ALMA Memo 537

Walsh Function Demodulation in the Presence of Timing Errors, leading to Signal Loss and Crosstalk

D.T. Emerson,
NRAO Tucson.

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Abstact: In the ALMA system, switched phase offsets of pi/2 radians are added to the first LO signal at each antenna as a means to separate upper and lower sidebands; these offsets are put on the LO phase following a Walsh switching cycle, and are decoded following correlation. Additional phase offsests of pi radians are applied to the first LO at each antenna, as a means of reducing spurious correlations between unwanted coherent signals that find their way into the IF at the antenna; these pi phase steps are demodulated at the antenna by corresponding sign change at the digitizer.

If there is a timing offset between the modulating and demodulating Walsh waveforms, then in general there are 2 unwanted effects:
(1) A loss of amplitude of the wanted signal, and
(2) Unwanted crosstalk introduced between otherwise orthogonal modulations.

This memo investigates the magnitudes of these effects in the ALMA context, and how the magnitudes are dependent on the specific choice of Walsh function. A strategy for choosing the optimum Walsh functions is suggested.

Introduction

The ALMA architecture incorporates nested Walsh function switching for two different purposes:

a) The first LO is switched between phase offsets of 0 and pi/2 radians, with the object of being able to separate out upper and lower sidebands of the receiver. The switching waveform is applied at the antenna to the First Local Oscillator Offset Generator (FLOOG), which controls the LO phase. The Walsh function used for this is different for each antenna, and at a given antenna is chosen from a set of 128 functions, with a shortest time element of 16 ms, corresponding to 16*128 ms = 2.048 seconds for the complete switching cycle. This switching is effectively demodulated in the correlator at the AOS Technical Building after the correlation process, where products from different phases of the switching cycle of a given interferometer pair are accumulated into different memory locations, enabling an eventual separation within the data reduction software into upper and lower sideband correlation terms.
b) In addition, the first LO is switched with further phase offsets of 0 and pi radians, to give a degree of immunity to spurious signals breaking into the receiver IF chain that could result in spurious correlated signals appearing at the output of the

the correlator. This modulation is similarly applied via the FLOOG at each antenna to the first LO, but is then demodulated at the Digitizer Demultiplexer (DGD), within the same antenna, by reversing the data sign bit in the data stream after demultiplexing to 250 MB/s. Beyond this point the signal is digital, so further spurious crosstalk signals are not likely to be introduced. A 128-element Walsh function with a shortest time element of 125 microseconds is used for this pi switching, giving a total time of 125*128 microseconds = 16 milliseconds. Note that this entire 128-element cycle is completed within a single element of the pi/2 switching sequence. Unlike the pi/2 switching, in this case the modulation and demodulation processes both occur at the antenna; however, any spurious signal appearing in the IF is now effectively **modulated** by the Walsh demodulation. To retain orthogonality with similar signals from other antennas, the precise timing of the modulation/demodulation function has to be matched at all antennas, after allowing for the total propagation time from antenna to correlator. This propagation delay is continuously changing, by a different amount for each antenna, as a celestial source is tracked.

In the absence of timing and other errors, the correct demodulation of a given Walsh function phase modulation gives the desired signal without attenuation, and in principle gives complete rejection of unwanted spurious or crosstalk terms. Emerson (1983) discusses optimum choice of Walsh function to minimize crosstalk in the presence of time-varying signals, but here we consider crosstalk that originates in modulation function timing errors. If there is a timing error in the demodulation relative to the modulation, in general the desired signal is attenuated, resulting in loss of signal-to-noise ratio, and different functions are no longer perfectly orthogonal so that rejection of crosstalk or spurious signals becomes imperfect.

For (a), the pi/2 modulation, the loss of orthogonality potentially leads to poorer rejection of the unwanted sideband signal. For (b), the pi modulation, rejection of spurious correlated signals appearing in the IF chain becomes poorer and so spurious correlations may appear at the output of the correlator.

Although the timing of the demodulation is critical with respect to the modulation applied to each respective Walsh function at each antenna, the two sets of Walsh functions – the inner loop with the pi switching and the outer loop with pi/2 switching – do not have to be precisely aligned with each other. In practice with the ALMA implementation they will be.

The ALMA architecture and timing requirements are described and discussed in *Fringe Tracking, Sideband Separation, and Phase Switching in the ALMA Telescope* (D'Addario, Feb 2000) and in the *ALMA System Technical Requirements,* July 2004.

Walsh Function Numbering Conventions

Consult any standard text on Walsh functions, such as Beacuchamp (1975).

Walsh functions may be generated very easily as the product of Rademacher functions. Rademacher functions R(nr,t) are simply square waves with frequency **fr** increasing by powers of 2, $\mathbf{fr}=2^{nr}$ or 1,2,4 … ; R(0,t) is equal to unity for the entire interval 0<t<T, where T is the duration of a complete cycle of the function set. Rademacher functions could be derived from sinusoidal functions which have identical zero crossing positions, $R(nr,t)=\text{sign}[\sin(2^{nr}. \quad .t)]$. A set of Walsh functions ordered according to the *Natural* or *Paley* order is conventionally written as PAL(n,t) where n is the Paley order number and t is the time between 0 and T, the period of the complete set. To generate such a set, n is first expressed in binary. The binary digits of n then define which Rademacher functions are to be multiplied together. For example, for PAL(13,t) n=13, or 1101 in binary; multiply together R(4,t), R(3,t) and R(1,t). The product of two Walsh functions is another Walsh function. The Paley index of the product can be derived from the modulo-2 addition of the binary digits of each of the component Paley indices.

Walsh functions are sometimes given in order of number of zero-crossings within the time-base T; **WAL(k,t)**. By analogy with cosine and sine pairs, Walsh functions can also be expressed as **CAL(n,t)** and **SAL(n,t),** even and odd waveforms, where for an even function **k =2.n** and for an odd function **k=2.n-1**. This is known as *Sequency* order, with Sequency defined as “one half of the average number of zero crossings per unit time interval.” It is often convenient to generate Walsh functions in Paley order (see Appendix 1) but WAL or sequency order often turns out to be convenient for signal analysis. It is found here that both loss of signal and RSS (root sum of squares) of crosstalk magnitudes have particularly simple forms if plotted in sequency order.

Figures 1 & 2 below are taken from Beauchamp, and for sake of example show the same 32-element set of Walsh functions arranged first in sequency order, and secondly in natural or Paley order.

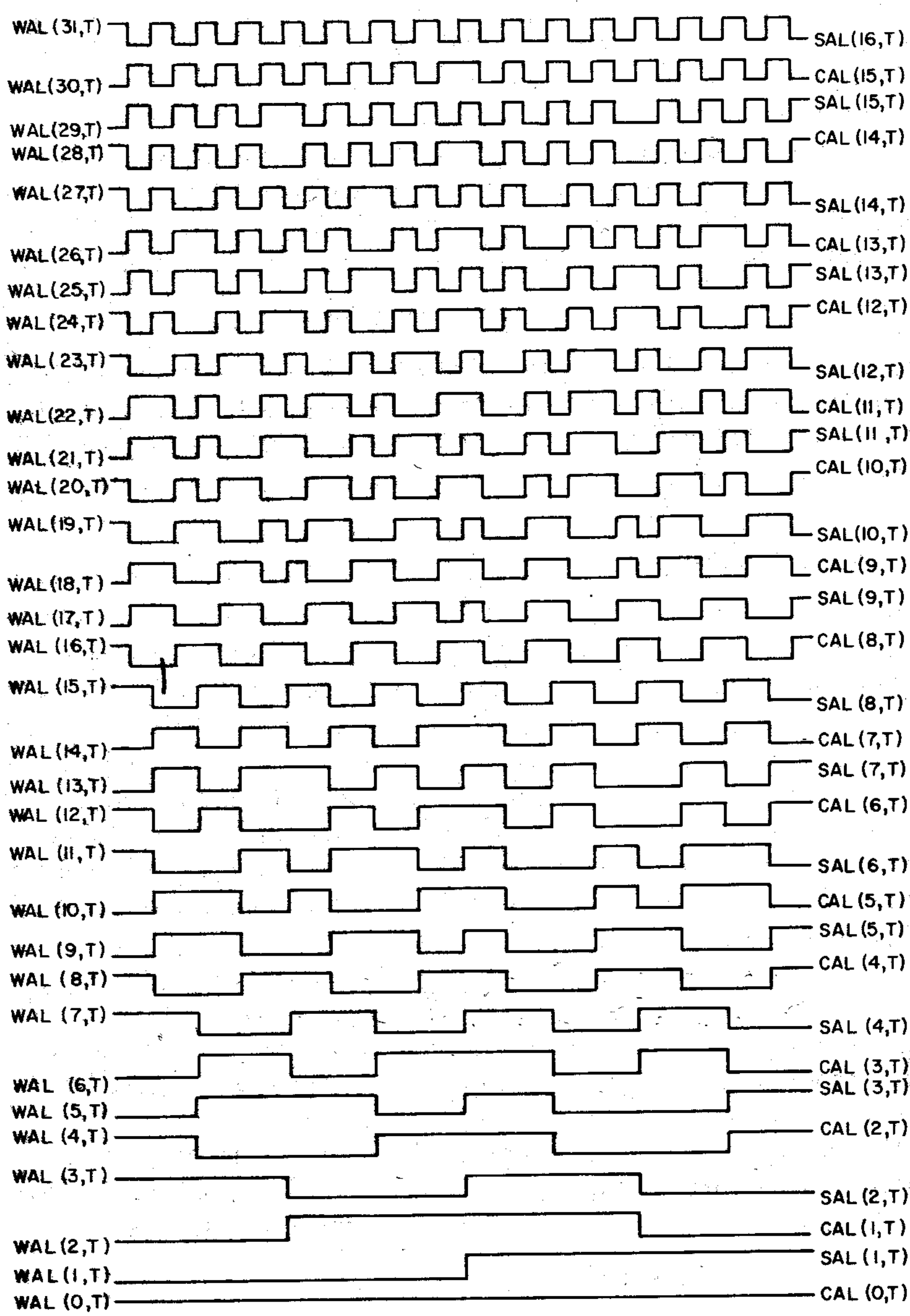

Figure 1, from Beauchamp (1975)

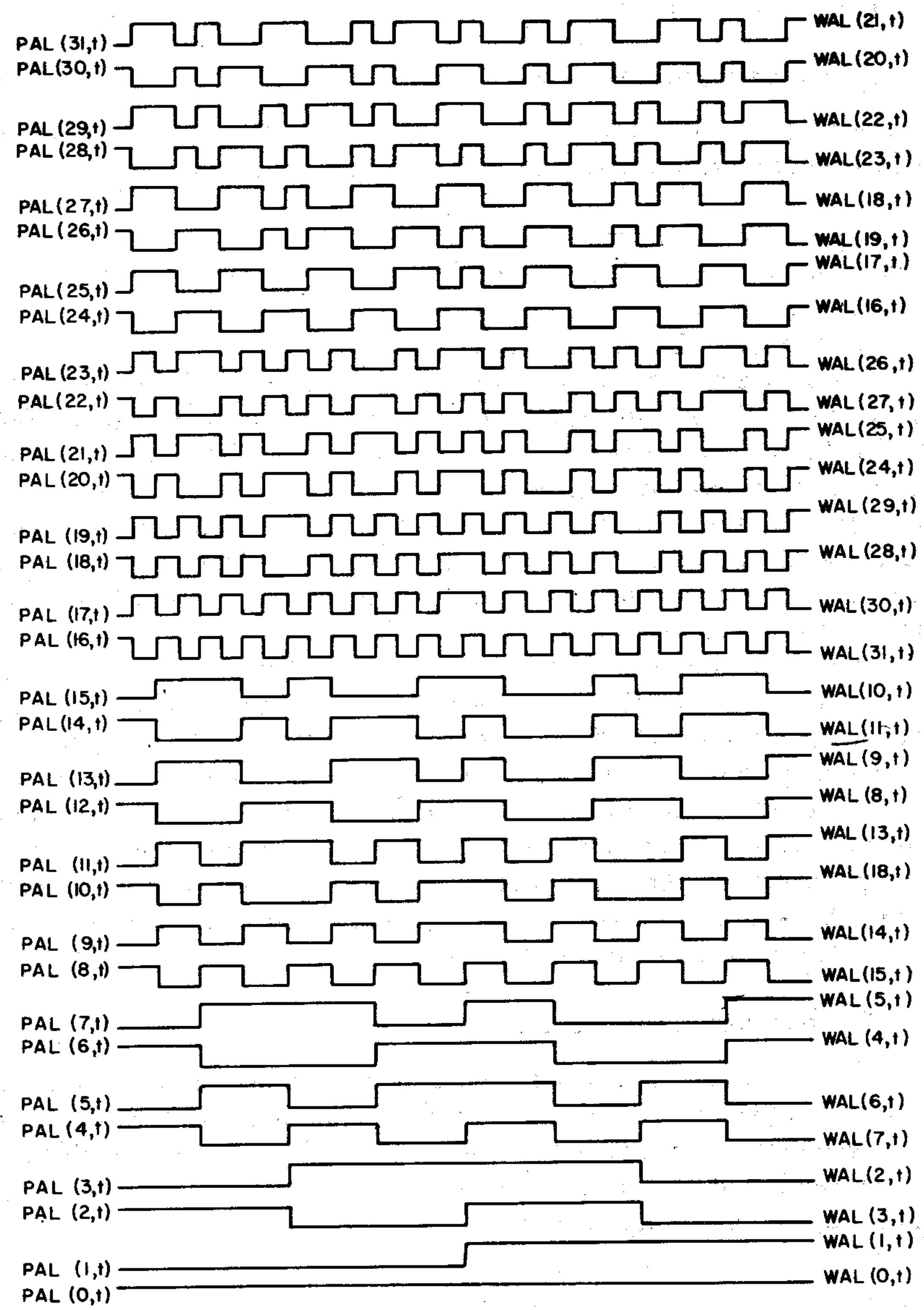

Figure 2, from Beauchamp (1975)

A Simulation

A simple numerical simulation was used to calculate the loss of sensitivity, and magnitude of crosstalk, between Walsh functions with a small timing offset. A 128-element set of Walsh functions, as will be used in ALMA, was used in the simulations. The Appendix below outlines the detailed procedure used. Figure 3 shows the amplitudes of all possible cross-products of one Walsh function with another function of the same set which has been shifted by 1% of the shortest bit length. Different symbols are shown for greater than -0.1 dB normalised amplitude, -0.1 to -20 dB and -20 to -30 dB. Cross-products with weaker than -30 dB amplitude are left blank. [1]

Sensitivity

Table I shows the loss of sensitivity caused by demodulating a Walsh function with an identical function but offset in time by 1% of the shortest bit duration; this is just the diagonal shown in Figure 3. The table is sorted in order of increasing degradation, which happens to coincide with Walsh WAL, or sequency ordering. This sensitivity degradation is plotted in Figure 4. As is seen, the loss of sensitivity increases linearly with sequency, with in the worst case 2% loss of sensitivity if there is a timing error of 1% of the shortest bit length – corresponding to about 1 microsecond in the ALMA pi-switching context.

Crosstalk

Figure 5 shows, for a given time-shifted Walsh function, the root-sum-of-squares (RSS) of all crosstalk values for all possible products with other Walsh functions, excluding the product with itself; this corresponds to the RSS of cross-products going across each row of Figure 3, or equivalently, down each column. As is seen, some Walsh functions are notably better than others; if crosstalk were the only issue, then we should avoid sequencies in the middle half of the set. The RSS values of products with a given Walsh function tend to be dominated by the few strongest cross-products; in Figure 3, the rows or columns showing many cross-products stronger than -30 dB, e.g. WAL(1,t) or WAL(126,t), tend to have a weaker RSS value than those rows or columns with very few, but nevertheless strong cross-products, such as WAL(63,t) or WAL(64,t). Overall, WAL(0,t) through WAL(32,t) and WAL(96,t) through WAL(127,t) are much less susceptible to crosstalk than WAL(33,t) through WAL(95,t). The only functions that retain perfect orthogonality with all other functions in the set, when subject to a timing offset, are WAL(0,t) and WAL(127,t).

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[1] The conversion to dB in Table 1 and Figure 3 was made using 10.log(V2/V1), where V2/V1 is the normalized cross-product amplitude ratio calculated by the simulation program. For ALMA, the crosstalk will appear after correlation as a voltage proportional to incoming flux density, so this is the correct conversion to use. In most communications applications, such crosstalk would appear as a voltage proportional to the square root of incoming receiver power, in which case 20.log(V2/V1) would be more appropriate.

More than half the possible cross-products are identically zero. A general rule is that the product of an odd Walsh sequency with an even sequency in the same set retains orthogonality in the presence of a timing shift; this gives the checker-board pattern seen in Figure 3.

Table 1. Loss of sensitivity with 1% shifted Walsh demodulation

PAL (N,t)	WAL (N,t)	% Signal	% loss	dB Loss	PAL (M,t)	WAL (M,t)	% Signal	% loss	dB loss
0	0	100	0	0	32	63	99	1	-0.04
1	1	99.97	0.03	0	99	66	98.97	1.03	-0.05
3	2	99.97	0.03	0	97	65	98.97	1.03	-0.05
2	3	99.94	0.06	0	98	67	98.94	1.06	-0.05
6	4	99.94	0.06	0	102	68	98.94	1.06	-0.05
5	6	99.91	0.09	0	103	69	98.91	1.09	-0.05
7	5	99.91	0.09	0	101	70	98.91	1.09	-0.05
12	8	99.88	0.13	-0.01	100	71	98.88	1.13	-0.05
4	7	99.88	0.13	-0.01	108	72	98.88	1.13	-0.05
13	9	99.84	0.16	-0.01	111	74	98.84	1.16	-0.05
15	10	99.84	0.16	-0.01	109	73	98.84	1.16	-0.05
10	12	99.81	0.19	-0.01	106	76	98.81	1.19	-0.05
14	11	99.81	0.19	-0.01	110	75	98.81	1.19	-0.05
11	13	99.78	0.22	-0.01	107	77	98.78	1.22	-0.05
9	14	99.78	0.22	-0.01	105	78	98.78	1.22	-0.05
8	15	99.75	0.25	-0.01	120	80	98.75	1.25	-0.05
24	16	99.75	0.25	-0.01	104	79	98.75	1.25	-0.05
27	18	99.72	0.28	-0.01	121	81	98.72	1.28	-0.06
25	17	99.72	0.28	-0.01	123	82	98.72	1.28	-0.06
26	19	99.69	0.31	-0.01	126	84	98.69	1.31	-0.06
30	20	99.69	0.31	-0.01	122	83	98.69	1.31	-0.06
29	22	99.66	0.34	-0.01	127	85	98.66	1.34	-0.06
31	21	99.66	0.34	-0.01	125	86	98.66	1.34	-0.06
20	24	99.63	0.38	-0.02	116	88	98.63	1.38	-0.06
28	23	99.63	0.38	-0.02	124	87	98.63	1.38	-0.06
21	25	99.59	0.41	-0.02	119	90	98.59	1.41	-0.06
23	26	99.59	0.41	-0.02	117	89	98.59	1.41	-0.06
18	28	99.56	0.44	-0.02	118	91	98.56	1.44	-0.06
22	27	99.56	0.44	-0.02	114	92	98.56	1.44	-0.06
19	29	99.53	0.47	-0.02	115	93	98.53	1.47	-0.06
17	30	99.53	0.47	-0.02	113	94	98.53	1.47	-0.06
48	32	99.5	0.5	-0.02	80	96	98.5	1.5	-0.07
16	31	99.5	0.5	-0.02	112	95	98.5	1.5	-0.07
49	33	99.47	0.53	-0.02	81	97	98.47	1.53	-0.07
51	34	99.47	0.53	-0.02	83	98	98.47	1.53	-0.07
54	36	99.44	0.56	-0.02	82	99	98.44	1.56	-0.07
50	35	99.44	0.56	-0.02	86	100	98.44	1.56	-0.07
55	37	99.41	0.59	-0.03	85	102	98.41	1.59	-0.07
53	38	99.41	0.59	-0.03	87	101	98.41	1.59	-0.07
60	40	99.38	0.63	-0.03	92	104	98.38	1.63	-0.07
52	39	99.38	0.63	-0.03	84	103	98.38	1.63	-0.07
61	41	99.34	0.66	-0.03	95	106	98.34	1.66	-0.07
63	42	99.34	0.66	-0.03	93	105	98.34	1.66	-0.07
62	43	99.31	0.69	-0.03	94	107	98.31	1.69	-0.07
58	44	99.31	0.69	-0.03	90	108	98.31	1.69	-0.07
57	46	99.28	0.72	-0.03	91	109	98.28	1.72	-0.08
59	45	99.28	0.72	-0.03	89	110	98.28	1.72	-0.08
40	48	99.25	0.75	-0.03	72	112	98.25	1.75	-0.08
56	47	99.25	0.75	-0.03	88	111	98.25	1.75	-0.08
41	49	99.22	0.78	-0.03	73	113	98.22	1.78	-0.08
43	50	99.22	0.78	-0.03	75	114	98.22	1.78	-0.08
46	52	99.19	0.81	-0.04	74	115	98.19	1.81	-0.08
42	51	99.19	0.81	-0.04	78	116	98.19	1.81	-0.08
45	54	99.16	0.84	-0.04	79	117	98.16	1.84	-0.08
47	53	99.16	0.84	-0.04	77	118	98.16	1.84	-0.08
36	56	99.13	0.88	-0.04	76	119	98.13	1.88	-0.08
44	55	99.13	0.88	-0.04	68	120	98.13	1.88	-0.08
37	57	99.09	0.91	-0.04	69	121	98.09	1.91	-0.08
39	58	99.09	0.91	-0.04	71	122	98.09	1.91	-0.08
38	59	99.06	0.94	-0.04	66	124	98.06	1.94	-0.08
34	60	99.06	0.94	-0.04	70	123	98.06	1.94	-0.08
35	61	99.03	0.97	-0.04	67	125	98.03	1.97	-0.09
33	62	99.03	0.97	-0.04	65	126	98.03	1.97	-0.09
96	64	99	1	-0.04	64	127	98	2	-0.09

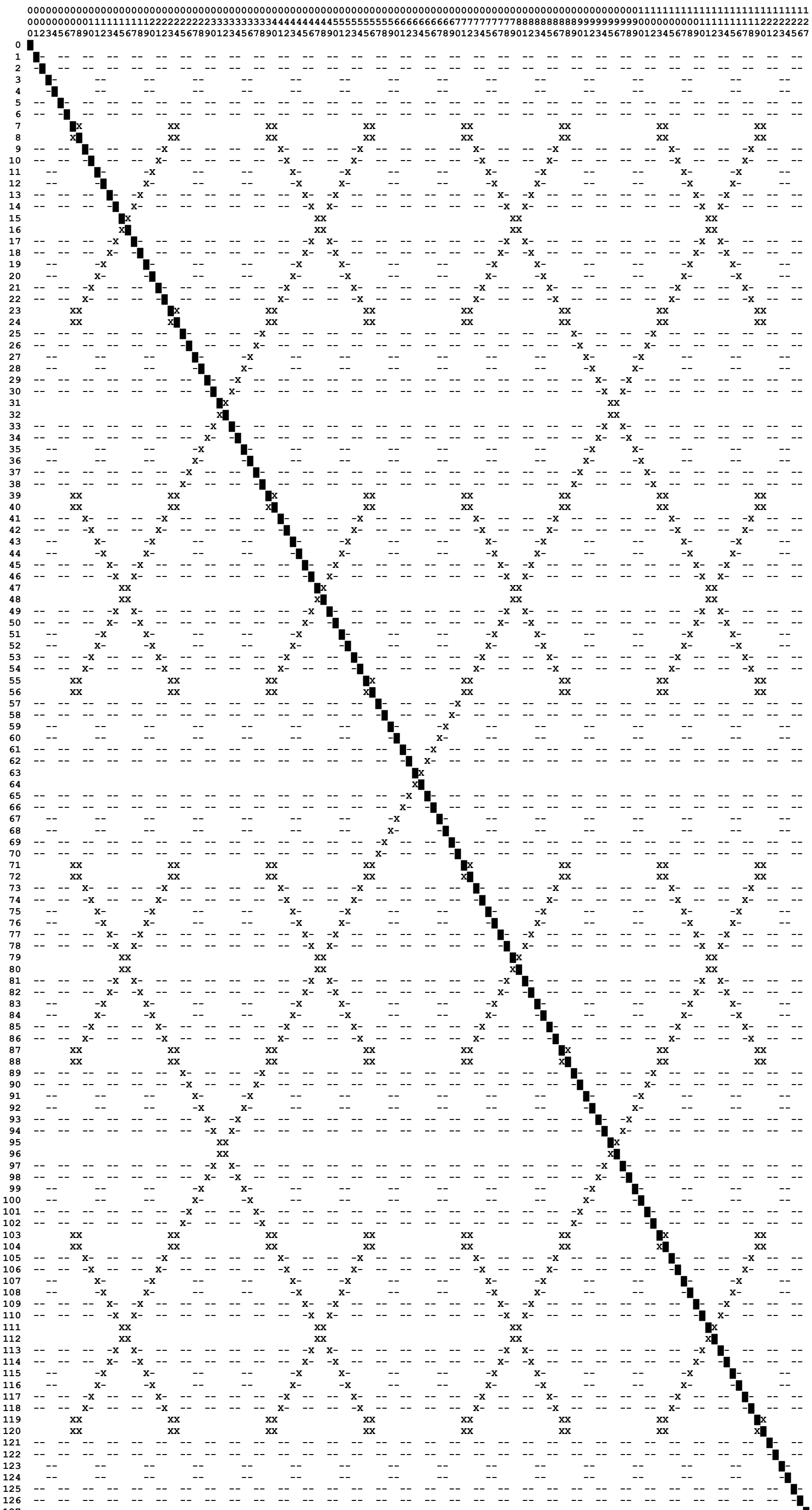

Figure 3: Amplitudes of cross-products of different 1% time shifted Walsh functions, in WAL order. Amplitudes of >-0.1 dB, -20 to -0.1 dB, and -30 to -20 dB are shown with "▮", "x" and "-". Weaker than -30 dB is left blank.

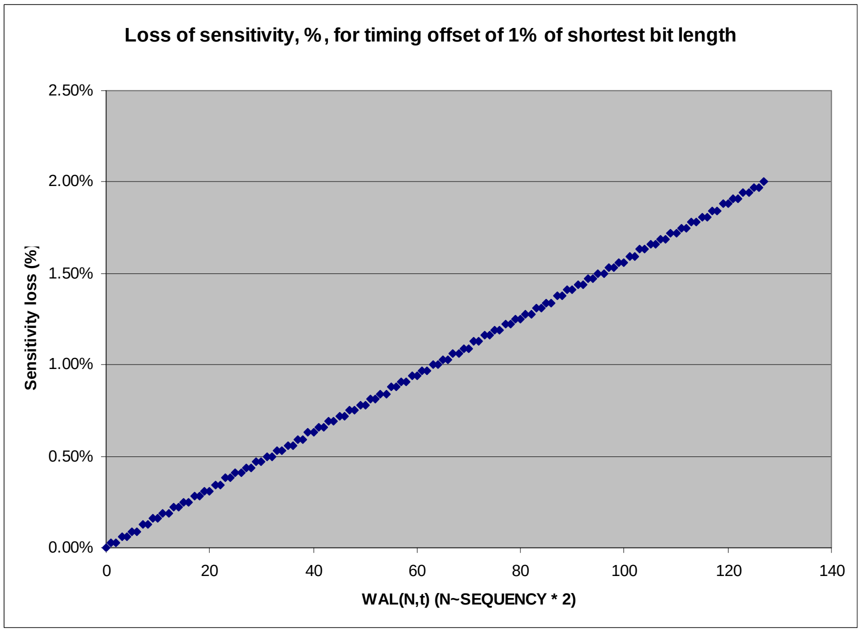

Figure 4: Sensitivity loss for different Walsh sequencies, with 1% timing error.

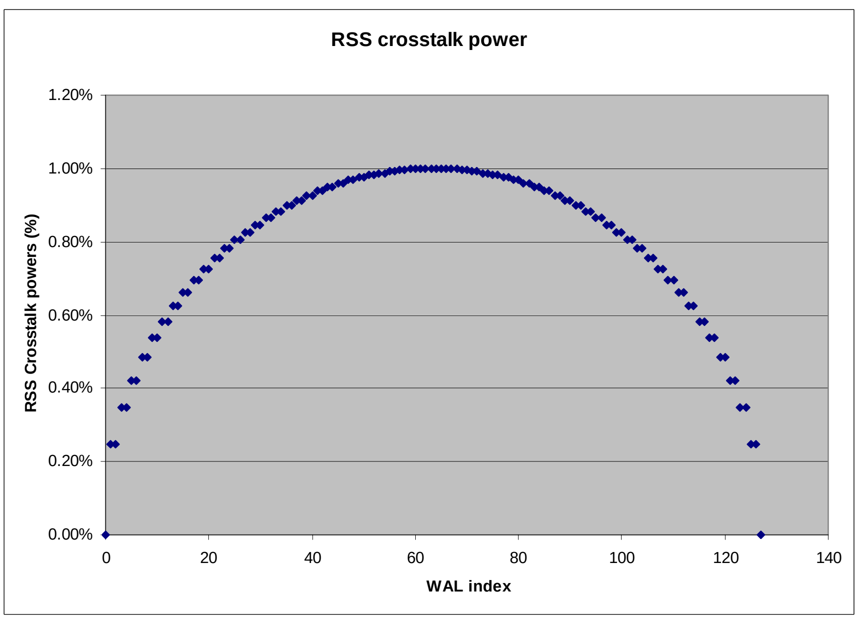

Figure 5: RSS crosstalk for different Walsh sequencies, with 1% timing error.

Summary of Results

1. If the demodulation waveform is shifted by 1% of the shortest element, then there is up to 2% loss of signal. The precise loss is proportional to Walsh sequency, and also proportional to timing offset. See Table 1 and Figure 4.
2. With a small timing shift, the different Walsh functions do not necessarily remain orthogonal, which potentially leads to crosstalk between different wanted signals, or poor rejection of spurious signals. See Figures 3 and 5.
3. With timing error of 1% of the shortest bit, the worst crosstalk is at a level of 1%. Not all possible Walsh products give crosstalk – more than half of the possible pairs of functions remain orthogonal in the presence of small timing shifts. However, with the exceptions of WAL(0,t) and WAL(127,t), all time-shifted Walsh functions have lost orthogonality with at least some of the other Walsh functions in the set. See Figure 3.
4. If the RSS crosstalk is calculated for a given Walsh function, using crosstalk magnitudes from all possible pairs of products with that function except for its own self-product, then if the RSS crosstalk plotted in sequency order, the result shows a smooth variation that peaks at half the maximum sequency. The worst crosstalk from the set of 128 functions is with WAL(63,t) and WAL(64,t) , i.e. SAL(32,t) and CAL(32,t). The crosstalk decreases symmetrically for WAL indices decreasing and increasing from this worst case, down to zero crosstalk for all products with shifted WAL(0,t) or WAL(127,t). See Figure 4. It should be noted that the RSS crosstalk is usually dominated by a very few strong crosstalk terms, rather than the cumulative effect of perhaps many more but much weaker terms.

Conclusions and Recommendation

A. To minimize signal (s/n) loss, if we need to choose 64 Walsh functions from the complete set of 128, we should choose the 64 functions with lowest sequency numbers, WAL(0,t) to WAL(63,t), rejecting WAL(64,t) through WAL(127,t). The loss is proportional to sequency number, so WAL(1,t) through WAL(64,t) is as good a choice.

B. To minimize crosstalk in the presence of spurious signals and drift, we should choose the 32 functions with lowest sequency numbers, and the 32 functions with highest sequency numbers, rejecting WAL(32,t) through WAL(95,t). This is not compatible with (A) above.

C. Loss of signal is probably more serious than reduction of crosstalk immunity, since other factors such as astronomical fringe rate add to the crosstalk immunity. It is suggested that an optimum set of functions to minimize signal loss is chosen initially, such as WAL(0,t) through WAL(63,t). If in ALMA operation it turns out that crosstalk or imperfect crosstalk rejection with certain antennas becomes a problem, then the Walsh modulation functions for those specific antennas may be changed to give better immunity in those specific cases. Figure 3 may help in the choice of function.

As the above figures show, for a timing error of 1% of the smallest bit length, the signal loss is up to 2%, although may be kept down to 1% if Walsh functions are chosen from the lowest half of sequency order. Similarly, the worst crosstalk rejection will be 20 dB in these circumstances. **For the pi/2 switching,** this timing error is the most tolerant, being 1% of 16 milliseconds, or **160 microseconds**. This will probably be easy to attain. For the **pi switching,** retaining orthogonality between modulation and demodulation within one antenna should be relatively easy; **a tolerance of 1.25 microseconds** gives a worst case 2% loss, or 1% worst case with a similar restriction in choice of Walsh sequency index. A good goal for the tolerance would be one tenth of this. To retain orthogonality with all antennas, to avoid crosstalk, the real time ALMA timing computation is much more complicated, depending on source position a well as precise antenna coordinates, but the same tolerance of 1.25 microseconds will retain 20 dB of rejection or better.

Acknowledgements

I thank Dick Thompson and Larry D'Addario for helpful comments on the manuscipt.

APPENDIX

Outline of program to simulate crosstalk and amplitude loss between time-shifted Walsh Functions

1. Tabulate a set of W orthogonal Walsh functions, in Natural or Paley order.
    a. Generate a set of R Rademacher functions, where $N=2^W$
    b. For each value of N=0 to W-1, use the binary representation of N to choose a subset of Rademacher functions to multiply together. The product is the desired Walsh function of Paley index N.

2. For each of the W tabulated Walsh functions,
    a. Oversample each function by 100:1. This allows simulation of offsets **dt** measured in percentage of the smallest bit length. Double (by copying) the function length, to allow cross-products time offsets **dt** up to W bits.
    b. For this oversampled Walsh function, sum the cross-product terms with each of the W tabulated Walsh functions, allowing for the oversampling and for the chosen percentage offset dt of one function with respect to the other. After normalization, this gives the desired signal (for identical but time-shifted functions) or crosstalk.

3. Sort the cross-products in order according to sensitivity loss or to crosstalk magnitude. It is found convenient to convert the PAL ordering into WAL or sequency, as described by Beauchamp (1975) and others.

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D'Addario, L.R.: ALMA System Technical Requirements, July 2004. See http://edm.alma.cl/forums/alma/dispatch.cgi/documents/showFile/100722/d20040825221327/No/ALMA-80.04.00.00-005-A-SPE.pdf. Section 4.5 discusses synchronization of the switching waveforms. See also ALMA System Technical Spec #444.