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Sideband Calibration of Millimeter-Wave Receivers

A. R. Kerr, S.-K. Pan and J. E. Effland
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Accurate calibration of a millimeter-wave heterodyne receiver requires a knowledge of the ratio of gains at the upper and lower sideband frequencies — *i.e.*, the *sideband ratio*. For an ideal double-sideband receiver, the sideband ratio is unity, but in practice it can be several dB, particularly when a high IF is used [1]. In the case of sideband-separating receivers, the sideband ratio is the *image rejection*, which is infinite in the ideal case but may be as low as 10 dB in practice. In principle, the sideband ratio of a receiver can be measured by injecting CW signals of known relative amplitudes into the upper and lower sidebands and measuring the IF response to each. At millimeter wavelengths, however, it is difficult to determine with sufficient accuracy the relative amplitudes of two low level RF signals separated in frequency by twice the IF ($2f_{IF}$ = 8–24 GHz in the case of ALMA receivers).

This note shows that the sideband ratio of a sideband-separating receiver can be determined *without knowledge of the RF signal levels* if the IF response to broadband RF noise sources at two temperatures is also measured. This is not the case for simple double-sideband receivers.

Unless the sideband ratio of a heterodyne receiver is close to the ideal value (less than ~ 0.1 dB for a double-sideband receiver, greater than ~ 20 dB for a sideband-separating receiver), a significant correction is required to the single-sideband receiver noise temperature deduced from the Y-factor measured using broadband hot and cold loads. If, in addition, there is significant conversion gain from one or more of the higher harmonic sidebands, $nf_{LO} \pm f_{IF}$ ($n \geq 2$), that must also be taken into account when evaluating the sideband ratio (or image rejection) and the single-sideband receiver noise temperature.

1. Determination of the Image Rejection of a Sideband-Separating Receiver

1.1. With No Conversion from Higher Harmonic Sidebands

In this section it is assumed that the only significant conversion in the sideband-separating receiver is from the upper and lower sidebands, $f_{LO} \pm f_{IF}$, to f_{IF}. Conversion from higher harmonic sidebands, $nf_{LO} \pm f_{IF}$ (n = 2, 3, ...), to f_{IF} is taken to be insignificant. Such a receiver is depicted in Fig. 1, with the conversion (power) gains from each RF input port to each IF output port denoted by the quantities $G_{i,j}$. The desired image rejection ratios are:

$$R_1 = \frac{G_{1U}}{G_{1L}} \text{ at IF port 1, and } R_2 = \frac{G_{2L}}{G_{2U}} \text{ at IF port 2.}$$

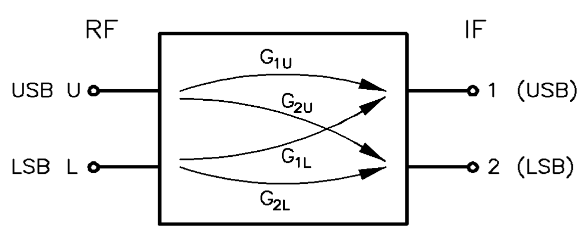

Fig. 1. Power gains of the sideband-separating receiver. The RF upper and lower sideband ports are normally the same waveguide or transmission line, but are shown separately here for clarity.

The following measurements are made:

(i) With a CW test signal (of unknown amplitude) in the upper sideband, the corresponding IF signals at IF ports 1 and 2 are measured. The ratio of these powers is

$$M_U = \frac{G_{1U}}{G_{2U}} \quad . \tag{1}$$

(ii) With a CW test signal (of unknown amplitude) in the lower sideband, the corresponding IF signals at IF ports 1 and 2 are measured. The ratio of these powers is

$$M_L = \frac{G_{2L}}{G_{1L}} \quad . \tag{2}$$

(iii) The changes, ΔP_1 and ΔP_2, of output power at IF ports 1 and 2, are measured when a cold load at the receiver input is replaced by a hot load. If the difference in noise temperatures of the hot and cold loads is ΔT, then

$$\Delta P_1 = kB\Delta T\left(G_{1U} + G_{1L}\right) , \tag{3a}$$

$$\Delta P_2 = kB\Delta T\left(G_{2U} + G_{2L}\right) \quad . \tag{3b}$$

Define

$$M_{DSB} \triangleq \frac{\Delta P_1}{\Delta P_2} = \frac{G_{1U} + G_{1L}}{G_{2U} + G_{2L}} \quad . \tag{4}$$

The measured quantities M_U, M_L, and M_{DSB} can now be used to deduce the sideband separation ratios R_1 and R_2.

First, it is convenient to normalize all the G_{ij} to G_{2L}, so $g_{1U} = \frac{G_{1U}}{G_{2L}}$, $g_{2U} = \frac{G_{2U}}{G_{2L}}$, $g_{1L} = \frac{G_{1L}}{G_{2L}}$,

and $g_{2L} = 1$. The desired image rejection ratios become:

$$R_1 = \frac{g_{1U}}{g_{1L}} \text{ and } R_2 = \frac{g_{2L}}{g_{2U}} = \frac{1}{g_{2U}} . \tag{5}$$

From (1):

$$g_{2U} = \frac{g_{1U}}{M_U} \quad . \tag{6}$$

From (2):

$$g_{1L} = \frac{1}{M_L} \quad . \tag{7}$$

From (4):

$$0 = g_{1U} + g_{1L} - M_{DSB} - M_{DSB}\, g_{2U} \, . \tag{8}$$

From (6), (7) and (8):

$$0 = g_{1U} + \frac{1}{M_L} - M_{DSB} - M_{DSB} \cdot \frac{g_{1U}}{M_U} \quad .$$

Therefore,

$$g_{1U} = \frac{M_{DSB} - \frac{1}{M_L}}{1 - \frac{M_{DSB}}{M_U}} . \tag{9}$$

From (9) and (6):

$$g_{2U} = \frac{M_{DSB} - \frac{1}{M_L}}{M_U - M_{DSB}} \quad . \tag{10}$$

So, in (5): $$R_1 = \frac{g_{1U}}{g_{1L}} = M_U \cdot \frac{M_L M_{DSB} - 1}{M_U - M_{DSB}} \ , \tag{11}$$

and $$R_2 = \frac{g_{2L}}{g_{2U}} = M_L \cdot \frac{M_U - M_{DSB}}{M_L M_{DSB} - 1} \ . \tag{12}$$

Note that $$R_1 R_2 = M_U M_L \ . \tag{13}$$

1.2. Correcting Image Rejection Measurements for Conversion from Higher Harmonic Sidebands

Now consider a sideband-separating receiver in which there is significant conversion from higher harmonic sidebands, $nf_{LO} \pm f_{IF}$ (n = 2, 3, ...), to f_{IF}. Such a receiver is depicted in Fig. 2, where input port N collectively represents all the higher harmonic sideband input ports. The conversion gains from the higher harmonic sidebands to IF output ports 1 and 2 are denoted $G_{1,N}$ and $G_{2,N}$. These are the (power) gains that would be measured if hot and cold loads were placed simultaneously at all the higher harmonic sideband input ports while the terminations on ports U and L are maintained at a steady temperature: $G_{1,N} = \sum_{m=2}^{\infty} G_{1,m}$ and $G_{2,N} = \sum_{m=2}^{\infty} G_{2,m}$.

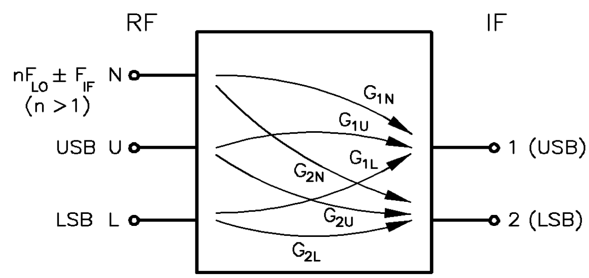

Fig. 2. Power gains of the sideband-separating receiver. RF port N represents all the higher harmonic sidebands $nf_{LO} \pm f_{IF}$ *(n = 2, 3, ...)*. The RF sideband ports are normally the same waveguide or transmission line, but are shown separately here for clarity.

As before, the desired image rejection ratios are:

$$R_1 = \frac{G_{1U}}{G_{1L}} \text{ at IF port 1, and } R_2 = \frac{G_{2L}}{G_{2U}} \text{ at IF port 2.}$$

The quantities M_U, M_L, ΔP_1, and ΔP_2 are measured as before:

(i) With a CW test signal (of unknown amplitude) in the upper sideband, the corresponding IF signals at IF ports 1 and 2 are measured. The ratio of these powers is

$$M_U = \frac{G_{1U}}{G_{2U}} \ . \tag{14}$$

(ii) With a CW test signal (of unknown amplitude) in the lower sideband, the corresponding IF signals at IF ports 1 and 2 are measured. The ratio of these powers is

$$M_L = \frac{G_{2L}}{G_{1L}} \ . \tag{15}$$

(iii) The changes, ΔP_1 and ΔP_2, of output power at IF ports 1 and 2, are measured when a cold load at the receiver input is replaced by a hot load. If the difference in noise temperatures of the hot and cold loads is ΔT, then

$$\Delta P_1 = kB\Delta T\left(G_{1U} + G_{1L} + G_{1N}\right) , \tag{16a}$$

$$\Delta P_2 = kB\Delta T\left(G_{2U} + G_{2L} + G_{2N}\right) . \tag{16b}$$

(iv) Measurement (iii) is repeated with an inclined dichroic plate between the receiver input and the hot/cold load. The dichroic plate should be designed to reflect all USB and LSB power, while transmitting all power at the higher harmonic sidebands $nf_{LO} \pm f_{IF}$ (n > 1). It is inclined so it does not affect the source impedance seen by the receiver. If the insertion gain of the dichroic plate at the higher harmonic sidebands is G_D ($\leq$ 1), then the resulting changes in output power are:

$$\Delta P_1^N = kB\Delta T G_{1N} G_D , \tag{17a}$$

$$\Delta P_2^N = kB\Delta T G_{2N} G_D . \tag{17b}$$

(G_D can be measured by observing the change in ΔP^N when a second identical inclined dichroic plate is introduced in the beam.) It is now possible to subtract the higher harmonic sideband contribution ΔP^N from ΔP to obtain the corrected power changes, ΔP_1^C and ΔP_2^C which would be measured if the hot and cold loads were visible only in the upper and lower sidebands:

$$\Delta P_1^C = \Delta P_1 - kB\Delta T G_{1N} = \Delta P_1 - \Delta P_1^N / G_D , \tag{18a}$$

$$\Delta P_2^C = \Delta P_2 - kB\Delta T G_{2N} = \Delta P_2 - \Delta P_2^N / G_D . \tag{18b}$$

From (16)-(18), $\Delta P_1^C = kB\Delta T\left(G_{1U} + G_{1L}\right)$ and $\Delta P_2^C = kB\Delta T\left(G_{2U} + G_{2L}\right)$. As before, define

$$M_{DSB} \triangleq \frac{\Delta P_1^C}{\Delta P_2^C} = \frac{G_{1U} + G_{1L}}{G_{2U} + G_{2L}} . \tag{19}$$

The measured quantities M_U, M_L, and M_{DSB} can now be used to deduce the sideband separation ratios R_1 and R_2 using eqns. (5)-(13).

2. Determination of the SSB Receiver Noise Temperature

2.1. With No Conversion from Higher Harmonic Sidebands

The noise temperature of a millimeter-wave receiver is usually measured by the Y-factor method with hot and cold loads, typically liquid nitrogen and room temperature, whose noise temperatures are accurately known. The IF response contains contributions from both RF sidebands, and the resulting noise temperature is the double-sideband quantity:

$$T_{R,DSB} = \frac{T_{hot} - Y\,T_{cold}}{Y - 1} . \tag{20}$$

This is the noise temperature of the RF source which, when connected at the input (*i.e.,* both upper and lower sidebands) of a noiseless but otherwise identical receiver, would produce the same output power at the IF port in question as the actual receiver would produce with its input connected to a source with zero noise temperature.*

The single-sideband noise temperature of a receiver is obtained by ascribing all the receiver noise to an equivalent input source in one sideband. It is the noise temperature of the RF source which, when connected to one

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* The concept of zero noise temperature, implying a thermal noise power density of zero W/Hz, is non-physical but is a convenient abstraction.

sideband input of a noiseless but otherwise identical receiver, with the other sideband input connected to a source with zero noise temperature, would produce the same output power at the IF port in question as the actual receiver with both sideband inputs connected to sources with zero noise temperature. The SSB noise temperature of a DSB receiver with sideband ratio R ($0 \le R \le \infty$) is obtained by correcting the DSB noise temperature for the image contribution to the IF output:

$$T_{R,\,SSB} = T_{R,\,DSB}\left(1+\frac{1}{R}\right) \quad . \tag{21a}$$

For a sideband-separating receiver, the SSB noise temperatures are likewise

$$T_{R,\,USB} = T_{R,\,DSB}\left(1+\frac{1}{R_1}\right) \quad \text{and} \quad T_{R,\,LSB} = T_{R,\,DSB}\left(1+\frac{1}{R_2}\right) \quad . \tag{21b}$$

where R_1 and R_2 are the image rejections at the two IF outputs. The dependence of the SSB receiver noise temperature $T_{R,\,SSB}$ on the image rejection (or sideband ratio) R (dB) is shown in the table below.

$T_{R,\,SSB} = T_{R,\,DSB}\,(1+1/R)$

R dB	(1 + 1/R)
-20	101
-10	11.0
-5	4.16
-1	2.26
-0.1	2.02
0	2
0.1	1.98
1	1.79
5	1.32
10	1.10
15	1.03
20	1.01
25	1.003
30	1.001

2.2. Correcting Noise Temperature Measurements for the Effect of Conversion from Higher Harmonic Sidebands

If there is significant conversion from the higher harmonic sidebands $nf_{LO} \pm f_{IF}$ ($n > 1$) to f_{IF}, the SSB receiver noise temperature determined from the Y-factor will require a further correction. Consider the output power P_1 at IF port 1 of a sideband separating receiver (or at the single IF port of a DSB receiver). When hot and cold loads are connected to the receiver input (without the dichroic plate),

$$P_{1,\,hot} = kB\left[\left(T_{R,\,DSB} + T_{hot}\right)\left(G_{1,U} + G_{1L}\right) + T_{hot}G_{1N}\right], \tag{22}$$

$$P_{1,\,cold} = kB\left[\left(T_{R,\,DSB} + T_{cold}\right)\left(G_{1,U} + G_{1L}\right) + T_{cold}G_{1N}\right]. \tag{23}$$

From (17a), $G_{1N} = \dfrac{\Delta P_1^{\,N}}{kB\Delta T G_D}$. Therefore, in (22) and (23):

$$P_{1,hot} - \frac{\Delta P_1^N T_{hot}}{\Delta T G_D} = kB\left(T_{R,DSB} + T_{hot}\right)\left(G_{1,U} + G_{1L}\right), \tag{24}$$

$$P_{1,cold} - \frac{\Delta P_1^N T_{cold}}{\Delta T G_D} = kB\left(T_{R,DSB} + T_{cold}\right)\left(G_{1,U} + G_{1L}\right). \tag{25}$$

Define the corrected Y-factor:

$$Y^C = \frac{P_{1,hot} - \dfrac{\Delta P_1^N T_{hot}}{\Delta T G_D}}{P_{1,cold} - \dfrac{\Delta P_1^N T_{cold}}{\Delta T G_D}}. \tag{26}$$

All the quantities on the right side of (26) are known from measurement, so Y^C can be evaluated.

From (24) and (25),

$$T_{R,DSB} = \frac{T_{hot} - Y^C T_{cold}}{Y^C - 1}. \tag{27}$$

This is the DSB receiver noise temperature of a DSB or sideband-separating receiver. For the sideband-separating receiver, the DSB receiver noise temperature at the second IF port is determined using the same procedure. Once the DSB receiver noise temperature for the IF port(s) is determined, the SSB noise temperatures are obtained from (21) using values of image rejection ratios determined as in section 1.2.

3. Conclusion and Discussion

The image rejection of a sideband-separating mixer can be measured accurately using CW test signals in the upper and lower sidebands, even when the relative power levels of the test signals are not known. This allows accurate determination of the upper- and lower-sideband gains and the single-sideband noise temperature of a sideband-separating receiver, even if it has poor image rejection. *In contrast, there is no simple and accurate way to determine the sideband ratio and SSB sensitivity of a DSB receiver.* This has implications for ALMA's single-dish mode of operation, in which sideband separation using LO phase switching is not possible, but high SSB measurement accuracy is required.

The frequencies of the upper- and lower-sideband CW test signals used in determining M_U and M_L must be chosen to give the same intermediate frequency f_0. During the measurements with the RF noise sources, a narrow-band IF filter centered at the same f_0 should be used. The bandwidth B of the filter should be smaller than the width of any features on the receiver gain, noise, or image rejection characteristics.

The levels of the CW test signals need not be known, but they must be low enough to avoid saturation of the mixer or the IF measuring system, and large enough to give a measurable response above the noise floor at the isolated port. The measured IF signal will include the noise of the measuring system. When a power meter or other square-law detector is used, an indicated signal level a factor H above the noise floor of the measuring system can be corrected by a factor (1 - 1/H) to obtain the actual IF signal level. The correction factor (1 - 1/H) is shown in the Fig. 2 (lower curve) as a function of the indicated signal-to-noise ratio H. If a spectrum analyzer is used to measure the IF signals, a different correction factor must be used [2, 3]. Most spectrum analyzers use an envelope detector rather than a square-law detector, and the appropriate correction also depends on whether a log or linear display is selected. If envelope detection is used with the usual log display, the correction factor is $-10.42 \times 10^{-0.333(H(dB))}$ dB. This is shown as the upper curve in Fig. 3.

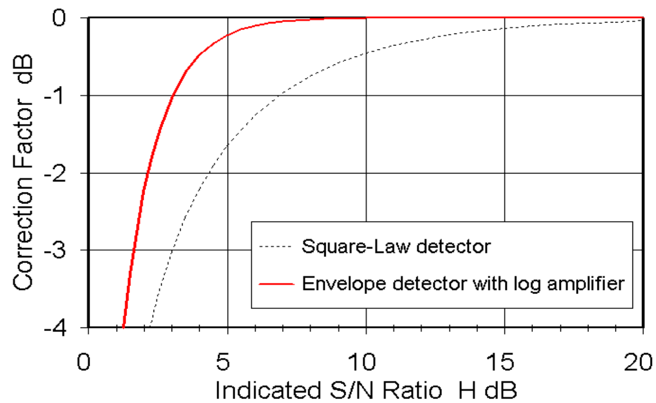

Fig. 3. Correction for system noise when measuring a CW test signal. The correction factor is plotted as a function of the indicated signal-to-noise ratio H. The upper curve applies to measurements using a spectrum analyzer with an envelope detector (usual in modern spectrum analyzers) and a log (dB) display. The lower curve is for measurements using a power meter with a square-law detector.

Depending on the details of the sideband-separating receiver in Figs. 1 or 2, it may be necessary to ensure that the terminations on IF ports 1 and 2 do not change during the measurements. If the receiver contains IF amplifiers or isolators in such a way that ports 1 and 2 are isolated from one another, then the measurement of M_U, M_L, and M_{DSB} can be done by connecting the IF measuring system to ports 1 and 2 without concern for the termination on the unused port (*e.g.*, using a simple coaxial SP2T switch). If, on the other hand, IF ports 1 and 2 are not isolated, as is the case when ports 1 and 2 are the output ports of the IF quadrature hybrid of a sideband-separating mixer, then care must be taken to ensure that both ports remain properly terminated throughout the measurement. This can be achieved using a coaxial changeover switch.

If an IF switch is used in these measurements and is connected directly to an SIS mixer, it must be chosen to have low static generation when operated. We have found that most mechanical coaxial switches generate a static pulse during switching sufficient to damage an SIS mixer.

When measuring the noise temperature (rather than just the image rejection) of a receiver using hot and cold loads, the noise temperatures of the loads must be accurately known[*]. At millimeter wavelengths, the noise temperatures of hot and cold loads are not exactly equal to their physical temperatures [4, 5]. This is because: (i) the Rayleigh-Jeans radiation law deviates from the Planck law at low temperatures and high frequencies, and (ii) the zero-point noise has a significant magnitude at millimeter wavelengths ($hf/2k = 5.5$ K at 230 GHz). In the millimeter-wave range, when liquid nitrogen and room temperature loads are used, these two corrections to the Rayleigh-Jeans law almost cancel out and sufficient accuracy is obtained for ALMA receiver measurements if it is assumed that the noise temperature of a load is equal to its physical temperature. This gives results close to those which would be obtained using the Callen and Welton radiation law in calculating the hot and cold load noise temperatures, and treats the zero-point noise as part of the noise temperature of the source rather than part of the receiver noise temperature. At 230 GHz, the difference between the Callen and Welton noise temperature and the physical temperature of black bodies at 77 K and 300 K is 0.13 K and 0.03 K, respectively.

[*] The noise temperature of a load is its available thermal noise power density (W/Hz) divided by Boltzmann's constant, k.

4. References

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