

ALMA MEMO # 468  
Designs of Wideband 3dB Branch-line Couplers  
for ALMA Bands 3 to 10

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**Abstract**

A guiding principle of design of wideband waveguide 3-dB 5-branch-line couplers for ALMA bands 3 to 10 is suitable for use in SSB heterodyne receivers, their designs and their performances are reported. The couplers have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band. The couplers for the ALMA band 3, 4, 5, 6, 7, 8, 9 and 10 include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides, respectively.

The couplers are designed by a numerical analytical method by using matrices based on the circuit theory and they are optimized with 3D EM simulator (HFSS). The numerical analytical method for seeking S-matrices mentioned in this memo enables quick calculations for performances of 3-dB branch-line couplers in the case of optimization and possesses advantages over electromagnetic simulation software in common use.

**Keywords**

ALMA bands 3 to 10, wideband waveguide quadrature hybrid, wideband waveguide 3dB 5-branch-line coupler, numerical matrix analysis, S-matrix

## 1 Introduction

Split-block branch-line couplers with strong coupling (e.g., 3 dB) are easily made and has already reported [1], however, no clear guiding principles to design wideband split-block branch-line couplers have been presented. Authors have already reported a wideband waveguide 3-dB 5-branch-line coupler suitable for use in SSB heterodyne receivers in ALMA Band 3 (84-116 GHz) [2][3]. The coupler is designed by a numerical analytical method by using matrices based on the circuit theory and has good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89.5-90.7 degrees over the frequency range of 84GHz-116GHz.

In this manuscript, a guiding principle to design wideband waveguide 5-branch-line couplers for ALMA bands 3 to 10, their designs and their performances are reported. Initially, the couplers are designed by a numerical analytical method by using matrices based on the circuit theory and they are optimized with

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Table 1: Waveguides for ALMA bands

Band	from - to (GHz)	$f_0$ (GHz)	$\Delta f$ (GHz)	$\frac{\Delta f}{f_0}$	EIA (VDI)	a (mm)	b (mm)	$f_c$ (GHz)
3	84 - 116	100	32	32.0%	WR-10	2.54	1.27	59.1
4	125 - 163	144	38	26.4%	WR-6	1.651	8.255	90.9
5	163 - 211	187	48	25.7%	WR-5	1.295	0.6477	115.8
6	211 - 275	243	64	26.3%	WR-3	0.8636	0.4318	173.7
7	275 - 370	322.5	95	29.5%	WR-2.8	0.7112	0.3556	210.9
8	385 - 500	442.5	115	26.0%	WR-2.2	0.5588	0.2794	268.4
9	602 - 720	661	118	17.9%	WR-1.5	0.3810	0.1905	393.7
10	787 - 950	868.5	163	18.8%	WR-1.2	0.3048	0.1524	492.1

3D EM simulator (HFSS). Being worked toward practical use, the designs for the ALMA bands 3, 4, 5, 6, 7, 8, 9 and 10 include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides as shown in Table 1, respectively.

## 2 A guiding principle to design wideband waveguide 3-dB couplers

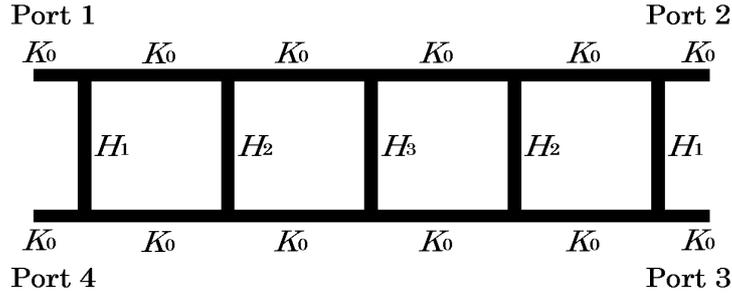


Figure 1: A schematic of 5-branch-line coupler with 10 E-plane T-junctions.

A schematic of 5-branch-line coupler with 10 E-plane T-junctions is shown in Fig.1, including two waveguides with the characteristic impedance  $K_0$ . The characteristic impedances of slits between the two waveguides are  $H_1$ ,  $H_2$ ,  $H_3$ ,  $H_2$  and  $H_1$ , respectively.

The S-matrix of waveguide 3-dB 5-branch-line coupler with E-plane junctions as shown in Fig.1 is as follows[2].

$$\begin{pmatrix} S_{11} & * & * & * & S_{12} & S_{13} & * & * & * & S_{14} \\ * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * \\ S_{21} & * & * & * & S_{22} & S_{23} & * & * & * & S_{24} \\ \hline S_{31} & * & * & * & S_{32} & S_{33} & * & * & * & S_{34} \\ * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * \\ S_{41} & * & * & * & S_{42} & S_{43} & * & * & * & S_{44} \end{pmatrix} = 2 \begin{pmatrix} \frac{1+H_1}{j \tan \theta} + 1 & \frac{1}{j \sin \theta} & 0 & 0 & 0 \\ \frac{1}{j \sin \theta} & \frac{2+H_2}{j \tan \theta} & \frac{1}{j \sin \theta} & 0 & 0 \\ 0 & \frac{1}{j \sin \theta} & \frac{2+H_3}{j \tan \theta} & \frac{1}{j \sin \theta} & 0 \\ 0 & 0 & \frac{1}{j \sin \theta} & \frac{2+H_2}{j \tan \theta} & \frac{1}{j \sin \theta} \\ 0 & 0 & 0 & \frac{1}{j \sin \theta} & \frac{1+H_1}{j \tan \theta} + 1 \\ \hline 0 & 0 & 0 & 0 & \frac{H_1}{j \sin \theta} \\ 0 & 0 & 0 & \frac{H_2}{j \sin \theta} & 0 \\ 0 & 0 & \frac{H_3}{j \sin \theta} & 0 & 0 \\ 0 & \frac{H_2}{j \sin \theta} & 0 & 0 & 0 \\ \frac{H_1}{j \sin \theta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{H_1}{j \sin \theta} \\
0 & 0 & 0 & \frac{H_2}{j \sin \theta} & 0 \\
0 & 0 & \frac{H_3}{j \sin \theta} & 0 & 0 \\
0 & \frac{H_2}{j \sin \theta} & 0 & 0 & 0 \\
\frac{H_1}{j \sin \theta} & 0 & 0 & 0 & 0 \\
\hline
\frac{1+H_1}{j \tan \theta} + 1 & \frac{1}{j \sin \theta} & 0 & 0 & 0 \\
\frac{1}{j \sin \theta} & \frac{1}{j \tan \theta} & \frac{1}{j \sin \theta} & 0 & 0 \\
0 & \frac{1}{j \sin \theta} & \frac{2+H_3}{j \tan \theta} & \frac{1}{j \sin \theta} & 0 \\
0 & 0 & \frac{1}{j \sin \theta} & \frac{2+H_2}{j \tan \theta} & \frac{1}{j \sin \theta} \\
0 & 0 & 0 & \frac{1}{j \sin \theta} & \frac{1+H_1}{j \tan \theta} + 1
\end{pmatrix}^{-1} \cdot \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} \quad (1)$$

In the above equation, the value of  $H_1$ ,  $H_2$  and  $H_3$  are normalized by  $K_0$  ( $H_1 \leftarrow \frac{H_1}{K_0}$ ,  $H_2 \leftarrow \frac{H_2}{K_0}$ ,  $H_3 \leftarrow \frac{H_3}{K_0}$ ), and the notes of "★" in the matrix do not have to be taken care of. The quantity  $\theta$  represents the electrical "length" of each transmission line. Here, the following conditions have to be satisfied for 3-dB coupling at center frequency. (Similar formulas for branch-line couplers up to six branches are shown in [4].)

$$H_2 = \frac{1}{H_1 + \sqrt{2} + 1} \quad (2)$$

$$H_3 = \frac{1 - 2H_1 - H_1^2}{\sqrt{2}} - \frac{(|S_{21}| - |S_{31}|)_{@f=f_0}}{18.6} \quad (3)$$

Especially, in the case of TE<sub>10</sub> waveguide branch-line couplers, the frequency dependence of  $\theta$  is;

$$\theta(x) = \frac{\pi}{2} \sqrt{\frac{x^2 - \alpha^2}{1 - \alpha^2}}, \quad (4)$$

where  $x = f/f_0$  and  $\alpha = f_c/f_0$ . The value of  $f$ ,  $f_0$  and  $f_c$  are the frequency of the applied signal, the center frequency and the cut-off frequency of the waveguide, respectively.

Figure 2 and 3 show the performances of a wideband 3dB 5-branch-line coupler. In this case, the values of  $H_1$ ,  $H_2$  and  $H_3$  are 0.222, 0.326(=  $\frac{1}{H_1 + \sqrt{2} + 1} - \frac{1}{18.6}$ ) and 0.358(=  $\frac{1 - 2H_1 - H_1^2}{\sqrt{2}}$ ), respectively. The value of  $\alpha$  is 0.59. This is the value of cut-off frequency  $f_c$  for WR-10 waveguide (2.54mm×1.27mm) to the center frequency of  $f_0=100$ GHz. As you can see in this figure, this coupler have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of  $0.8f_0 < f < 1.2f_0$ . Thus, we decided the guiding principle of design of wideband waveguide 3-dB couplers for all ALMA bands as stated above.

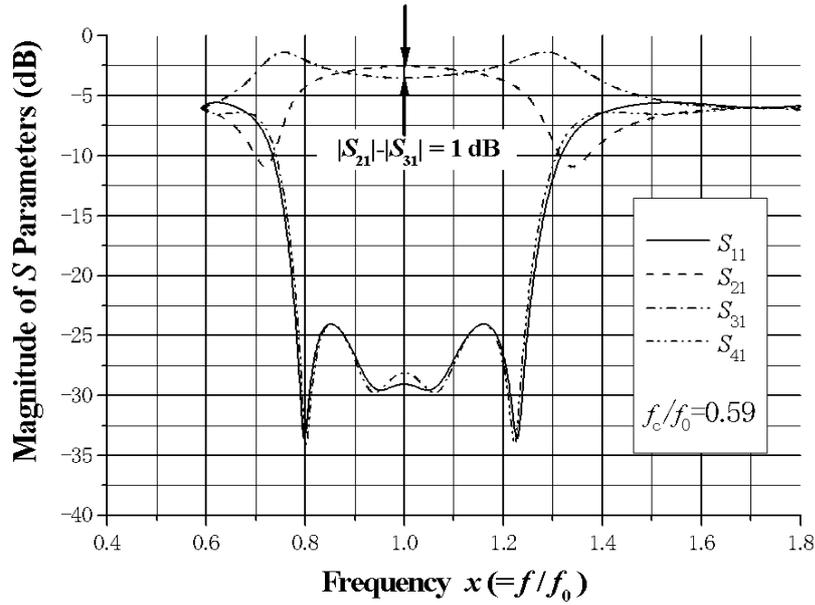


Figure 2: Numerical results for the frequency dependence of S parameters of 3dB 5-branch-line coupler in the case of  $H_1 = 0.222$  and  $|S_{21}| - |S_{31}| = 1\text{dB}$  at the center frequency ( $H_2 = 0.326$  and  $H_3 = 0.358$ ). The value of  $\alpha = f_c/f_o$  is 0.59.

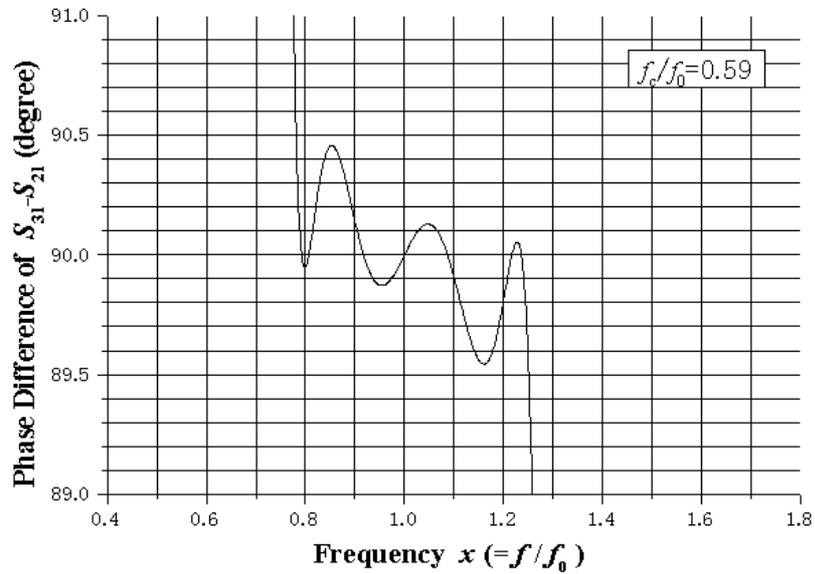


Figure 3: Numerical results for the frequency dependence of a phase difference of 3dB 5-branch-line coupler in the case of  $H_1 = 0.222$  and  $|S_{21}| - |S_{31}| = 1\text{dB}$  at the center frequency ( $H_2 = 0.326$  and  $H_3 = 0.358$ ). The value of  $\alpha = f_c/f_o$  is 0.59.

### 3 Practical designs of 3-dB couplers for ALMA bands 3 to 10

Figures 4, 5, 6, 7, 8, 9, 10 and 11 show the designs of the 3-dB couplers for the ALMA band 3, 4, 5, 6, 7, 8, 9 and 10, respectively. They include WR-10, WR-6, WR-5, WR-3, WR-2.8, WR-2.2, WR-1.5 and WR-1.2 waveguides, respectively. The couplers in the figures have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band.

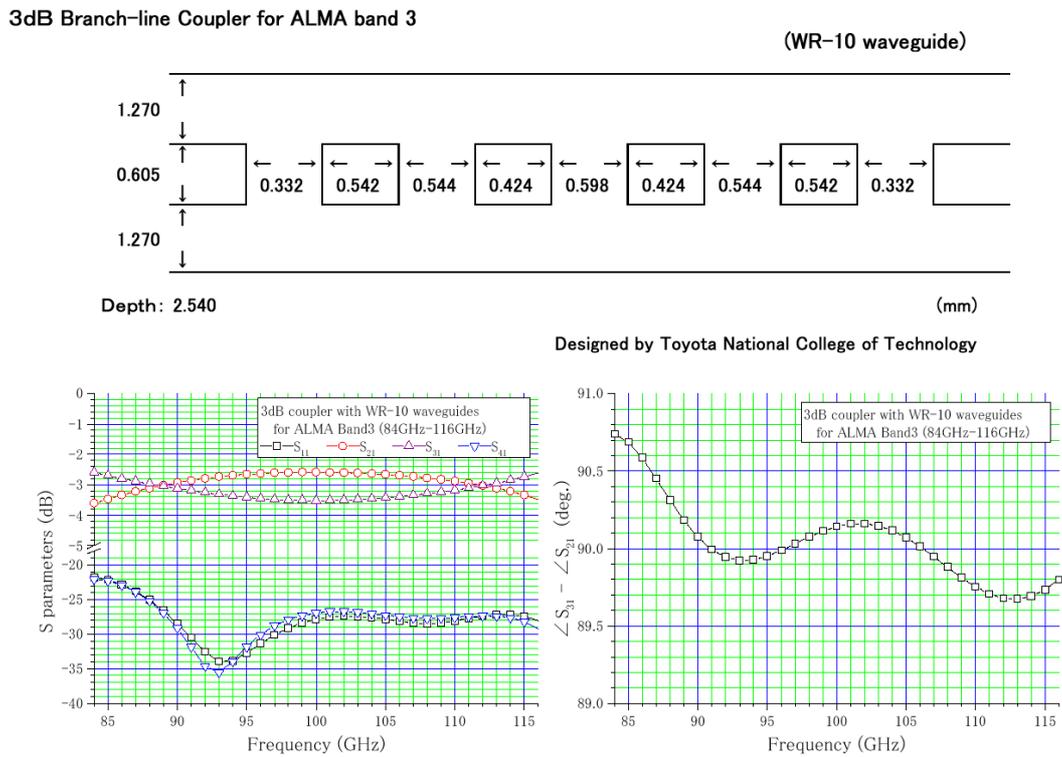


Figure 4: The design and the performances of a waveguide 3-dB coupler for ALMA band 3

3dB Branch-line Coupler for ALMA band 4

(WR-6 waveguide)

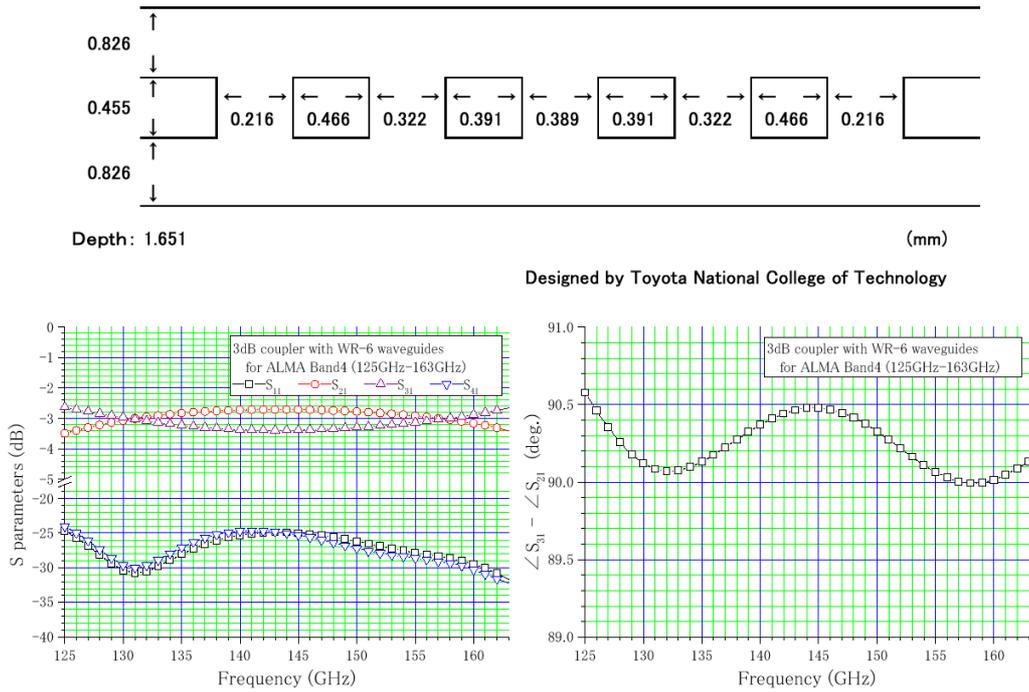


Figure 5: The design and the performances of a waveguide 3-dB coupler for ALMA band 4

3dB Branch-line Coupler for ALMA band 5

(WR-5 waveguide)

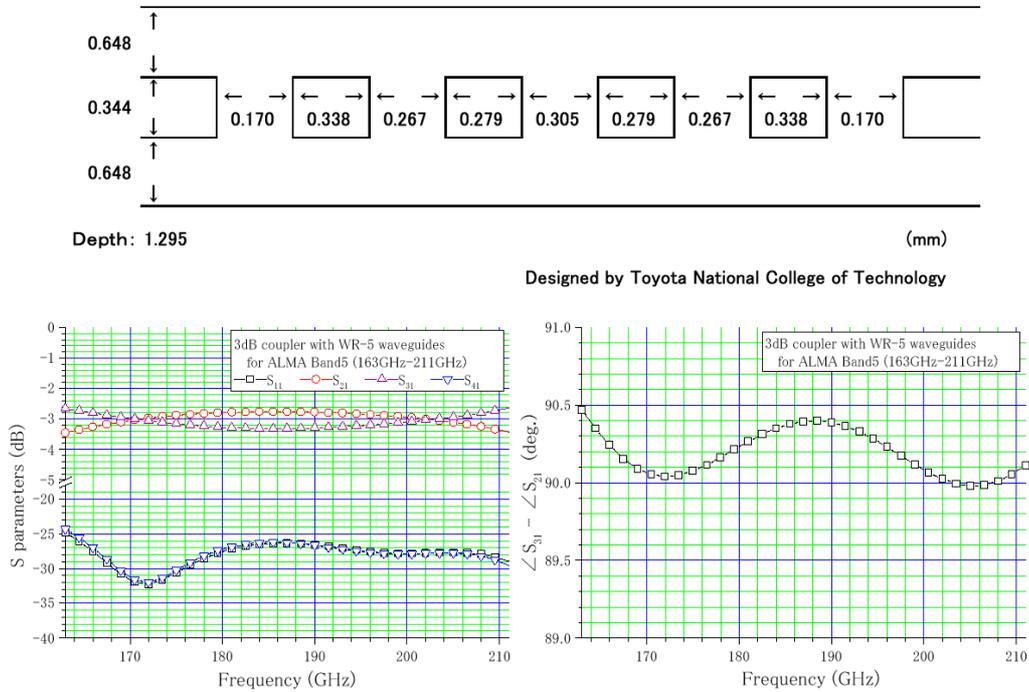


Figure 6: The design and the performances of a waveguide 3-dB coupler for ALMA band 5

### 3dB Branch-line Coupler for ALMA band 6

(WR-3 waveguide)

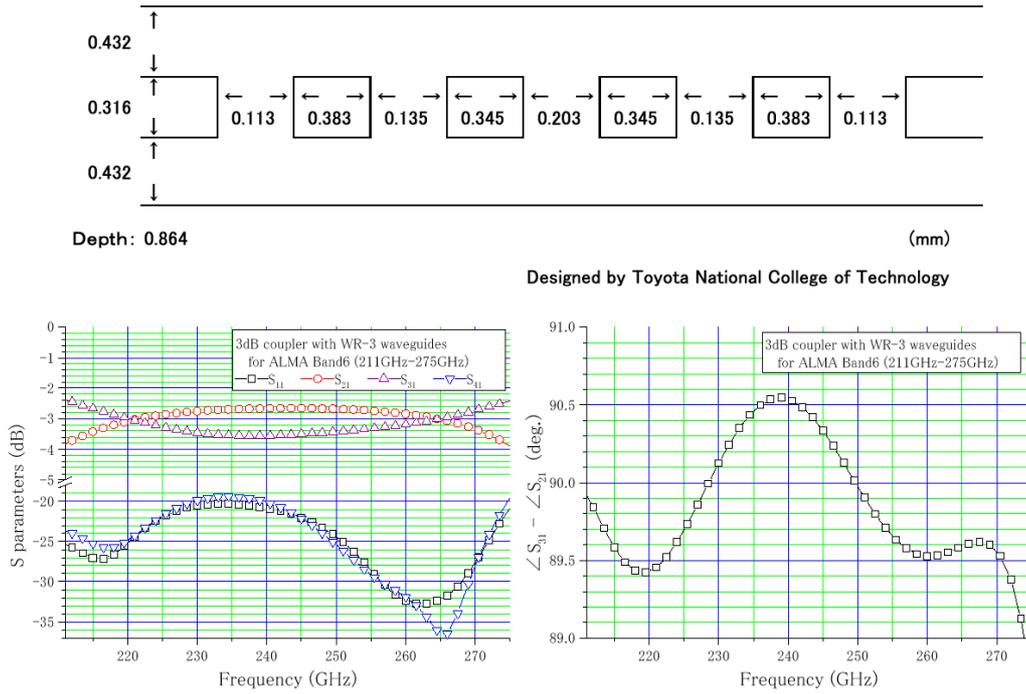


Figure 7: The design and the performances of a waveguide 3-dB coupler for ALMA band 6

### 3dB Branch-line Coupler for ALMA band 7

(WR-2.8 waveguide)

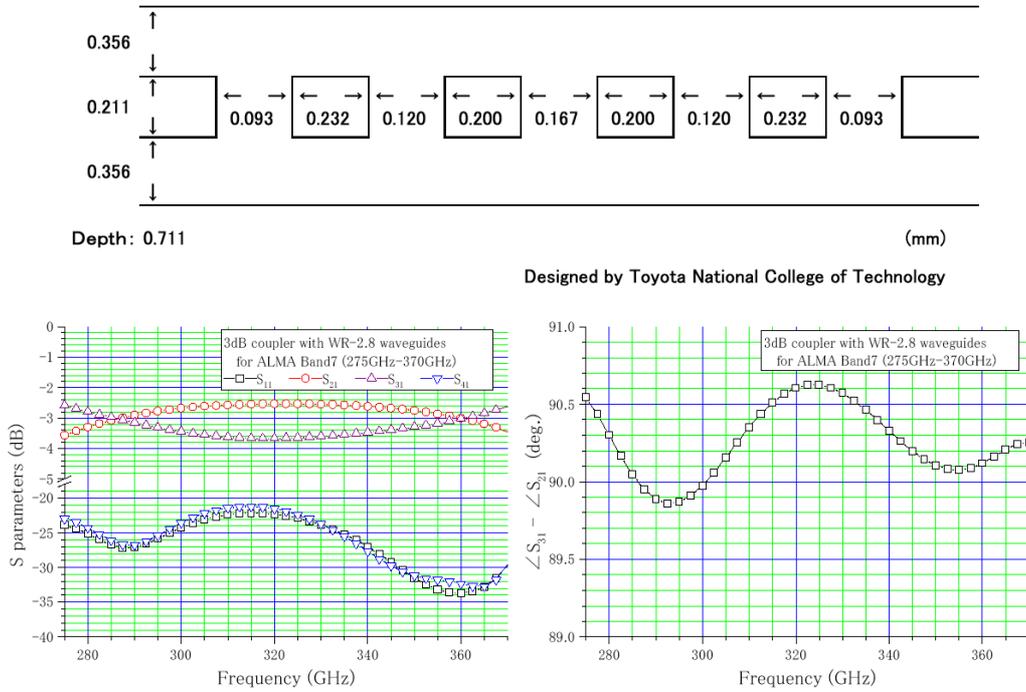


Figure 8: The design and the performances of a waveguide 3-dB coupler for ALMA band 7

**3dB Branch-line Coupler for ALMA band 8**

(WR-2.2 waveguide)

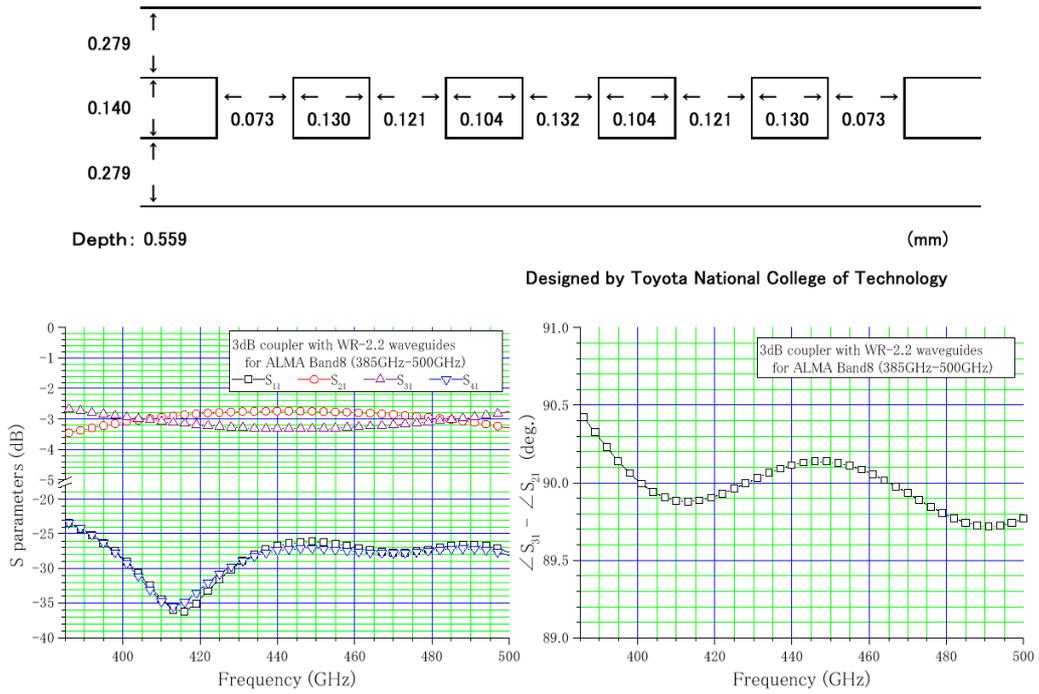


Figure 9: The design and the performances of a waveguide 3-dB coupler for ALMA band 8

**3dB Branch-line Coupler for ALMA band 9**

(WR-1.5 waveguide)

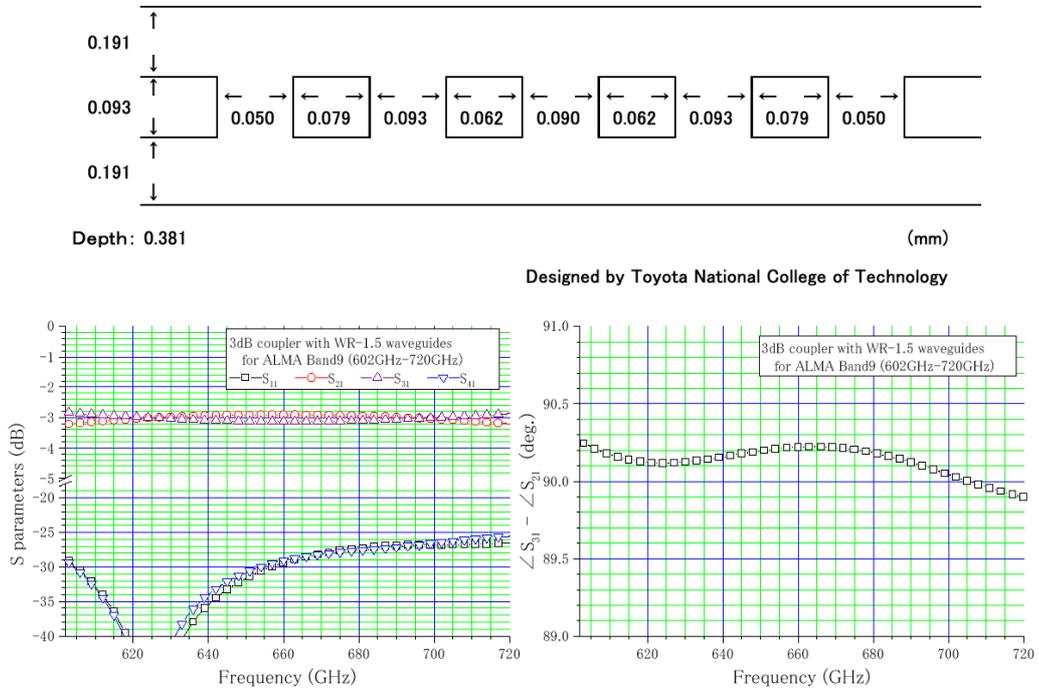


Figure 10: The design and the performances of a waveguide 3-dB coupler for ALMA band 9

### 3dB Branch-line Coupler for ALMA band 10

(WR-1.2 waveguide)

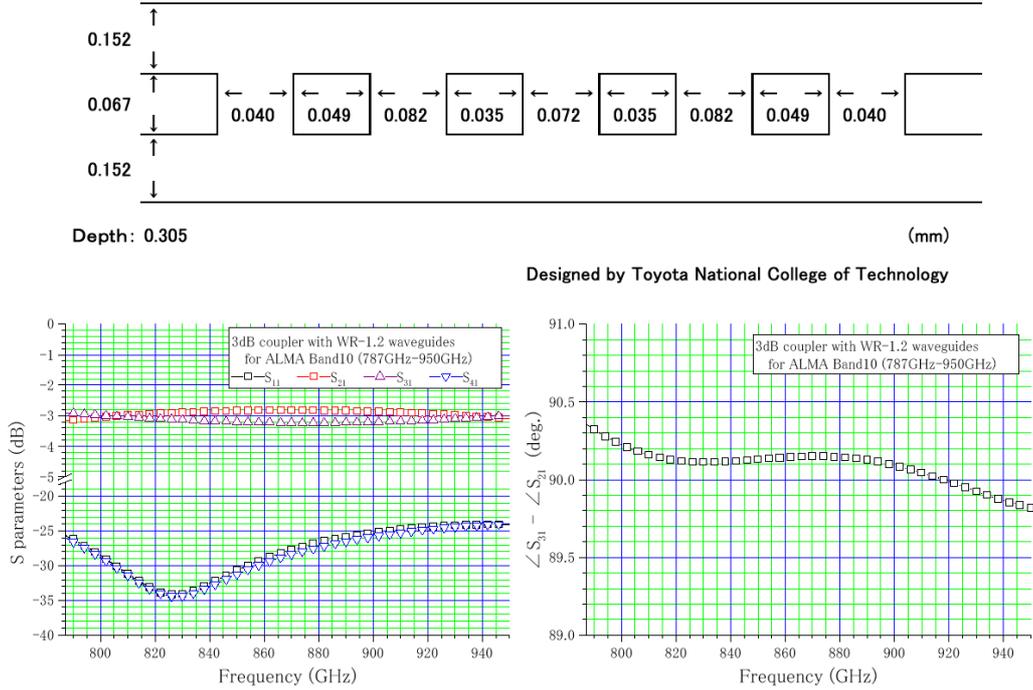


Figure 11: The design and the performances of a waveguide 3-dB coupler for ALMA band 10

## 4 Summary

We presented a guiding principle of design of wideband waveguide 3-dB 5-branch-line couplers for ALMA bands 3 to 10 in suitable for use in SSB heterodyne receivers and also showed the designs and the performances. The couplers have good performances, which maintain an amplitude balance to within 1dB and a phase difference of 89-91 degrees over the frequency range of each ALMA band.

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