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Mode conversion and Resonant Absorption in Bent Overmoded Waveguide

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Abstract

Mode conversion losses are a concern when an oversize waveguide is used to minimize ohmic losses. We present computations of mode conversion in oversize rectangular waveguide bends.

The numerical results are based on a published analytical formulation of the problem. They are presented in a dimensionless form applicable to a range of parameters believed to cover practical cases of interest for engineers that need to design a signal transport involving oversized waveguide and bends.

A few cases have been cross validated using a 3D EM simulation software; the numerical and analytical results are in good agreement.

Oversized waveguide bends are sometimes used to minize ohmic losses when a signal must be transported over a significant distance. The waveguide devices at both ends are usually in a smaller, single-mode waveguide size, and are connected to the overmode waveguide by suitable transitions. The combination forms a resonant cavity for higher order modes. We examine how the transmission of the fundamental mode is affected by resonances in the higher order modes.

Contents:

1. Introduction

For millimeter waves, oversized (overmoded) waveguides are sometimes used in order to minimize ohmic losses, when a signal must be transported over a significant distance such as local oscillator distribution, for instance in a cryogenic system. With a suitable tapered transition, the TE_{10} mode can be transmitted to the oversized waveguide with good efficiency; many higher order modes are decoupled by symmetry considerations alone.

Depending on the physical layout and other design constraints, the signal transport may need to follow a curved path, requiring waveguide bends.

Waveguide bends are known to induce coupling between modes. We could not find in the literature a comprehensive set of numerical results in a form suitable for practical applications. This motivated us to produce such a set of results, and to analyze practical consequences, including overmode trapping between waveguide transitions.

2. Analytical formulation of mode conversion

2.1 Statement of the problem

Given a section of standard (b/a = ½) waveguide with uniform curvature in either the E- or the H-plane, excited with the fundamental TE_{10} mode, we aim to determine the amplitude coupled into the various overmodes as a function of the frequency υ and the radius of curvature R of the centerline.

2.2 General form of the equations

We used the results derived in Ref [1], where a waveguide bend is represented by *n* coupled transmission lines, *n* being the number of propagating modes. The normalized coupled mode voltages E_n obey a set of coupled ordinary differential equations (ODE) that involve the propagation constants k_{nn} (k_{nn} = β_n is the propagation constant of the mode n) and their coupling coefficients k_{mn} (k_{mn} is the coupling coefficient between modes m and n).

The equations below represent the case of a H-plane bend:

$$\begin{aligned} &\frac{d}{dz}E_{10} - jk_{11}E_{10} + jk_{12}E_{20} + ... + jk_{1n}E_{n0} = 0 \\ &... \\ &\frac{d}{dz}E_{n0} - jk_{n1}E_{10} + jk_{n2}E_{20} + ... + jk_{nn}E_{n0} = 0 \end{aligned} \qquad (1)$$

where z is the distance measured along the centerline of the bend (z = R.α), R is the radius of the centerline of the bend, and α is the angle of the bend. In the present work, we define the k_{mn} coefficients as real, to avoid some ambiguity present in Ref [1]. Only propagating modes (at a given frequency) are considered.

2.3 Dimensionless variables

In order to present general results in a compact form, and to reduce the volume of calculations to the essential minimum, we have treated the problem in non-dimensional form. The parameters are therefore:

- Φ = a/λ = (a/c) υ, the reduced frequency

- ρ = R/a, the reduced radius of curvature
- α, the angle of the bend

The equations of the problem were also cast in non-dimensional form.

2.4 Number of modes considered

If only one extra mode has a significant coupling to the fundamental mode TE_{10}, the problem is simplified, with power being transferred cyclically between the two coupled modes along the coupling region (Ref [2]). So, a possible solution is to adjust the bend dimensions in order to cancel the amplitude of the first spurious mode at the output of the bend (Ref [1]). However, such a cancellation can only be obtained for a discrete value of the frequency. For many practical applications, the waveguide is used over a significant frequency range, over which one attempts to minimize the power coupled to spurious modes.

Besides, a treatment with just one overmode is strictly valid only up to the cutoff frequency of the next overmode. Having set a/λ=4 as the maximum value of the reduced frequency, six spurious mode could appear for a H-plane bend (TE_{20}, TE_{30}, TE_{40}, TE_{50}, TE_{60} and TE_{70}), and six (degenerated) spurious modes could appear for an E-plane bend ([TE_{11}, TM_{11}], [TE_{12}, TM_{12}], [TE_{13}, TM_{13}]). Some exploratory finite difference time domain (FDTD) simulations showed that three overmodes at most were significant over the region of parameter space that we cover in the case of an H-plane bend. Accordingly, the coupled set of ODE's (1) was solved for a maximum of four modes (the fundamental, and three spurious modes) for H-plane bends, and seven spurious modes (the fundamental, and the three couples of degenerated spurious modes) for E-plane bends, the actual number of modes being dependent on the frequency.

2.5 Equations for H-plane bends

In the case of a H-plane bend, the coupling between TE_{m0} and TE_{n0} is zero if m-n is even. So, for a bend excited by his dominant mode TE_{10}, only the TE_{20}, TE_{40}…modes are directly coupled. The odd-numbered TE_{30}, TE_{50}… modes are coupled through the even-numbered modes.

The coupling coefficient between two TE modes (TE_j and TE_m) is given in Ref [3]:

$$k_{jm} = -\frac{4}{\pi^2} \times \frac{1}{\sqrt{\xi_{nj}\xi_{nm}}} \times \frac{\beta_j + \beta_m}{2\sqrt{\beta_j \beta_m}} \times \frac{a}{R} \times \frac{1}{(n_j^2 - n_m^2)^2} \times \left(\beta_m n_m^2 \sqrt{\frac{l_j^2 + 4n_j^2}{l_m^2 + 4n_m^2}} + \beta_j n_j^2 \sqrt{\frac{l_m^2 + 4n_m^2}{l_j^2 + 4n_j^2}}\right) \tag{2}$$

With:

$\xi_n = 1 + \delta_{0n}$ i.e. $\xi_0 = 2$ and $\xi_n = 1$ for $n \neq 0$

a : long dimension of the rectangular waveguide

l_j, l_m, n_j, n_m: Indexes of the modes (the mode j could be written $TE_{n_j l_j}$, and the mode m could be written $TE_{n_m l_m}$), $l_j = l_m$, $n_j + n_m$ odd.and

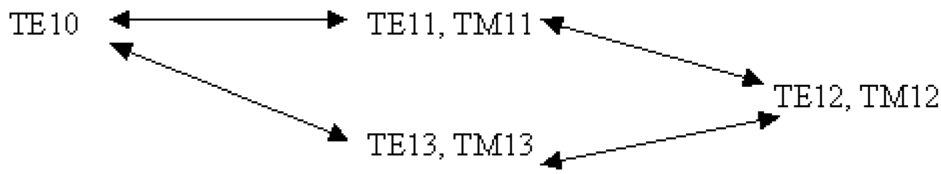
$$\beta_{\ln} = \frac{2\pi}{\lambda}\sqrt{1 - \left(\frac{l\lambda}{2a}\right)^2 - \left(\frac{n\lambda}{2b}\right)^2} \tag{3}$$

When three spurious modes are considered, the expressions of the coupled amplitudes are too complicated to be written literally. For each of a set of values of the reduced frequency $\Phi = a/\lambda$ and the reduced radius $\rho = R/a$, spaced over a regular grid, the coefficients of the propagation equations (1) were evaluated numerically. A closed form solution of the constant coefficient linear ODE's was then evaluated for discrete values of the bend angle α.

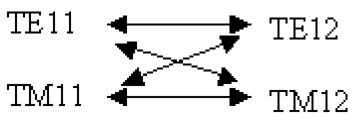
2.6 Equations for E-plane bends

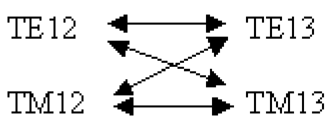
In the case of an E-plane bend, the coupling between the TE_{10} mode and the TE_{1n} and TM_{1n} modes is zero when n is even. So, for E-plane bends excited by TE_{10} mode, the first spurious modes which appear in the same time is the couple of degenerated modes TE_{11} and TM_{11}. The others couples of degenerated modes which could appear are (TE_{12}, TM_{12}), (TE_{13}, TM_{13})…

The coupling between the fundamental mode and the spurious modes for an E-plane bend can be represented by the scheme below:



With:





This case is a little bit more complicated than the case of H-plane bends, because both the coupling between two TE modes, the coupling between two TM modes and the coupling between TE and TM modes must be considered.

The expressions of the coupling coefficients used to calculate mode conversion losses for an E-plane bend are:

Coupling coefficient between two TE modes (TE_j and TE_m):

$$k_{jm} = -\frac{4}{\pi^2}\times\frac{1}{\sqrt{\xi_{nj}\xi_{nm}}}\times\frac{\beta_j+\beta_m}{2\sqrt{\beta_j\beta_m}}\times\frac{a}{R}\times\frac{1}{(n_j^2-n_m^2)^2}\times \left(\beta_m n_m^2\sqrt{\frac{l_j^2+4n_j^2}{l_m^2+4n_m^2}}+\beta_j n_j^2\sqrt{\frac{l_m^2+4n_m^2}{l_j^2+4n_j^2}}\right) \quad (4)$$

Coupling coefficient between two TM modes (TM_j and TM_m):

$$k_{jm} = -\frac{4}{\pi^2} \times \frac{\beta_j + \beta_m}{2\sqrt{\beta_j \beta_m}} \times \frac{a}{R} \times \frac{n_j n_m}{(n_j^2 - n_m^2)^2} \times \left(\beta_m \sqrt{\frac{l_j^2 + 4n_j^2}{l_m^2 + 4n_m^2}} + \beta_j \sqrt{\frac{l_m^2 + 4n_m^2}{l_j^2 + 4n_j^2}}\right) \quad (5)$$

Coupling coefficient between a TE mode (mode m) and a TM mode (mode j) :

$$k_{jm} = \frac{4}{\sqrt{\xi_{nm}}} \times \frac{\beta_j + \beta_m}{2\sqrt{\beta_j \beta_m}} \times \frac{1}{R} \times \frac{l \times n_j}{(n_j^2 - n_m^2)} \times \frac{a^2}{b\lambda} \times \frac{1}{\sqrt{l_m^2 + 4n_m^2} \times \sqrt{l_j^2 + 4n_j^2}} \times \frac{2}{\pi} \quad (6)$$

With:

$\xi_n = 1 + \delta_{0n}$ i.e. $\xi_0 = 2$ and $\xi_n = 1$ for $n \neq 0$

a : short dimension of the waveguide (unlike for the H-plane case)

b: long dimension of the waveguide (b=2a)

l_j, l_m, n_j, n_m: Indexes of the modes

$$\beta_{\ln} = \frac{2\pi}{\lambda} \sqrt{1 - \left(\frac{l\lambda}{2a}\right)^2 - \left(\frac{n\lambda}{2b}\right)^2}$$

The rest of the treatment is the same as for H-plane bends.

3. Results for mode conversion

The results presented here show the total power lost in the bend by mode conversion, considering all the calculated spurious modes, for different discrete values of bend’s angle, versus R/a and a/λ.

For H-plane bends, we have represented different contour values of the total power lost in TE_{20}, TE_{30} and TE_{40} modes (Figure 1)

For E-plane bends, we have represented different contour values of the total power lost in TE_{11}, TM_{11}, TE_{12}, TM_{12}, TE_{13} and TM_{13} modes (Figure 2)

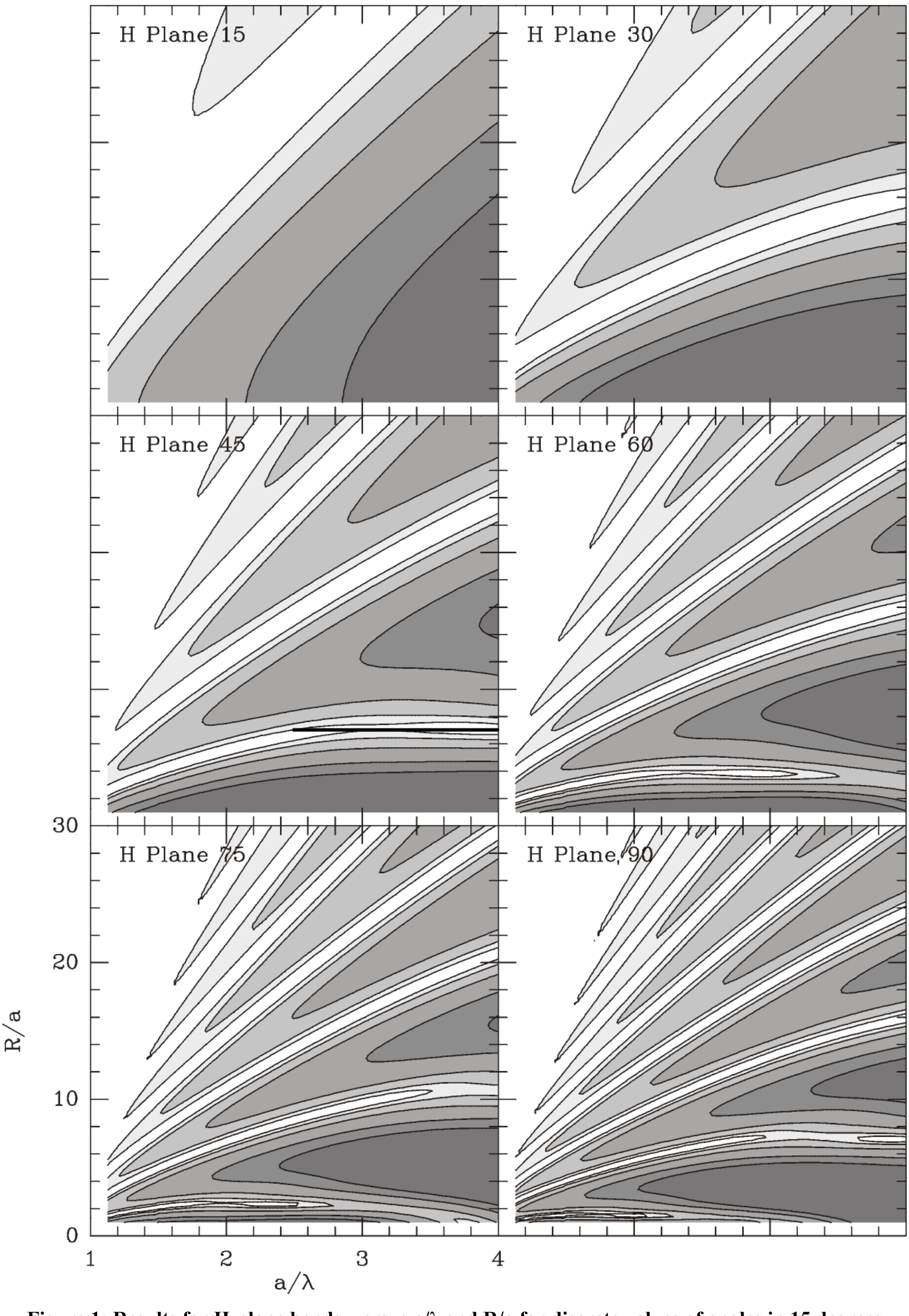


Figure 1: Results for H-plane bends, versus a/λ and R/a for discrete values of angles in 15 degrees increments. The contour values represent the fractional power lost in the 3 first spurious modes. The contour values are 0.01 (-20dB), 0.03 (-15dB), 0.1 (-10dB), 0.3 (-5dB), and 0.5 (-3dB). The horizontal line overlaid on the 45° box points to a "quiet zone"; see text for comments.

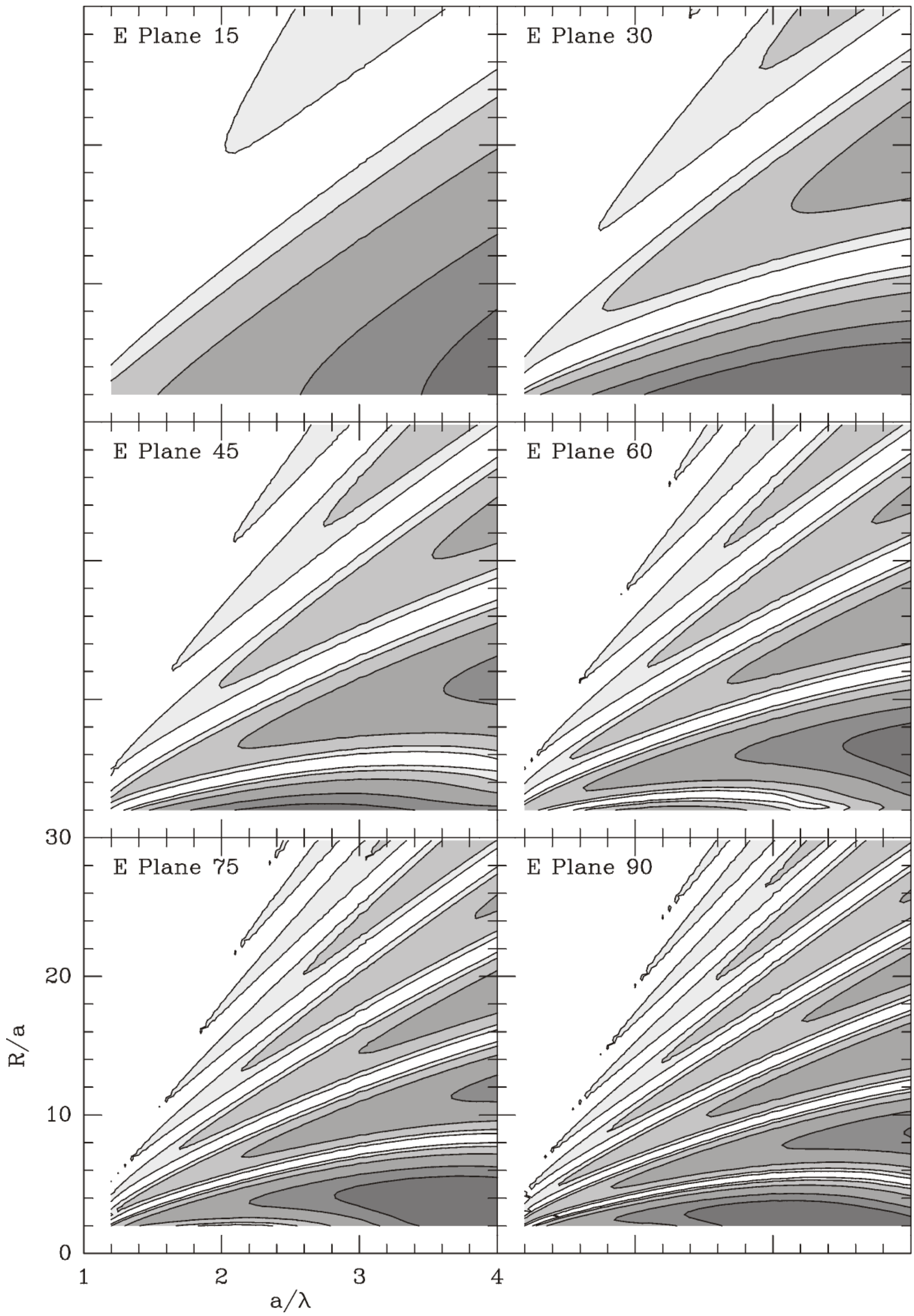


Figure 2: Results for E-plane bends, versus a/λ and R/a for discrete values of angles in 15 degrees increments. The contour values represent the fractional power lost in the 6 first spurious modes. The contour values are 0.01 (-20dB), 0.03 (-15dB), 0.1 (-10dB), 0.3 (-5dB), and 0.5 (-3dB).

It is apparent that, as the bend angle increases, the diagram of converted power presents an increasingly complex alternance of regions of high and low coupling; this is due to the difference in propagation constants of the fundamental mode and overmode, and to direct and reverse coupling.

While, for large bend angles, the complexity of the diagram seems to exclude usable solutions for small bend radii and high frequencies, a closer examination shows that interesting "quiet zones" exist deep inside the diagram; for instance, see for an H plane 45° bend, a fairly compact bend with R/a=7 has low conversion between $a/\lambda = 2.5$ and $a/\lambda = 4$.

4. Comparison with FDTD simulations

We have compared analytical results with the results obtained by FDTD electromagnetic simulations of 90 degrees angle bends.

On the simulator (Microwave Studio, CST), we designed a 90 degrees H-plane bend, in waveguide standard WR10, with a reduced radius R/a = 5, excited by its fundamental mode TE_{10}, and we have compared the FDTD simulation of the amplitude coupling of the first spurious mode TE_{20}, with analytical calculations of amplitude coupling in both the case of two modes consideration and four mode consideration.

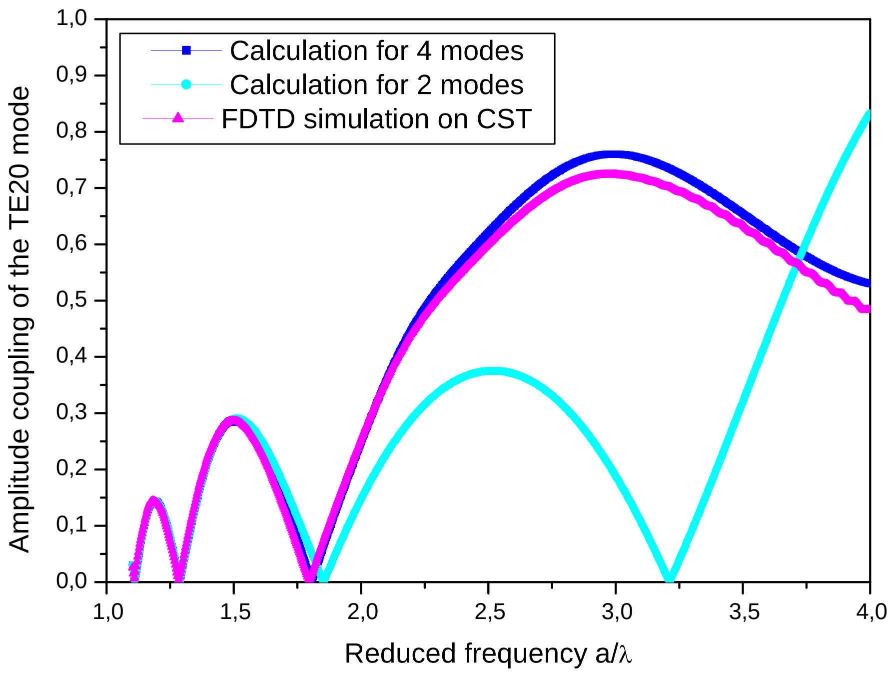


Figure 3: Comparison of FDTD simulation of TE_{20} amplitude coupling with analytical calculations for one and three spurious modes

We also simulated a 90 degrees E-plane bend in waveguide standard WR10, with a reduced radius R/a=5, excited by its fundamental mode, and we have compared the FDTD simulation of the amplitude coupling of the first couple of degenerated spurious modes TE_{11}, TM_{11} with analytical calculations considering three couples of degenerated spurious modes.

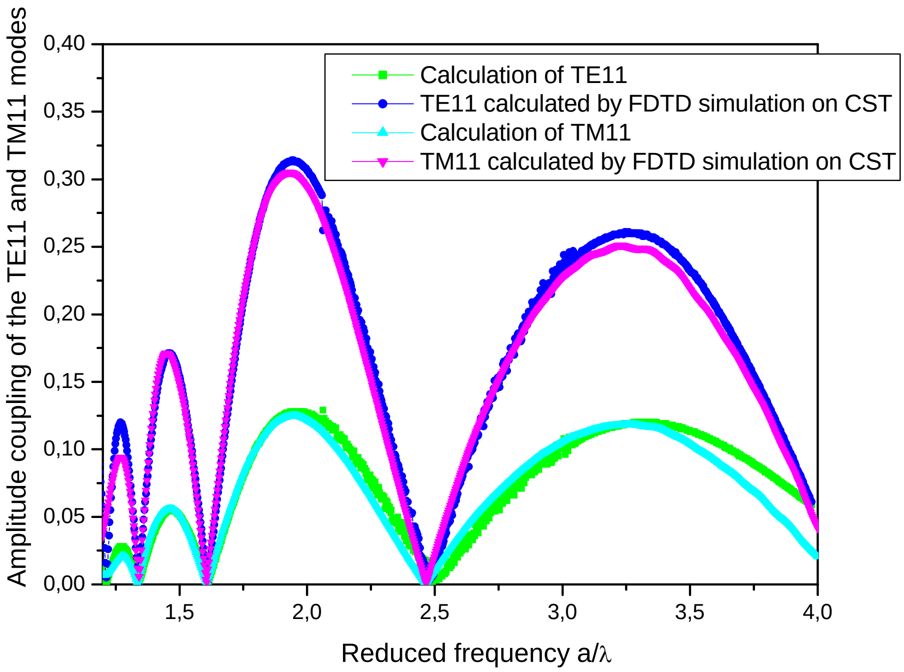


Figure 4: Comparison of FDTD simulation of TE11 and TM11 amplitudes coupling with analytical calculations for six spurious modes

Comparing results obtained by the calculation of one and three spurious modes on Figure 3, we could see that the contributions of higher order spurious modes are not negligible above their cutoff frequencies. We also see that the consideration of three spurious modes in the H-plane case (Figure 3), and three couples of degenerated spurious modes in the E-plane case (Figure 4), is sufficient to obtain a good agreement between analytical calculations and FDTD simulations, for a value of reduced frequency $a/\lambda<4$.

5. Resonant Mode Trapping

Oversized waveguide bends are sometimes used to minize ohmic losses when a signal must be transported over a significant distance. The waveguide devices at both ends are usually in a smaller, single-mode waveguide size, and are connected to the overmode waveguide by suitable transitions. Any overmode generated by bends in the oversize waveguide is trapped between the transitions. The main mode is therefore coupled to a resonant cavity. In the present section, we evaluate the resonant loss affecting the main transmission line.

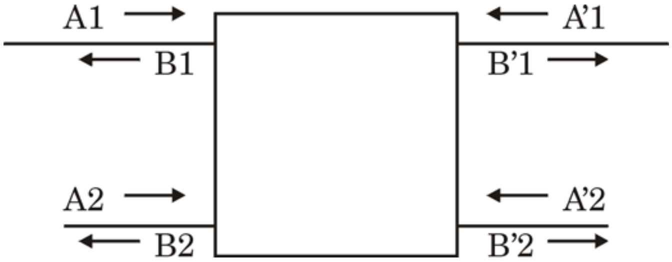


Figure 5: Four-port schematic of mode conversion in the bend. The index 1 belongs to the fundamental mode, and index 2 to the dominant trapped overmode.

The bend in the oversize waveguide, and the associated couplings is represented as a four-port symmetric lossless passive device, as shown in Figure 5.As in Ref [5], we neglect reflection coupling, and we assume that ohmic losses in the bend itself can be neglected (or treated separately from mode coupling). The amplitudes shown in Figure 5 are related by the following equations:

$$\begin{bmatrix} B1 \\ B'1 \\ B2 \\ B'2 \end{bmatrix} = \begin{bmatrix} 0 & q & 0 & ip \\ q & 0 & ip & 0 \\ 0 & ip & 0 & q \\ ip & 0 & q & 0 \end{bmatrix} \cdot \begin{bmatrix} A1 \\ A'1 \\ A2 \\ A'2 \end{bmatrix} \tag{7}$$

Where p is the *amplitude* of the mode coupling determined above (note: in Figures 1 and 2, the contours values are in *power* units), and:

$$q = \sqrt{1 - p^2} \tag{8}$$

These equations are supplemented by the following ones:

$$\begin{aligned} &A2 = r\ B2 \\ &A'2 = r\ B'2 \\ &A1 = 1 \quad A2 = 0 \quad A'1 = 0 \quad A'2 = 0 \end{aligned} \tag{9}$$

where $r = \rho \exp(i\theta)$ is the complex propagation coefficient for one half round-trip of the over mode from the bend to the end of the oversize section and back to the bend.

We solved numerically the above system of 8 equations for various values of p, ρ, and θ. The result of primary interest in the solution is $B'1$. When varying θ alone, resonant dips in $|B'1|^2$ are apparent, with minimum transmission for $\theta = n\pi$, and maximum transmission for $\theta = (2n+1)\frac{\pi}{2}$. We have computed the minimum ($\theta = 0$) and maximum ($\theta = \frac{\pi}{2}$) transmission for an array of values of p and ρ, and the results are shown graphically on Figure 6 and Figure 7.

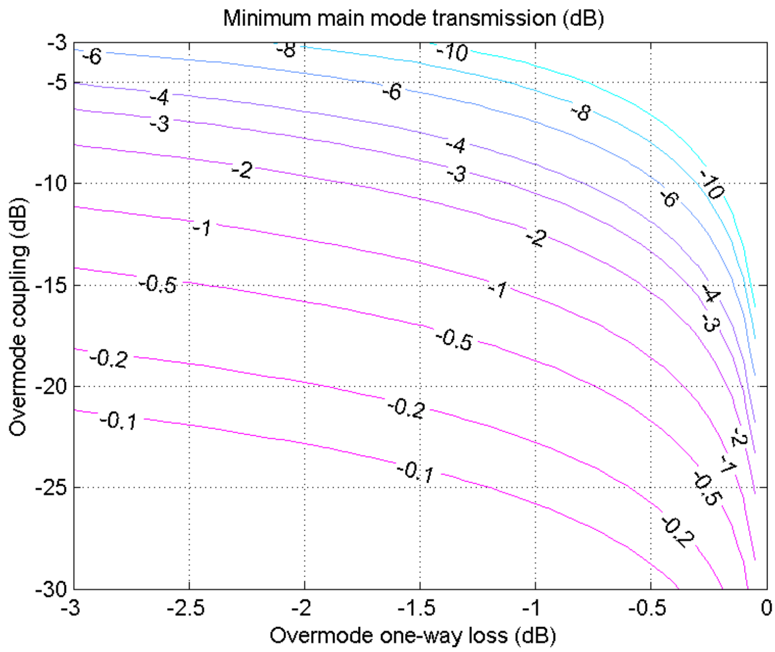


Figure 6. Minimum transmission contours (dB), versus the one-way (one half round trip) loss of the overmode, expressed in dB (20 log(ρ) and the coupling between the fundamental and the overmode, also expressed in dB (20 log(p)).

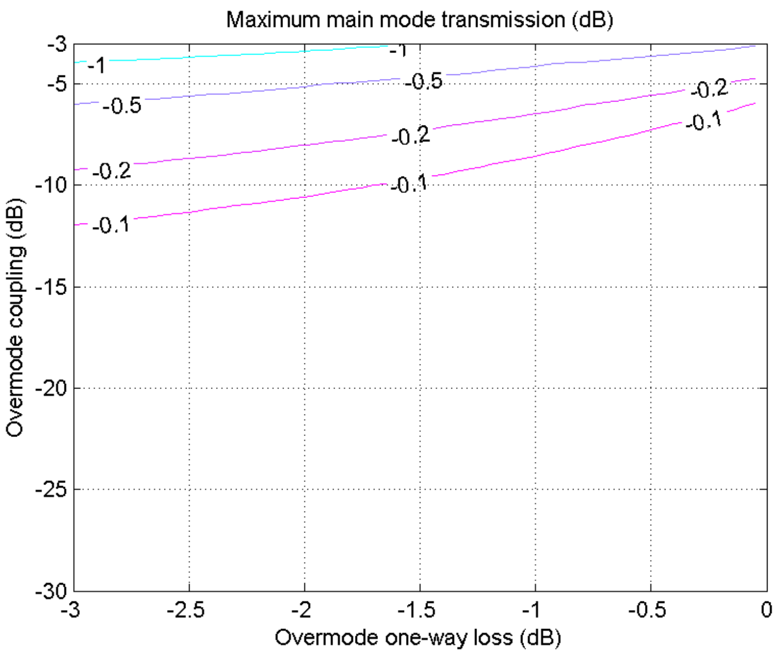


Figure 7. Maximum transmission, same units as in Figure 6. The loss at maximum transmission is generally small enough that, for practical purposes, it is not necessary to distinguish between the minimum transmission and the amplitude of the resonant dip.

An approximate equation for the depth of the resonant dip is given in Ref [5] as eqn.A52; a similar equation is given in Ref [6]. Both contain (different) typographical errors; a correct form is given in Ref [7]. In our notation, that equation reads:

$$P_{\min} / P_{\max} = \frac{1 - 2\,p^2\,\rho^2 /(1+\rho^2)}{1 + 2\,p^2\,\rho^2 /(1-\rho^2)} \qquad (10)$$

Motivated by the doubt and uncertainity introduced by these differences, we compared the results of our numerical solution with those of the approximate equation. The results are shown on Figure 8, cast in a form similar to Figure 2 of Ref [7].

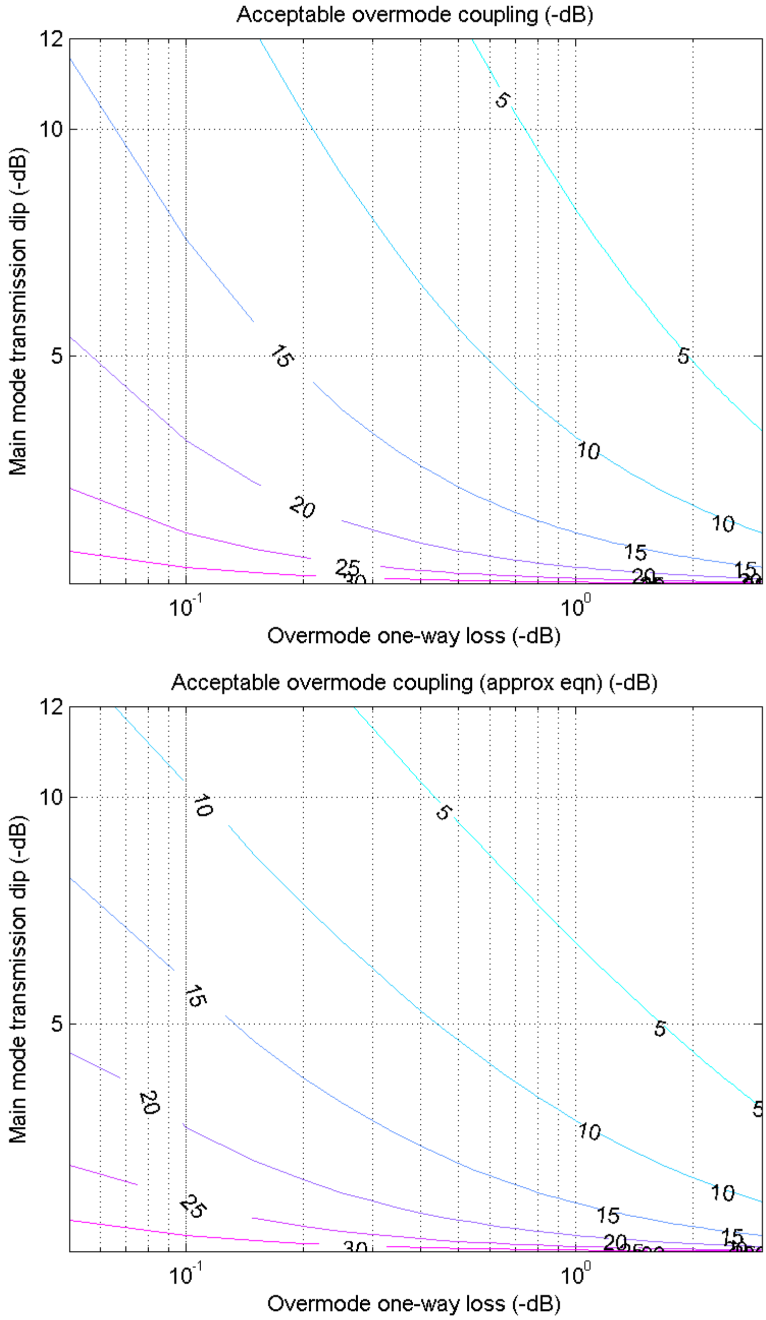


Figure 8. Comparison of the results of Equation 10. Top: "exact" result from our numerical solution of the network equations; bottom: approximate equation. Three changes in the presentation of the data were made compared with Figure 6 and Figure 7: a) the overmode coupling is now the dependent variable; b) the second independent variable is the resonant dip instead of minimum transmission; c) loss is counted as positive dB, in keeping with the presentation of figure 2 of Ref [7]. Substantial differences between the "exact" solution and the approximate equation are apparent for overmode couplings stronger than 15dB, presumably because the approximate equation neglects second-order couplings.

6. Conclusion

Based on published analytical results, we have generated diagrams of the conversion from the fundamental mode to overmodes, in a dimensionless form applicable to a wide range of practical configurations. Together with an analysis of the coupled resonance of the trapped overmode, these results allow the design engineer to select a configuration such that the resonant power loss of the fundamental mode stays below a prescribed limit. In some cases, in particular for 45° and 90° angles, it is possible in principle to design compact bends that have a fairly low conversion coupling over a substantial frequency range.

It is a pleasure to acknowledge informative exchanges with G.Ediss, regarding, among other, the correct form of equation 10.

7. References

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