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Phase Correction using Submillimeter Atmospheric Continuum Emission

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Abstract

We discuss a phase correction method using total power from submillimeter/millimeter SIS receivers. The discussion is based on FTS measurements and model calculations of submillimeter/millimeter atmospheric emission under various weather conditions. Liquid water in the atmosphere (cloud or fog) limits the phase correction accuracy using the total power measurements. Comparison between millimeter and submillimeter total power phase correction shows that the sensitivity requirement is more easily met at submillimeter wavelengths under good atmospheric conditions (precipitable water vapor [*pwv*] < 1 mm), and the phase correction error due to liquid water is much less at these wavelengths. Simultaneous operation of millimeter and submillimeter receivers will further reduce the phase correction error appreciably.

1 Introduction

In modern ground-based millimeter/submillimeter interferometry, water vapor fluctuations in the Earth atmosphere limit the achievable resolution, similar to “seeing” in optical astronomy. Therefore it is of uttermost importance to devise a successful method to correct for the effect of water vapor fluctuations. If there is a different amount of water vapor in the line of sight of one antenna as compared to another, a path difference is introduced, which will result in a phase offset in an interferometric measurement. If the path difference fluctuates there will be phase fluctuations that deteriorate the data quality noticeably. To compensate for the excess paths caused by water vapor, radiometric measurements of atmospheric temperature fluctuations around water vapor lines or in "atmospheric window" regions are used. Water vapor affects both the amplitude and phase of the radio waves in a related and well-known manner, thus allowing to correct the phase at any given frequency by measuring the intensity at a different one. However, this method has a problem when liquid water (clouds or fog) exists in the atmosphere, since it largely affects the amplitude of the wave but less its phase. Therefore, the presence of liquid water alters the relationship between amplitude and phase. Hence, it is important for the phase correction to separate water vapor from liquid water, and retrieve the water vapor fluctuation.

The Atacama Large Millimeter Array (ALMA) aims for a spatial resolution of less than 10 milliarcsec at a frequency around 900 GHz, which requires that the path length fluctuations are less than 11.5 μm. To achieve this goal routinely, a carefully designed phase correction method is absolutely essential. At present, the most convincing phase correction method uses 183 GHz water vapor radiometers [16]. In dry weather (precipitable water vapor [pwv] $\leq$ 10 mm), the water vapor line at 183 GHz has much higher temperature sensitivity to changes in the amount of water vapor than the line at 22 GHz. Phase correction at 183 GHz is a very promising method for dry sites such as the Atacama dessert, where ALMA will be built. Since the 183 GHz line center saturates with about 2 mm of pwv, and since the continuum part far away from the line is affected by liquid water, the radiometers measure the wing component of the line. This method is being tested extensively at the Caltech Submillimeter Observatory (CSO) - James Clark Maxwell Telescope (JCMT) interferometer [17] as well as at the Sub-Millimeter Array (SMA), and currently a second edition of the radiometers is being built with a substantially improved design [1].

The sky brightness temperature measurements at the ALMA site with a Fourier Transform Spectrometer (FTS) suggest that it is possible to separate the measured emission into water vapor and liquid water contributions using simultaneous measurements at millimeter and submillimeter wavelengths [8, 9]. The FTS measurements also suggest that the submillimeter emission is less affected by liquid water than the millimeter emission. From these facts, we suggest two different phase correction methods; (1) measurement of the submillimeter emission alone which is less affected by the liquid water, and (2) measurements of the emission at two frequencies (millimeter and submillimeter) and separation of the contribution from water vapor and liquid water. Both of these phase correction methods only require measurements of the total power output from one or two astronomical receivers on each antenna, and there is no need to build additional instruments. Therefore it is possible to apply this technique to any submillimeter interferometer.

In this memo, we discuss these two phase correction methods. In section 2 and 3, we will look at the total power phase correction at only one frequency: In section 2, we evaluate whether the receiver temperature of state-of-the-art receivers is low enough to achieve the ALMA requirements for the phase correction. In section 3, we estimate the additional error in the phase correction caused by the presence of liquid water. In section 4, we discuss how much this error could be reduced by using a second frequency for phase correction.

2 Sensitivity Requirements for Single Frequency Total Power Phase Correction for ALMA

In order to determine the sensitivity requirements of the total power phase correction with ALMA, we first calculate the derivative of the atmospheric temperature ($T_{\rm atm}$) with respect

Table 1: Atmospheric temperature changes ($\Delta T_{\rm atm}$) that would be observed for a change in path length of 11.5 μm (ALMA requirement) assuming it is caused by water vapor fluctuations. This temperature change depends not only on the observing frequency, but also on the weather condition.

	$pwv = 0.25$ mm (excellent τ_{220})			$pwv = 0.70$ mm (25% quartile of τ_{220})			$pwv = 1.38$ mm (50% quartile of τ_{220})		
Freq. (GHz)	$T_{\rm atm}$ (K)	$\frac{\partial T_{\rm atm}}{\partial pwv}$ (K/mm)	$\Delta T_{\rm atm}$ (K)	$T_{\rm atm}$ (K)	$\frac{\partial T_{\rm atm}}{\partial pwv}$ (K/mm)	$\Delta T_{\rm atm}$ (K)	$T_{\rm atm}$ (K)	$\frac{\partial T_{\rm atm}}{\partial pwv}$ (K/mm)	$\Delta T_{\rm atm}$ (K)
220	4.7	8.8	0.016	8.7	8.8	0.016	15	8.8	0.016
345	13	34	0.059	28	32	0.056	49	30	0.052
410	27	62	0.10	53	55	0.093	88	47	0.079
492	76	160	0.25	140	110	0.17	200	60	0.090
675	69	190	0.33	140	120	0.22	200	61	0.11
691	77	210	0.35	150	120	0.21	220	57	0.097
809	95	240	0.44	180	120	0.23	230	45	0.084
875	82	210	0.34	160	120	0.20	220	54	0.089

to pwv in the atmosphere ($\partial T_{\rm atm}/\partial pwv$). These calculations were performed for selected frequencies that are either located at the center of atmospheric windows and/or are frequencies of special astronomical interest [7] using the radiative transfer model ATM (Atmospheric Transmission at Microwaves [11]; Table 1). The reference atmosphere for the calculation is the 1976 U.S. standard atmosphere for tropical latitude and we use an altitude of 5000 m. We require the r.m.s. excess path length fluctuation not to exceed 11.5 μm, which is the specification for ALMA. We calculate $\partial T_{\rm atm}/\partial pwv$ for pwv amounts of 0.25 mm, 0.70 mm, and 1.38 mm, which correspond to excellent conditions (or a 220 GHz opacity $\tau_{220} \sim 0.016$; [7]), to the 25% quartile ($\tau_{220} \sim 0.036$; [12]), and to the 50% quartile ($\tau_{220} \sim 0.061$; [12]) at the ALMA site, respectively. To derive the change of the atmospheric temperature ($\Delta T_{\rm atm}$) from $\partial T_{\rm atm}/\partial pwv$, we use the Δpwv and excess path length (ΔL) relation shown in Sutton & Hueckstaedt (1996) [14], which is approximately $\Delta L = 6.3\Delta pwv$ [15] at millimeter-wave, but changes close to water vapor lines (Fig. 1).

We then determine whether the noise of the state-of-the-art receivers is low enough to achieve the ALMA requirements for phase correction. Here we will only take the receiver temperature into account, and assume that the receivers are either very stable or calibrated frequently such that the gain of the receiver is well known (Table 2). Currently, the best receiver temperatures are a few times higher than the quantum limit at frequencies below 700 GHz, but are much worse above this frequency (10 times or more higher than the quantum limit; [10]). The receiver temperature ($T_{\rm RX}$) and $T_{\rm atm}$ constitute the more or less constant background temperature that causes the noise in the measurement:

$$T_{\rm noise} = T_{\rm RX} + T_{\rm atm} \ . \qquad (1)$$

We do not consider any antenna loss, ground pick-up, cosmic microwave background, etc. nor receiver instability. The r.m.s. noise temperature ($\Delta T_{\rm rms}$) is

$$\Delta T_{\rm rms} = \frac{T_{\rm noise}}{\sqrt{\Delta\nu\Delta t}} \ , \qquad (2)$$

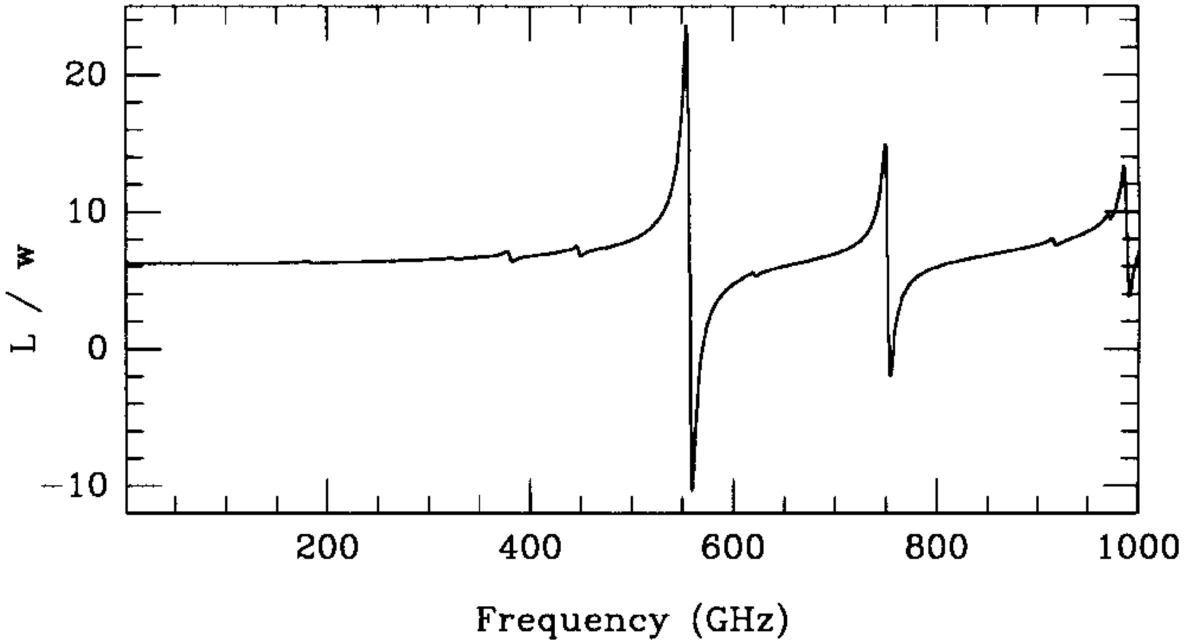

Figure 1: Optical path delay per millimeter of *pwv* for millimeter/submillimeter frequencies. From Figure 1 of Sutton & Hueckstaedt (1996) [14].

where $\Delta\nu$ and Δt are the band-width and the integration time of a measurement, which are 4 GHz and 1 second (ALMA specifications) in the calculations, respectively. We also evaluate the ratios of $\Delta T_{\rm atm}/\Delta T_{\rm rms}$ using $\Delta T_{\rm atm}$ values in Table 1, which are signal-to-noise (S/N) ratios to detect $\Delta T_{\rm atm}$ with the system.

The frequency with the highest S/N ratio will vary depending on the moisture of the atmosphere. For $pwv = 0.25$ mm, the highest sensitivity (i.e., the best S/N ratio) is achieved around 492 GHz, and similar sensitivity is also achieved around the 675 GHz atmospheric window. For larger amounts of *pwv* (or higher opacity) the highest sensitivity shifts to lower frequencies. This is because the atmospheric emission becomes saturated first in the strongest atmospheric emission lines as *pwv* increases (see $\Delta T_{\rm atm}$ in Table 1). Table 2 also suggests that state-of-the-art receivers with operating frequencies below 700 GHz can detect $\Delta T_{\rm atm}$ with S/N ratios larger than 10 even in medium weather condition (50% quartile). Receivers operating above 700 GHz are less sensitive and would not achieve a good phase correction.

For the phase correction using the total power outputs from the receivers, frequencies above 450 GHz have several advantages over lower ones: Firstly, they have higher S/N ratios to detect a given $\Delta T_{\rm atm}$ (see Table 2). Since the submillimeter opacity is higher than the millimeter opacity (typically $\tau_{220} \approx (22-24)\tau_{\rm submm}$ at the center of the atmospheric windows; [8, 7]), the change in atmospheric temperature caused by a certain *pwv* change in the submillimeter is larger than that in the millimeter, when $\tau_{\rm submm} \leq 1$. Secondly, liquid water fluctuations cause approximately the same temperature changes at all frequencies, consequently the relative contribution of liquid water to the sky brightness temperature is much lower at higher frequencies (Fig. 2; see also [13, 3, 4]). Indeed, the FTS measurements at the ALMA site clearly show

Table 2: Receiver temperatures ($T_{\rm RX}$), noise temperatures ($T_{\rm noise}$), and r.m.s. noise temperatures for an integration time of 1 second and 4 GHz bandwidth ($\Delta T_{\rm rms}$), as well as the signal-to-noise ratios ($\Delta T_{\rm atm}/\Delta T_{\rm rms}$) to detect water vapor fluctuations equivalent to 11.5 μm of optical paths, as evaluated in Table 1.

		$pwv = 0.25$ mm (excellent τ_{220})			$pwv = 0.70$ mm (25% quartile of τ_{220})			$pwv = 1.38$ mm (50% quartile of τ_{220})		
Freq. (GHz)	$T_{\rm RX}$ (K)	$T_{\rm noise}$ (K)	$\Delta T_{\rm rms}$ (K)	$\frac{\Delta T_{\rm atm}}{\Delta T_{\rm rms}}$	$T_{\rm noise}$ (K)	$\Delta T_{\rm rms}$ (K)	$\frac{\Delta T_{\rm atm}}{\Delta T_{\rm rms}}$	$T_{\rm noise}$ (K)	$\Delta T_{\rm rms}$ (K)	$\frac{\Delta T_{\rm atm}}{\Delta T_{\rm rms}}$
220	64	69	0.0011	15	73	0.0012	14	79	0.0013	13
345	100	120	0.0018	32	130	0.0021	27	150	0.0024	22
410	120	150	0.0023	44	170	0.0028	34	210	0.0033	24
492	150	230	0.0037	68	290	0.0046	36	350	0.0055	16
675	270	350	0.0055	61	420	0.0066	33	480	0.0075	15
691	280	360	0.0057	61	440	0.0069	31	490	0.0078	12
809	1300	1400	0.022	20	1500	0.023	10	1500	0.024	3
875	1400	1500	0.024	15	1600	0.025	8	1600	0.026	3

this effect [8, 9]. Since liquid water affects the phase much less than water vapor, it is desirable to either choose a frequency above 450 GHz where the contribution of liquid water to the brightness temperature is negligible (Fig. 2), or, alternatively, to find means of distinguishing between liquid water and water vapor (see Sect. 4).

In summary, these discussions suggest that the frequency range from 450 GHz to 700 GHz is best for the phase correction using the receiver total power at a single frequency. Table 2 also suggests that the total power phase correction at high frequencies is easier (has higher S/N ratio) in dry weather.

3 Estimation of Phase Correction Error due to Liquid Water

FTS measurements of millimeter and submillimeter wave atmospheric opacities show linear correlations between 220 GHz and submillimeter opacities [5, 6, 7, 8]. All of these observed correlations have a scatter of about 10-15% [8, 7]. These deviations might be due to the presence of liquid water. In the following discussion, we assume these deviations are caused by the effect of liquid water, and use the ratio of 220 GHz and 675 GHz opacity to estimate the amount of liquid water and its impact on the phase correction. If these deviations are caused by the error of the FTS measurements, the effect of liquid water should be smaller than we assumed, and the phase correction error should be smaller than we calculated below.

The observed opacity correlation between 220 GHz and 675 GHz is approximately $\tau_{675}/\tau_{220} = 22$ for a hydrometer-free atmosphere (Fig. 3a; [8]). However, the value of that opacity ratio can drop to a few for a hydrometer-dominated atmosphere, which is shown on Fig. 3(b) [7]. The following discussion is based on the assumption that we attribute all of the measured total power to water vapor emission and use the total amount of *pwv* derived for the phase correction, whereas in fact the *pwv* is lower since part of the total power is caused by liquid water emission. Note that in this discussion we do not take the effect of ice into account, but

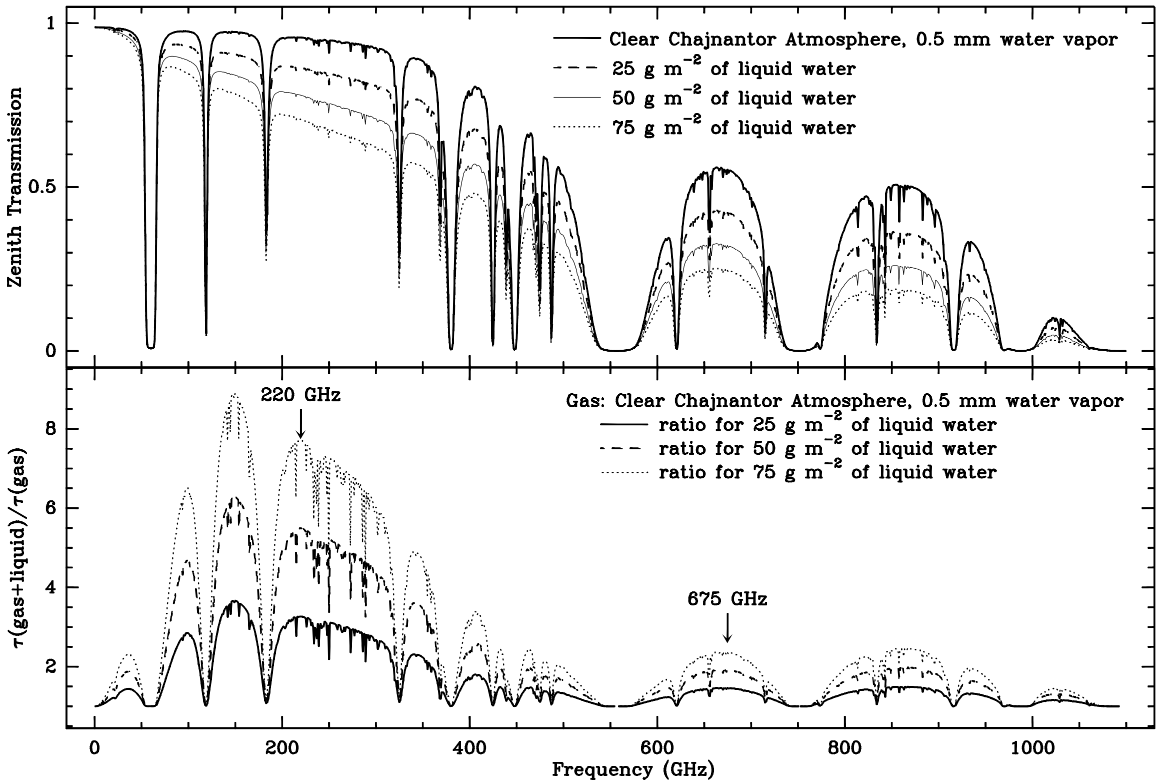

Figure 2: (*Top*) Atmospheric transmission spectra under various liquid water contents. (*Bottom*) Ratios of hydrometer-dominated to hydrometer-free opacity spectra. It is clear that the effect of liquid water is larger on the millimeter-wave emission than on the submillimeter-wave emission. The spectra are calculated using a radiative transfer model ATM [11].

leave this topic to a separate paper. We express the liquid water opacity as $\Delta\tau$. The upper limit of the fluctuation of liquid water opacity at 220 GHz can be estimated from Fig. 6(c) in Matsushita et al. (2000) [8], which shows the liquid water opacity separated from the 220 GHz total opacity. The r.m.s. of this opacity fluctuation is about 0.01 around 220 GHz for a total opacity of 0.1 (we did not use the liquid water dominant data; see [8] for detail). Under the assumption that calibration errors and inaccuracies in separating liquid water and water vapor are small, we can attribute this scatter to fluctuation of the liquid water opacity at 220 GHz of $\Delta\tau_{220}/\tau_{220} = 0.1$.

Using this relation, $\Delta\tau_{220}$ was calculated for different amounts of pwv. Further, $\Delta\tau_{675}$ was determined using the empirical relationship of $\Delta\tau_{675}/\Delta\tau_{220} = 1.9$ from our FTS data (Fig. 3b; [7]). These $\Delta\tau$ would have led us to (over)correct the phase by ΔL as listed in Table 3, when mistaking $\Delta\tau$ for fluctuations in water vapor and when applying the $\partial\tau/\partial pwv$ relationship for water vapor described in Sect. 2 to $\Delta\tau$. Since liquid water has a negligible effect on the phase, this would have resulted in an overcorrection in the phase and therefore an error in the phase correction.

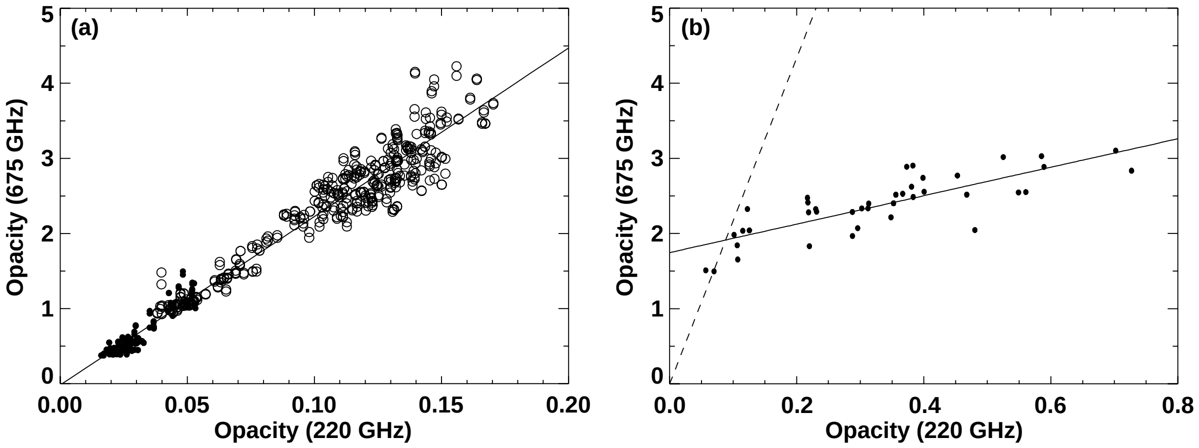

Figure 3: Opacity correlations between 220 GHz and 675 GHz. The circles and dots are FTS data taken in 1998 and 1997, respectively. (a) The opacity correlation under low liquid water content (i.e., dominated by water vapor emission). The solid line is the 220 GHz and 675 GHz opacity correlation of $\tau_{675}/\tau_{220} \sim 22$ [8]. (b) The opacity correlation under large liquid water content. The solid line suggest an opacity correlation of $\Delta\tau_{675}/\Delta\tau_{220} \sim 2$ [7]. The dashed line is the same correlation as in (a).

The calculated ΔL for 675 GHz for different amounts of pwv are an order of magnitude smaller than those for 220 GHz. Though the calculations were performed using a specific set of FTS data, nevertheless it is true in general that phase correction using 675 GHz is less sensitive to liquid water than that using 220 GHz. It is also clear that the presence of liquid water may limit the accuracy of the total power method. As mentioned above, if the measured opacity fluctuation is caused not only by liquid water but also by the FTS measurements error, ΔL could be smaller than that listed in Table 3.

4 Dual Frequency Operation

To reduce the phase correction error caused by liquid water in the atmosphere, it may be better to use a two channel (millimeter and submillimeter) system, i.e., use total power outputs from two simultaneously operating (or observing) receivers, and separate the water vapor and

Table 3: Errors of the total power phase correction in excess path length (ΔL) caused by liquid water in the atmosphere.

pwv (mm)	τ_{220}	τ_{675}	$\Delta\tau_{220}$	$\Delta\tau_{675}$	$\frac{\partial\tau_{220}}{\partial pwv}$	$\frac{\partial\tau_{675}}{\partial pwv}$	ΔL_{220} (μm)	ΔL_{675} (μm)
0.25	0.016	0.358	0.0016	0.0030	0.035	1.021	295.4	18.8
0.70	0.036	0.806	0.0036	0.0068	0.035	1.029	654.7	42.3
1.38	0.061	1.366	0.0061	0.0116	0.036	1.040	1084.4	71.4

liquid water components. Since the FTS data show the possible effect of liquid water in the atmospheric emission even under good weather conditions, the separation of water vapor and liquid water would greatly improve the phase correction accuracy. It might, however, not be straightforward to separate liquid water from water vapor, because it requires an accurate knowledge of the emission properties of liquid water as a function of frequency, which will depend on droplet size and temperature, and is likely to change from night to night. The 1998 FTS measurements sampled just one specific type. Nevertheless, the general trend does not change and submillimeter measurements are less affected by liquid water. If the liquid water model uncertainty is 10%, the limit of the 675 GHz total power phase correction, ΔL_{675}, will reduce the values in Table 3 by 1/10, and will achieve the ALMA requirement of 11.5 μm.

In summary, the advantages of this dual frequency technique are (1) lower requirements on receiver temperatures and stabilities, and (2) smaller phase correction error when the submillimeter opacity is lower. This phase correction method requires (a) simultaneous operation (millimeter and submillimeter), and (b) temperature calibration of two receivers.

In the future we hope to test these two total power phase correction methods at the Sub-Millimeter Array (SMA; [2]) on Mauna Kea, which will be capable of dual frequency operation, and we will further compare it to the radiometric phase correction using the 183 GHz radiometers already installed [16] and possibly using the improved radiometers [1].

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