 5 for $z \gg 1$. The 3-mm and 1.3-mm cases are shown by different curves.

1.1 General properties

For no separation ($s = 0$), the overlap is $4w_1^2w_2^2/(w_1^2 + w_2^2)^2$ (Figure 2(a)). In the 'far-field' limit ($z \to \infty$), this tends to a constant value of $4(\lambda_1\lambda_2)^2/(\lambda_1^2 + \lambda_2^2)^2$. Numerically this is equal to 0.19 for 3-mm and 1.3-cm beams. However, this limit is not reached until a distance of 30 km from the telescope (Figure 2(c)). These results are not significantly altered when the wavelengths are 1.3 mm and 1.3 cm (Figure 2(b) and (c)).

When the beams are offset then there are two limiting regimes where the equation simplifies. The 'near-field' behavior is difficult to simplify as there are three limits which must be satisfied: $z \ll \pi w_0^2/\lambda_1$, $z \ll \pi w_0^2/\lambda_2$ and $z^2 \ll 2A_{\rm w}^2/(\lambda_1^2 + \lambda_2^2)$. If all three are satisfied then the overlap decreases as a gaussian in z.

In the limit where $z^2 \gg 2A_{\rm w}^2/(\lambda_1^2 + \lambda_2^2)$, the overlap becomes constant, the value of which is determined by the angular separation of the two beams. For an angular separation which satisfies $\theta_s^2 \gg (\lambda_1^2 + \lambda_2^2)/(2\pi A_{\rm w})$, this limiting overlap is vanishingly small. The point of equality for this expression occurs at $\theta_s \simeq 9$ arcmin for BIMA antennas operating at wavelengths of 1.3 cm and 3 mm.

2 Results

2.1 BIMA

For the current setup at BIMA $\lambda_1 = 3$ mm, $\lambda_2 = 1.3$ cm, $F = 10$ dB (for both wavelengths) and $D = 6.1$ m, which gives a beamwaist of $w_0 = 2.87$ m. The offset between the astronomical and water-vapor radiometer (WVR) beams (θ_s) is set by the size of the 1.3-cm optics and other structural constraints to be 28 arcmin at 3 mm (and 22 arcmin at 1.3 mm). Referring to equation 5, $A_{\rm w} = 25.34$ m^2.

Figure 3(a),(b) shows the overlap between the astronomical and WVR beams as a function of height in the atmosphere for wavelengths of 3 mm and 1.3 mm. In each panel the variation is plotted for an airmass of 1 (looking at the zenith) and 2 (corresponding to an elevation of 30 degrees) to show the range of dependence on source elevation.

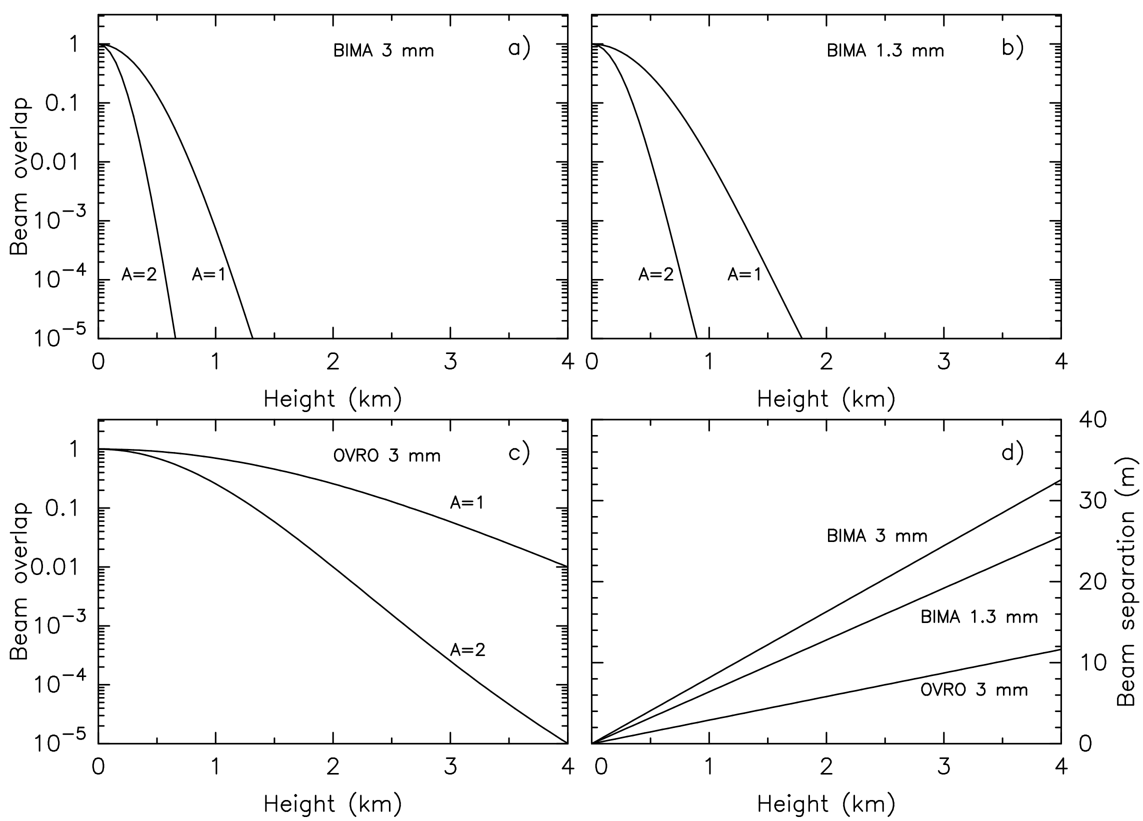

Figure 3: (a) Beam overlap at 3 mm and 1.3 cm for BIMA. The two curves are for airmasses of 1 and 2 (correspnding to an elevation of 90° and 30° respectively). (b) As (a) but for 1.3 mm. (c) As (a) but for OVRO where the beam offset is only 10 arcmin. (d) Linear separation as a function of height for the 3-mm and 1.3-mm beams at BIMA as well as the 3-mm beam at OVRO assuming the telescope is pointed towards the zenith.

The striking result is that for altitudes above about 800 m, the overlap becomes less than 1%, and even smaller for higher airmass. The beam overlap is considerably smaller when observing at an airmass of 2 because the path length through the atmosphere increases with increasing airmass, and therefore the overlap decreases more rapidly with height in the atmosphere. This fact suggests that, in order to minimize errors, future instruments should have the beams offset in azimuth to ensure that both beams are looking along the atmospheric paths of equal length.

Figure 3(d) plots the beam separation as a function of height (at the zenith). For heights below about 4 km, the beam separation is less than $\sim$35 m at 3 mm (and smaller at 1.3 mm). Therefore if the dominant source of water-vapor fluctuations has a scale less than this separation then the two beams will not be observing through the same amount of water vapor, and there will therefore be a small atmospheric phase difference between the two beams. However, if the water-vapor content of the atmosphere shows variations on a larger scale then the fact that the beams do not overlap may not be critical (but see Section 3).

2.2 OVRO

The overlap is significantly increased for the setup at OVRO, although it is still only a few per cent at a height of 3 km (and much lower for when the telescope is pointed away from the zenith: see Figure 3(c)). Here the antenna diameter is 10.4 m and the offset between the beams is only 10 arcmin. The edge-taper has been assumed to be 10 dB at both wavelengths. The dominant source of this improvement is the smaller offset between the two beams at OVRO. The fact that the OVRO antennas are larger also increases the overlap (as the near-field regime extends further for large antennas), but this is a secondary effect.

Figure 4 demonstrates these points where the beam overlap as a function of height (assuming an airmass of 1) is plotted for cases when the beam offset for BIMA is reduced to 10 arcmin, and the OVRO offset increased to 28 arcmin. Also plotted are a number of examples where the beam offset for BIMA varies between 5 and 20 arcmin, demonstrating the sensitivity to this parameter. (Note however that these are artificial cases for BIMA and if implemented would cause the WVR beam to be truncated, thus degrading the WVR beam.)

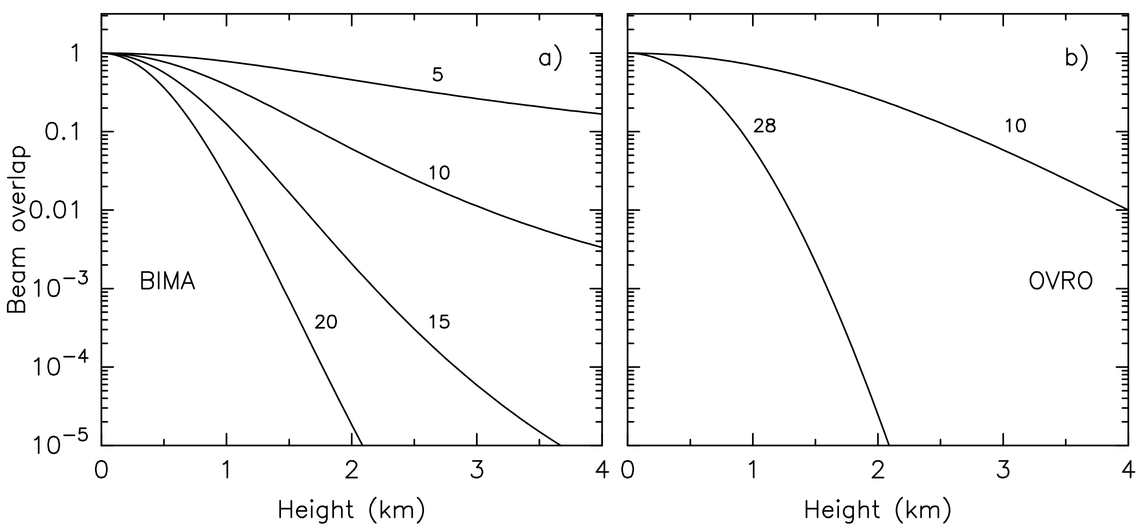

Figure 4: (a) Overlap of the astronomical and WVR beams at BIMA as a function of height for beam offsets of between 5 and 20 arcmin. (b) The beam overlap for OVRO assuming offsets of 10 arcmin (current) and 28 arcmin for comparison. Note that in the case that the beam offset is 10 arcmin for both instruments, the decrease in the overlap with height is more rapid with the smaller BIMA antennas.

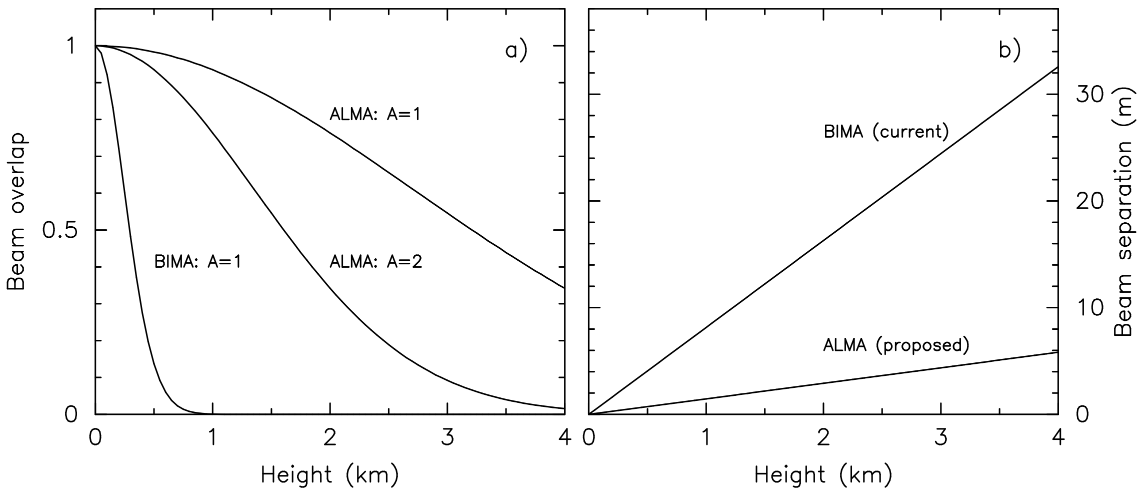

Figure 5: (a) Overlap of the astronomical and WVR beams as a function of height for the arrangement recently proposed for ALMA. The cases for an airmass of 1 and 2 are shown. Also plotted is the beam overlap for the current BIMA setup (for an airmass of unity). Note that, unlike many of the previous figures, the overlap is plotted on a linear scale. (b) A comparison of the beam separation as a function of height for the proposed ALMA and current BIMA setups.

2.3 ALMA

Despite the high quality of the Chajnantor site, ALMA will also employ water-vapor radiometers to attain the highest resolutions proposed, although these will measure the water line at 183 GHz (1.64 mm). Observation of this line has the advantage that the WVR optics are more compact than at 22 GHz and so a smaller offset can be realised, and the line is stronger because its optical depth is higher. One current proposal for such an instrument by Cambridge and Onsala has a beam offset of only 5 arcmin. Given that the proposed diameter for the ALMA antennas is 12 m, it is clear from the previous results that such a system will have a considerably greater beam overlap than either of the current BIMA or OVRO systems. This is shown graphically in Figure 5(a) where the zenith beam overlap is still as high as 0.35 at a height of 4 km. For an airmass of 2, the overlap is somewhat smaller but remains greater than a few per cent at 4 km.

Similarly, a smaller beam offset directly translates into a smaller linear separation: at a (zenith) height of 4 km above the array (or about 9 km altitude in the atmosphere), the separation will be of order half a telescope diameter (i.e. $\sim$ 6 m – see Figure 5b).

3 Discussion

The single most striking result from the previous section is the essentially negligible overlap between the astronomical and WVR beams at BIMA (and OVRO). At first this appears to be a serious deficiency, and here we explore the consequences of the question: does it matter that the beam overlap is so small?

The simplest way to examine this is to determine whether the two beams 'see' the same atmosphere within the integration time of the radiometer. Over an integration time $t_{\rm int}$, the atmosphere moving with speed v will traverse a distance $d = vt_{\rm int}$. This implies the speed at which the atmosphere moves between the beams must be at least $d/t_{\rm int}$ in order that both beams sample the same parcel of water vapor, assuming that the atmosphere moves parallel to the line joining the centers of the beams and that the properties of the atmosphere moving through each beam are, on average, identical (which may be true if the relevant scales are much larger than the beam separation). For an integration time of 10 s (Staguhn et al. 1998), and a beam separation of 35 m, the wind speed must be at least 3.5 m s^{-1}. Radiosonde measurements at Reno (NV) airport (located approximately 150 miles SE from Hat Creek) show that for most of the time, the wind speed is higher than 3.5 m s^{-1} (at an altitude greater than 500–1500 m above ground), and in general the wind direction is constant with increasing altitude. If the wind speed condition is not satisfied, a time lag may be observed between the water-vapor and astronomical phase data. This analysis may seem somewhat naive, but experimental data do show a time lag for the cases when the beams are indeed aligned along the direction of the prevailing wind (Tahmoush & Rogers 2000). Furthermore, the lag is observed in both the positive and negative sense depending on whether the wind blows the atmosphere through the WVR beam before or after the astronomical beam.

A more rigorous approach is to examine the expected phase fluctuations along a baseline between the two beams. On a baseline of length b (in km) which is smaller than the width of the turbulent layer responsible for the fluctuations in water vapor content, the root-mean-square phase fluctuations (in degrees) at a wavelength λ (in mm) are given by

$$\Phi_{\rm rms}(b) = \frac{K}{\lambda}\, b^{5/6}\,, \qquad (6)$$

where K depends on the site and weather (a better site has a lower value of K). For example, a typical value for the VLA is 300 (Carilli & Holdaway 1999). The corresponding value for Hat Creek is unknown. In the absence of experimental data, we assume that conditions at Hat Creek are similar to those at the VLA. This equation may also be expressed in terms of electrical path length, $L_{\rm e}$,

$$L_{\rm e} = \frac{K}{360}\, b^{5/6} \qquad (7)$$

in millimeters (Carilli & Holdaway 1999). For a baseline between the two beams of 20 m (corresponding to a zenith height of 2.5 km, or only 1.25 km at an airmass of 2), the mean path length fluctuation is $\sim 30\ \mu$m, close to the 35-μm design specification for APHID (Harris 2000). This suggests that the significant beam separation is in fact a limiting factor in the correction that can be achieved, although it obviously depends on the height at which the water vapor fluctuations occur. Thus it would seem that knowledge of the altitude of the layer responsible for the fluctuations is a valuable quantity to determine.

Another way to look at the effect of the beam separation is to estimate the fractional error in the path-length correction. On a baseline b between two antennas, the beam separation s gives rise to an additional uncertainty of $(s/b)^{5/6}$ in the phase correction. Thus for short baselines, the beam separation will have an impact on the level of achievable phase correction, while long baselines ($b \gg s$) will be less affected. Again, the height of the dominant turbulent layer strongly affects this result as it determines the value of s. Therefore it is the beam separation at the height of the dominant turbulent layer rather than the beam overlap which is the more important in determining the accuracy of the phase correction.

For the ALMA antennas, the beam separation is less than the antenna diameter so the beam offset does not limit the sensitivity. Instead, the antenna diameter itself is the limiting factor. If $K \simeq 100$ (Carilli & Holdaway 1999) then the limiting path length is only 7 μm. At a wavelength of 450 μm, for example, this path length corresponds to a phase difference of 5.6°, and thus a coherence loss of only 1%.

4 Conclusion

In this memo we have determined the overlap between the astronomical and WVR beams at BIMA (as well as OVRO and ALMA) and explored the consequences with respect to reducing the uncertainty in phase difference between the two beams.

The primary result is that for the current BIMA system, the beams are not well coupled above an altitude of 800 m, with the center-to-center distance increasing to 35 m at 4 km. Reducing the offset between the beams significantly increases the overlap, although from a practical viewpoint, this is limited by the size of the 1.3-cm optics. The linear separation of the beams introduces a path-length error of order 30 μm (at an altitude of 2.5 km) and is therefore a potentially limiting factor in achieving the level of path-length correction specified for APHID (i.e. 35 μm per antenna), although the uncertainty is lower for long baselines. To minimize errors introduced by the beam offset, we suggest that future instruments should have beams which are offset in the azimuthal direction to ensure that the atmospheric paths are of equal length.

The prospects for ALMA, however, are somewhat brighter as operation at 183 GHz allows for smaller optics and therefore a smaller angular offset – 5 arcmin has been proposed for a joint Cambridge-Onsala instrument – while the larger antenna diameter also contributes by increasing the size of the ‘near field’. The beam coupling for such a system remains greater than 1 per cent even for an airmass of 2 at an altitude of 4 km. The beam separation is less than the antenna diameter for altitudes up to 8 km above the antenna and will therefore not limit the path-length corrections.

Acknowledgments

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References

[1] Carilli C.L., Holdaway M.A., 1999, Radio Sci, 34, 817

[2] Harris A.I., 2000, BIMA Memo 80 (ALMA Memo 307)

[3] Staguhn J., Harris A.I., Plambeck R.L., Welch W.J., 1998, in Phillips T.G., ed, SPIE 3357: Advanced Technology MMW, Radio and Terahertz Telescopes. SPIE, Bellingham (WA), p432.

[4] Tahmoush D.A., Rogers A.E.E., 2000, Radio Sci (submitted)