ALMA Memo 565

Walsh Function Definition for ALMA

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Abstract: ALMA has chosen to use a set of 128 orthogonal Walsh functions, both for the sideband separation using 90-degree phase switching at the first local oscillator, and for spurious rejection using 180-degree phase switching at the first local oscillator with demodulation by means of a digital sign change shortly after the digitizer. A set of orthogonal Walsh functions can be defined and ordered in many different ways (PAL, Natural or Paley ordering, WAL, CAL & SAL sequency ordering) and with different phasing definitions (e.g. Harmuth phasing, or "positive phasing".) It is proposed here that ALMA standardize on a set of Walsh functions in WAL order with positive phasing. A numerical table is included, and is available in machine readable form.

Introduction

ALMA will use orthogonal Walsh functions for two purposes:

(1) Sideband separation: by switching a 90-degree phase offset into the first local oscillator at each antenna, following a different orthogonal switching sequence at each antenna, after correlation it is possible to derive independently the complex visibility terms of signals in upper and lower sidebands, for each interferometer antenna pair. Details of this process are explained in reference (1). In the ALMA case, the Walsh functions used for this will be drawn from a 128-element set, where the shortest state has a duration of 16 milliseconds, and the complete set takes 2.048 seconds.

(2) Spurious signal and DC offset rejection: A 180-degree phase offset is switched into the first local oscillator at each antenna, in addition to the 90-degree offset described above. This 180-degree offset is similarly switched following a different orthogonal Walsh function at each antenna, where each function is drawn from the same 128-element set as in (1). The shortest state has a duration of 125 microseconds, with the complete 128-element set lasting 16 milliseconds. This complete switching cycle takes place within each of the 16-milliseconds corresponding to the shortest 90-degree state

described in (1). For convenience the nested Walsh function sequences in (1) and (2) are made to be synchronous, although in principle this is not essential. This 180-degree modulation of the signal is decoded digitally shortly after the digitizer, as a sign change of the data stream at the antenna, using the same Walsh function as was chosen for the 180-degree phase modulation at that antenna. This means that a DC offset in the digitizer, for example, now appears as a Walsh function superposed on the data stream. Since this Walsh function is orthogonal to that of every other antenna, at the output of the correlator the unwanted DC offset is suppressed.

Walsh Ordering and Phasing

Several different conventions of Walsh function numbering are in common use.
One convenient way of generating Walsh functions is by products of chosen Rademacher functions. [A Rademacher function R(n,t) may be defined by R(n,t)=sign{sin(2^n. .t)} (see e.g. reference (2), Beauchamp p.7)]. If R(n,t) is used in the product, then a "1" is placed in the nth binary digit of what becomes the natural or Paley index pn of the resulting Walsh function, PAL(pn,t).

Walsh functions may be ordered according to the number of zero crossings nw within one complete cycle t of the Walsh function WAL(nw,t); this ordering scheme has an appealing analogy with cosine and sine functions, and leads naturally to the CAL and SAL definitions. The term *sequency* is defined as "one half of the average number of zero crossings per unit time interval." Specifically,

$$\mathrm{WAL}(2.k,t) = \mathrm{CAL}(k,t)$$

and

$$\mathrm{WAL}(2.k\text{-}1,t) = \mathrm{SAL}(k,t)$$

where *k* is the *sequency* of the Walsh functions. See for example Beauchamp, p.12.

Even having decided on which ordering scheme to use, a particular set of Walsh functions is still not uniquely defined. Walsh functions are normally defined to have the amplitude either of +1 or -1, but the *phasing* of the function set has to be defined. Beauchamp gives diagrams (Figs 1.4 and 2.2 in that book – also reproduced as Figures 1 and 2 in ALMA Memo 537) of a 32-element set of Walsh functions in sequency (WAL) and in Natural (PAL) order. An inspection of those diagrams shows that some functions begin with +1, some with -1. This reflects the choice of *phasing* of the set of functions. Quoting Beauchamp (Section IIB1, p.18):

"PHASE OF THE ORDERED SET

The diagrams given in Figs 1.4 and 2.2 are arranged to emphasize the phase similarity with an ordered set of sine-cosine functions and will be referred to as Harmuth phasing.

However, if the series is derived directly from the Hadamard matrix or from Rademacher products, as described later, then the functions will be phased such that they all start at a +1 level and this will be referred to as positive phasing. It involves a reversal of sign for some of the functions shown in Figs 1.4 and 2.2."

Choice of Walsh Functions for ALMA

The ALMA Walsh functions are used to retrieve the desired signals, while rejecting unwanted effects such as DC offsets.

Not all Walsh functions are equally effective. ALMA Memo 537 (reference (3)) investigates the loss of signal, and the loss of orthogonality, in the presence of timing errors in application of the Walsh function; some Walsh functions are more immune to such errors than others. ALMA Memo 537 finds that in order to minimize loss of signal in the presence of timing errors, the WAL(N,t), or sequency numbers should preferably be the lowest numbers in the set. In order to minimize crosstalk, the mid-range values of WAL or sequency index should be avoided, making as much use as possible of just the higher and lower WAL values.

Emerson (1983), (reference (4)) describes how some Walsh functions are more effective than others at suppressing offsets that drift in amplitude as a function of time. Reference (4) shows that in order to maintain highest immunity to time-varying offsets, a PAL index with the highest number of component Rademacher products, that is with the highest number of non-zero digits in the binary representation of the PAL index, should be chosen. From this criterion, the best Walsh function chosen from a 128-element set would be PAL(127,t), being composed of 7 non-trivial Rademacher products. Next best, in order, would be:
PAL(126,t), PAL(125,t), PAL(123,t), PAL(119,t),PAL(111,t),PAL(95,t) and PAL(63,t), each of which is comprised of the product of 6 Rademacher functions, listed here in order of highest frequency Rademacher components first. These Paley indices are derived from N, N-1, N-2, N-4, N-8, N-16, N-32 and N-64, where N here is the highest index of the set, in this case 127; the corresponding WAL indices (see Table 2) are 85, 84, 86, 82, 90, 74, 106 and 42.

For ALMA, overall the ***WAL ordering*** is most convenient.

In the ALMA context, it is also convenient if all functions commence consistently with +1, which defines *positive phasing.* Our set can then be described as:

"***A set of Walsh functions with positive phasing arranged in WAL order."***

Table 1 shows all 128 functions that will be used in ALMA, defined in and numbered according to their WAL index. This table is also available as an ASCII format file in machine readable form as a text attachment at the end of this PDF file. Although a Walsh function is normally defined to have amplitude of +1 or -1, in order to save space in

Table 1 the functions have been shown with amplitudes +1 or 0. Figure 2 (courtesy of Gene DuVall and Michael Zaw) shows the same functions in an easier-to-view graphical form.

Conversion of WAL index to PAL index

Beauchamp (reference 2) describes (p.23) an algorithm for converting the Paley index to and from a WAL index, using binary modulo-2 addition and Gray code. Table 2, listed in order of WAL index, gives a lookup table for conversion from WAL to PAL ordering.

Conclusion

Walsh functions are built into the ALMA system, for sideband separation and for rejection of DC offsets and spurious signals. Walsh functions may be defined in several different ordering and phasing conventions. A standard convention is proposed here, using **WAL ordering and Positive Phasing**. A Walsh function lookup table is included here in the text, as well as a conversion table between WAL and PAL indices. A machine-readable ASCII table is attached to this document.

References

(1). See the section on "*Sideband Separation,* pp.181-183 in "*Interferometry and Synthesis in Radio Astronomy*, Second Edition, A. Richard Thompson, James M. Moran & George W. Swenson, Jr:, published by John Wiley & Sons, Inc., ISBN 0-471-25492-4 (2001).

(2) *Walsh Functions and their Applications*, K.G. Beauchamp, published by Academic Press, ISBN 0-12-084050-2 (1975).

(3) *Walsh Function Demodulation in the Presence of Timing Errors, leading to Signal Loss and Crosstalk*, ALMA Memo #537, available at http://www.alma.nrao.edu/memos/html-memos/alma537/memo537.pdf .

(4) *The Optimum Choice of Walsh Functions to Minimize Drift and Cross-Talk*, D.T. Emerson, Working Report 127, IRAM, Grenoble (1983). For convenience, this report is reproduced as an Appendix to the current document.

(5) **Table 1** is also attached to the current PDF file as a separate ASCII text file.

Table 1. ALMA Walsh functions in WAL order, with Positive Phasing.

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 0:11111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
 1:11111111111111111111111111111111111111111111111111111111111111110000000000000000000000000000000000000000000000000000000000000000
 2:11111111111111111111111111111111000000000000000000000000000000000000000000000000000000000000000011111111111111111111111111111111
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 6:11111111111111110000000000000000111111111111111100000000000000000000000000000000111111111111111100000000000000001111111111111111
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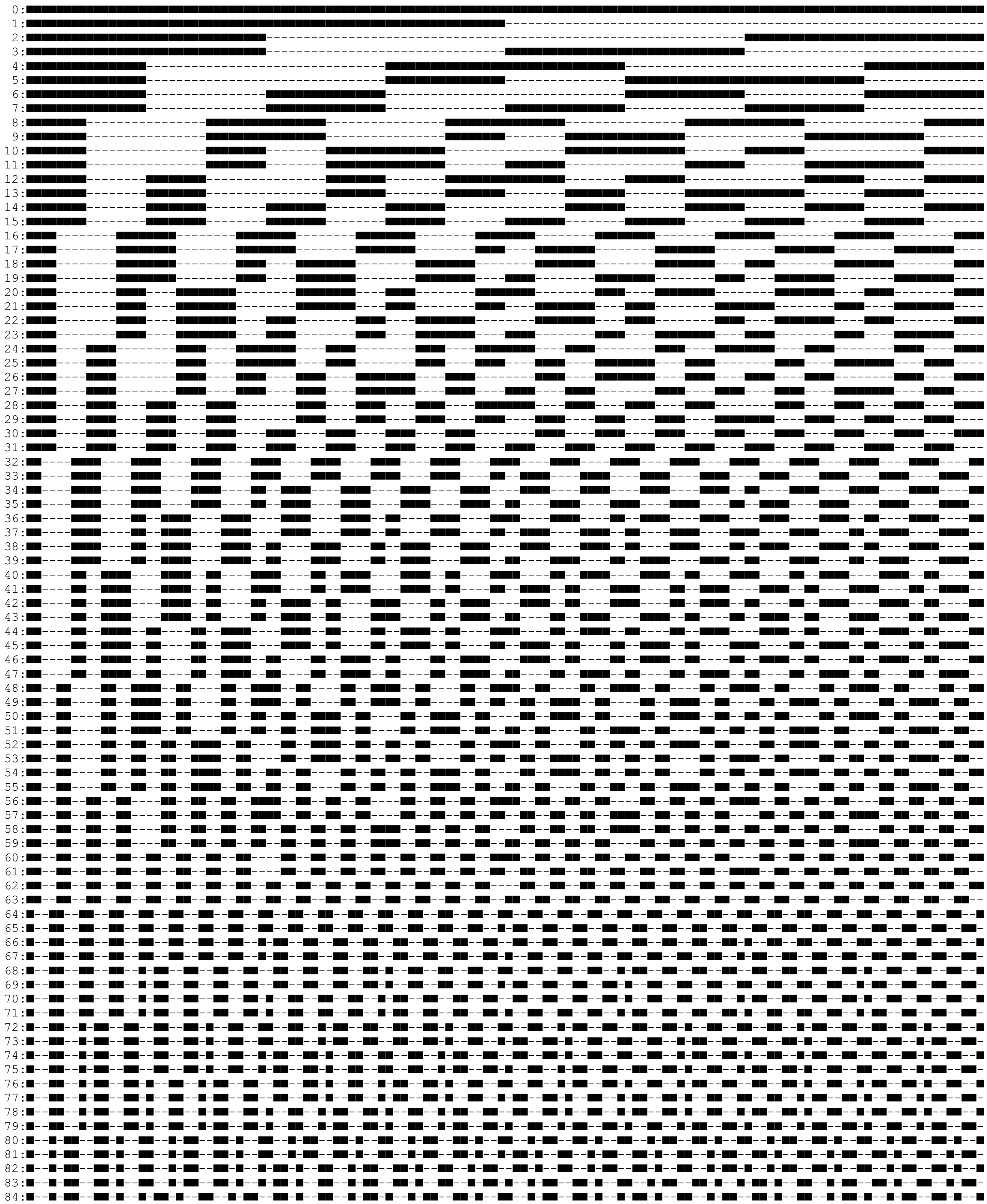
The above table of 128 Walsh functions is in WAL order. It was generated initially using products of Rademacher functions, which leads to natural or Paley order. Those functions were then reordered according to their WAL index, as described by Beauchamp (p.23) using modulo-2 addition and Gray code.

The functions have been depicted here with values *1/0*, rather than *+1/-1*, to save printed space.

Figure 1: ALMA Walsh functions in WAL order.



85:■--■-■■--■■-■--■-■■-■--■■--■-■■--■■-■--■■--■-■■-■--■-■■--■■-■--■■-■--■■-■--■■--■-■■-■--■■--■■-■--■■--■-■■--■■-■--■-■■-■--■■--■-■■-
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90:■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■
91:■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■-■--■-■■-
92:■--■-■■-■--■-■■-■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■-■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■
93:■--■-■■-■--■-■■-■--■-■■-■--■-■■--■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-
94:■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■
95:■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-
96:■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■
97:■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-
98:■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■
99:■-■--■-■■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■■-
100:■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■
101:■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-
102:■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■
103:■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-
104:■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■-■-■--■-■■-■-■-■--■-■
105:■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■■-■-■-■--■-■-■-■■-■--■-■■-■-■-■--■-■■-■-■-■--■-■■-■--■-■-■-■■-■-■-■--■-■-■-■■-
106:■-■--■-■-■-■■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-■--■-■-■-■■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-■--■-■-■-■■-■-■-■--■-■-■-■--■-■
107:■-■--■-■-■-■■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-
108:■-■--■-■-■-■■-■-■-■--■-■-■-■■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■
109:■-■--■-■-■-■■-■-■-■--■-■-■-■■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-
110:■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■
111:■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-
112:■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■
113:■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-
114:■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■■-■-■-■--■-■-■
115:■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-
116:■-■-■-■--■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■
117:■-■-■-■--■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-
118:■-■-■-■--■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■--■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■
119:■-■-■-■--■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-
120:■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■
121:■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-
122:■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■
123:■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■■-■-■-■-■-■-■-
124:■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■--■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■
125:■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-■--■■-■-■-■-■-■-■-■-■-■-■-■-■-■-■-
126:■--■
127:■-

TABLE 2: Conversion between WAL and PAL ordering

Given the WAL index, what is the corresponding PAL index for that function?

```
WAL(  0,t) = PAL(  0,t)
WAL(  1,t) = PAL(  1,t)
WAL(  2,t) = PAL(  3,t)
WAL(  3,t) = PAL(  2,t)
WAL(  4,t) = PAL(  6,t)
WAL(  5,t) = PAL(  7,t)
WAL(  6,t) = PAL(  5,t)
WAL(  7,t) = PAL(  4,t)
WAL(  8,t) = PAL( 12,t)
WAL(  9,t) = PAL( 13,t)
WAL( 10,t) = PAL( 15,t)
WAL( 11,t) = PAL( 14,t)
WAL( 12,t) = PAL( 10,t)
WAL( 13,t) = PAL( 11,t)
WAL( 14,t) = PAL(  9,t)
WAL( 15,t) = PAL(  8,t)
WAL( 16,t) = PAL( 24,t)
WAL( 17,t) = PAL( 25,t)
WAL( 18,t) = PAL( 27,t)
WAL( 19,t) = PAL( 26,t)
WAL( 20,t) = PAL( 30,t)
WAL( 21,t) = PAL( 31,t)
WAL( 22,t) = PAL( 29,t)
WAL( 23,t) = PAL( 28,t)
WAL( 24,t) = PAL( 20,t)
WAL( 25,t) = PAL( 21,t)
WAL( 26,t) = PAL( 23,t)
WAL( 27,t) = PAL( 22,t)
WAL( 28,t) = PAL( 18,t)
WAL( 29,t) = PAL( 19,t)
WAL( 30,t) = PAL( 17,t)
WAL( 31,t) = PAL( 16,t)
WAL( 32,t) = PAL( 48,t)
WAL( 33,t) = PAL( 49,t)
WAL( 34,t) = PAL( 51,t)
WAL( 35,t) = PAL( 50,t)
WAL( 36,t) = PAL( 54,t)
WAL( 37,t) = PAL( 55,t)
WAL( 38,t) = PAL( 53,t)
WAL( 39,t) = PAL( 52,t)
WAL( 40,t) = PAL( 60,t)
WAL( 41,t) = PAL( 61,t)
WAL( 42,t) = PAL( 63,t)
WAL( 43,t) = PAL( 62,t)
WAL( 44,t) = PAL( 58,t)
WAL( 45,t) = PAL( 59,t)
WAL( 46,t) = PAL( 57,t)
WAL( 47,t) = PAL( 56,t)
WAL( 48,t) = PAL( 40,t)
WAL( 49,t) = PAL( 41,t)
WAL( 50,t) = PAL( 43,t)
WAL( 51,t) = PAL( 42,t)
WAL( 52,t) = PAL( 46,t)
WAL( 53,t) = PAL( 47,t)
WAL( 54,t) = PAL( 45,t)
WAL( 55,t) = PAL( 44,t)
WAL( 56,t) = PAL( 36,t)
WAL( 57,t) = PAL( 37,t)
WAL( 58,t) = PAL( 39,t)
WAL( 59,t) = PAL( 38,t)
WAL( 60,t) = PAL( 34,t)
WAL( 61,t) = PAL( 35,t)
WAL( 62,t) = PAL( 33,t)
WAL( 63,t) = PAL( 32,t)
WAL( 64,t) = PAL( 96,t)
WAL( 65,t) = PAL( 97,t)
WAL( 66,t) = PAL( 99,t)
WAL( 67,t) = PAL( 98,t)
WAL( 68,t) = PAL(102,t)
WAL( 69,t) = PAL(103,t)
WAL( 70,t) = PAL(101,t)
WAL( 71,t) = PAL(100,t)
WAL( 72,t) = PAL(108,t)
WAL( 73,t) = PAL(109,t)
WAL( 74,t) = PAL(111,t)
WAL( 75,t) = PAL(110,t)
WAL( 76,t) = PAL(106,t)
WAL( 77,t) = PAL(107,t)
WAL( 78,t) = PAL(105,t)
WAL( 79,t) = PAL(104,t)
WAL( 80,t) = PAL(120,t)
WAL( 81,t) = PAL(121,t)
WAL( 82,t) = PAL(123,t)
WAL( 83,t) = PAL(122,t)
WAL( 84,t) = PAL(126,t)
WAL( 85,t) = PAL(127,t)
WAL( 86,t) = PAL(125,t)
WAL( 87,t) = PAL(124,t)
WAL( 88,t) = PAL(116,t)
WAL( 89,t) = PAL(117,t)
WAL( 90,t) = PAL(119,t)
WAL( 91,t) = PAL(118,t)
WAL( 92,t) = PAL(114,t)
WAL( 93,t) = PAL(115,t)
WAL( 94,t) = PAL(113,t)
WAL( 95,t) = PAL(112,t)
WAL( 96,t) = PAL( 80,t)
WAL( 97,t) = PAL( 81,t)
WAL( 98,t) = PAL( 83,t)
WAL( 99,t) = PAL( 82,t)
WAL(100,t) = PAL( 86,t)
WAL(101,t) = PAL( 87,t)
WAL(102,t) = PAL( 85,t)
WAL(103,t) = PAL( 84,t)
WAL(104,t) = PAL( 92,t)
WAL(105,t) = PAL( 93,t)
WAL(106,t) = PAL( 95,t)
WAL(107,t) = PAL( 94,t)
WAL(108,t) = PAL( 90,t)
WAL(109,t) = PAL( 91,t)
WAL(110,t) = PAL( 89,t)
WAL(111,t) = PAL( 88,t)
WAL(112,t) = PAL( 72,t)
WAL(113,t) = PAL( 73,t)
WAL(114,t) = PAL( 75,t)
WAL(115,t) = PAL( 74,t)
WAL(116,t) = PAL( 78,t)
WAL(117,t) = PAL( 79,t)
WAL(118,t) = PAL( 77,t)
WAL(119,t) = PAL( 76,t)
WAL(120,t) = PAL( 68,t)
WAL(121,t) = PAL( 69,t)
WAL(122,t) = PAL( 71,t)
WAL(123,t) = PAL( 70,t)
WAL(124,t) = PAL( 66,t)
WAL(125,t) = PAL( 67,t)
WAL(126,t) = PAL( 65,t)
WAL(127,t) = PAL( 64,t)
```

APPENDIX

The Optimum Choice of Walsh Functions to Minimize Drift and Cross-Talk

Array Design Study
Working Report Nr 127 D4

D.T.Emerson,

IRAM, Grenoble

July 18th 1983

Array Design Study
Working Report Nr. 127
D4

THE OPTIMUM CHOICE OF WALSH FUNCTIONS, TO MINIMIZE DRIFT AND CROSS-TALK

D.T. Emerson,
July 18th 1983.

For the interferometer electronics, we are using Walsh function modulation as a means of avoiding cross-talk between different parts of the system, and for extracting different component signals (e.g. cosine/sine) from the correlations of various antenna pairs. Walsh functions form an orthogonal set, and for time-invariant signals or offsets there is theoretically perfect isolation between the different Walsh-modulated signals. However, (e.g. see note of April 1982) for time-varying signals or time-dependent offsets, modulation by orthogonal Walsh functions no longer gives perfect isolation between the different components.

The degree of isolation to be expected from different combinations of Walsh-function modulation is considered here. The parts of the interferometer system where cross-talk or inadequate interference rejection are most serious should make use of the particular Walsh-function sub-set that gives the best protection. As shown below, the isolation to be expected may vary by MANY ORDERS OF MAGNITUDE. In all cases the Walsh functions are assumed to be perfect (no timing errors, etc.) . The residual drift component and the cross-talk are due only to the time-variance of the signals.

Sequency, Natural Order, and Rademacher functions

Walsh functions are commonly ordered according to either their "Natural order", or their "Sequency" (see e.g. "Walsh Functions and their Applications, by K.G. Beauchamp, pp 8 - 19). Sequency ordering corresponds to the mean frequency of zero-crossings of the set of Walsh functions. This ordering gives CAL and SAL functions, a close analogy to Fourier cosine and sine pairs, and the Walsh transform using this ordering is closely analogous to the Fourier transform.

A complete set of Walsh functions may be generated by taking products of Rademacher functions; the Rademacher function R(n,T) is simply a square-wave of half-period T/(2**n), with 2**(n-1) complete cycles of the square wave in the time T . R(0,T) is simply a constant (d.c.) term. The Walsh function PAL(n,T) generated by the product of R(i,T),R(j,T) ..... R(m,T) has a Natural order number of n= 2**(i-1)+2**(j-1)+ ..... 2**(m-1) . i.e., the component Rademacher functions correspond to the binary representation of the Natural order number. The product of two (or more) Walsh functions yields another Walsh function, whose Natural order number is given by modulo-two addition (i.e. no-carry addition of the binary Natural order numbers) of the component Walsh functions .

For all IRAM applications, I suggest that NATURAL ORDER (also known as PALey order) is more relevant than Sequency. Calculation of the product of functions, estimation of the degree of cross-talk, and even hardware generation of the functions, is all easier in Natural order. The conveniences of a Sequency representation are not relevant in the current IRAM context.

For reference, the table below shows the relation between Natural order, the component Rademacher functions and the Sequency for the first 15 Walsh functions. Multiplication by PAL(0,T) or by R(0,T) is equivalent to multiplication by unity - essentially a null operation.

TABLE I

Natural order	Rademacher products					Sequency
	R(4,T)	R(3,T)	R(2,T)	R(1,T)	R(0,T)	
PAL(0,T)					1	WAL(0,T)
PAL(1,T)				1		WAL(1,T)
PAL(2,T)			1			WAL(3,T)
PAL(3,T)			1	1		WAL(2,T)
PAL(4,T)		1				WAL(7,T)
PAL(5,T)		1		1		WAL(6,T)
PAL(6,T)		1	1			WAL(4,T)
PAL(7,T)		1	1	1		WAL(5,T)
PAL(8,T)	1					WAL(15,T)
PAL(9,T)	1			1		WAL(14,T)
PAL(10,T)	1		1			WAL(12,T)
PAL(11,T)	1		1	1		WAL(13,T)
PAL(12,T)	1	1				WAL(8,T)
PAL(13,T)	1	1		1		WAL(9,T)
PAL(14,T)	1	1	1			WAL(11,T)
PAL(15,T)	1	1	1	1		WAL(10,T)

The product of any Walsh or Rademacher function with itself gives unity. (Note that the Rademacher functions form a subset of Walsh functions.) As an example, the product of Walsh functions PAL(7,T) and PAL(10,T) is equivalent to:

R(3,T)*R(2,T)*R(1,T) * R(4,T)*R(2,T)

i.e.

R(4,T)*R(3,T)*R(2,T)*R(2,T)*R(1,T)

or:

R(4,T)*R(3,T)*R(1,T)

which is seen from the above table to correspond to PAL(13,T)

The same operation is performed by modulo-two binary addition:

PAL(7,T)	0111
PAL(10,T)	1010
Modulo-2 addition:	1101

i.e. PAL(13,T)

Rejection of Time-dependent Interference or Drifts

Consider the following model:

```
              |                     |
     D(t)  V                     |  PAL(n,T)
              |                     V
              |                     |
              |                    ---                ---                      S+X(D)
 ----->----------->---------| X |------->-----|lpf|------------------->-----
   S.PAL(n,T)                      ---                ---
                                                low-pass filter
                                             (integrates over time T)
```

A Walsh-modulated signal S.PAL(n,T) has some time-varying interference or drift D(t) super-posed on it before de-modulation. Multiplying the voltage (S.PAL(n,T) + D(t)) by the demodulating Walsh function PAL(n,T) gives S + D(t).PAL(n,T), or
S + X(D)
where S is the desired signal, and X(D) the unwanted cross-talk or residual component of drift or interference. Because of the time-variance of D(t), the product D(t).PAL(n,T) does not in general average to zero. (An essential part of complete Walsh demodulation is the summation or integration over period T after the multiplication, to remove the (high-frequency) fluctuations due to the presence of orthogonal walsh-functions. For simplicity this integration, common to any form of synchronous separation of orthogonal signals, is not shown in the following diagrams.)

The above model may be replaced by:

```
              |                |            |                          |
     D(t)  V               |R(i,T)      |R(j,T)     ......         |R(k,T)
              |                V            V                          V
              |                |            |                          |
              |               ---   D1     ---   D2                   ---   Dm=S + X(D)
 ----->----------->----| X |--->--| X |-->- .....-->--| X |-->------
   S.PAL(n,T)                 ---   (B)    ---   (C)                  ---
```

where demodulation (i.e. multiplication) by the Walsh function PAL(n,T) has been replaced by successive multiplications by the m component Rademacher functions (R(i,T),R(j,T), ... R(k,T)) from which PAL(n,T) is derived (see for example the Table above). The residual cross-talk or interference D1 and D2 at points (B), (C) etc. may now be derived.

The mean residual spurious voltage D1 at (B) in one cycle of the square wave of the Rademacher function R(i,T) is given by:

$$D1 = \frac{D(t+deltat) - D(t)}{2}$$

where D(t) is the mean of the interfering signal for the first half of the square-wave cycle, and D(t+deltat) is the mean of the interfering signal

for the second half-cycle. The time-difference of samples is the half-cycle time of the square-wave, which is (see above) T/(2**i) .
Making the usual approximations, for interfering signals which are varying slowly compared with the time T, the residual interference may be written as:

$$D1 = \frac{\left(D(t) + \text{deltat} . \frac{dD}{dt} \right) - D(t)}{2}$$

i.e.

$$D1 = \frac{\text{deltat}}{2} . \frac{dD}{dt}$$

or:

$$D1 = \frac{1}{2} . \frac{T}{(2**i)} . \frac{dD}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This mean spurious residual is calculated for one cycle of the 2**(i-1) square-wave cycles in the complete Rademacher cycle time T . Within the approximation stated above (i.e. signals changing slowly w.r.t. time T) this mean spurious signal will be the same throughout the time T. i.e. the above expression represents the mean residual signal at point (B), after multiplication by R(i,T) .

After the second multiplication, by R(j,T) , the residual spurious signal D2 at point (C) is given by the right-hand-side of expression (1) , but with D1 replacing D and R(j,T) replacing R(i,T).

i.e. :

$$D2 = \frac{1}{2} . \frac{T}{(2**j)} . \frac{d(D1)}{dt}$$

or, substituting for D1:

$$D2 = \frac{1}{2} . \frac{T}{(2**j)} . \frac{d}{dt} \left(\frac{1}{2} . \frac{T}{(2**i)} . \frac{dD}{dt} \right)$$

Similarly, after being multiplied by the m Rademacher functions R(i,T),R(j,T) .... R(k,T) corresponding to the appropriate Walsh function PAL(n,T) (e.g. from TABLE I above) the residual spurious response Dm will be given by:

$$Dm = X(D) = \left(\frac{T}{2}\right)^{**m} . \frac{1}{2**(i+j...+k)} . \frac{d^m D}{d D^m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This is the level of residual spurious signal after demodulation, where interference or drift D(t) is added to a Walsh-modulated signal S.PAL(n,T) . This expression may also be used to derive the cross-talk between two Walsh-modulated signals S1.PAL(p,T) and S2.PAL(q,T) .

e.g. the following system:

```
                               |
                               | PAL(p,T)
                               V
                               |
                              ---           S1
-----}--------->-------| X |------>------
  S1.PAL(p,T)       |     ---
+ S2.PAL(q,T)       |
                    V
                    |
                    |         ---           S2
                    ----| X |------>------
                              ---
                               |
                               |
                               ^
                               | PAL(q,T)
                               |
```

is exactly equivalent to:

```
                         |                        |
                         |PAL(p,T)                |PAL(r,T)
                         V                        V
  S1.PAL(p,T)            |                        |
+ S2.PAL(q,T)           ---   S1+S2.PAL(r,T)     ---
----->--------->----| X |-------->--------| X |-->----- S2+X(S1)
                        ---          |           ---
                                     V
                                     |
                                     |
                                      -------->---------- S1
```

where the Walsh function PAL(r,T) = PAL(p,T) . PAL(q,T) . This may easily be verified by considering the component Rademacher functions, such as shown in TABLE I . (As above, the summation or integration over the time T which is required to remove the component S2.PAL(r,T) to leave S1 alone is not shown.) The cross-talk X(S1) of some component of S1 interfering with the S2 signal is given by the expression (2) above, substituting D=S1 and the m Rademacher components i, j, ... k of PAL(r,T), = PAL(p,T).PAL(q,T) .

Some Practical Results

TABLE II shows the degree of immunity to drift D in the system, or the degree of cross-talk expected between time-varying signals. The degree of drift or cross-talk rejection in this example varies from approximately 1 % for PAL(1,T) to about 10E-13 for PAL(31,T) . The high ultimate rejection indicated in many cases will be degraded in practice by imperfections of the Walsh waveform (e.g. finite switching times). However, it is in general to be expected that the degree of rejection will be highest for those Walsh functions which are the products of the highest number of component Rademacher functions - e.g. PAL(n,T) with n= 31,15,23,27,29, or 30 will always be excellent, while n= 1,2,4,8 or 16 should be avoided in critical applications.

TABLE II below shows:

Column i, the Walsh function (in Natural order) PAL(n,T)

Column ii, the residual of drift or interference D(t) which remains after the demodulation by PAL(n,T)

Column iii, the numerical amplitude of crosstalk after the signal
S1.PAL(p,T) + S2.PAL(q,T)
has been demodulated to give separately S1 and S2.
S1 amd S2 are both assumed to be sinusoids, with frequencies of order .006 Hz, and the characteristic Walsh-function period T is 1 second. This is typical of the signals to be expected from the IRAM correlator (e.g. see Working Report Nr. 106).
In this case, PAL(n,T) in Column i corresponds to the Walsh function given by the product of PAL(p,T) and PAL(q,T) ;
i.e. n= modulo-two binary addition of p and q .

TABLE II

Walsh function (Natural order)	Residual component of unwanted drift D	Cross-talk amplitude from sine signal with f=0.006 Hz, T=1.0 sec.	Walsh function (Natural order)	Residual component of unwanted drift D	Cross-talk amplitude from sine signal with f=0.006 Hz, T=1.0 sec.
PAL(0,T)	D	1.	PAL(16,T)	$\frac{T}{64}\cdot\frac{dD}{dt}$	5.9E-04
PAL(1,T)	$\frac{T}{4}\cdot\frac{dD}{dt}$	9.4E-03	PAL(17,T)	$\frac{T^2}{256}\cdot\frac{d^2D}{dt^2}$	5.6E-06
PAL(2,T)	$\frac{T}{8}\cdot\frac{dD}{dt}$	4.7E-03	PAL(18,T)	$\frac{T^2}{512}\cdot\frac{d^2D}{dt^2}$	2.8E-06
PAL(3,T)	$\frac{T^2}{32}\cdot\frac{d^2D}{dt^2}$	4.4E-05	PAL(19,T)	$\frac{T^3}{2048}\cdot\frac{d^3D}{dt^3}$	2.6E-08
PAL(4,T)	$\frac{T}{16}\cdot\frac{dD}{dt}$	2.4E-03	PAL(20,T)	$\frac{T^2}{1024}\cdot\frac{d^2D}{dt^2}$	1.4E-06
PAL(5,T)	$\frac{T^2}{64}\cdot\frac{d^2D}{dt^2}$	2.2E-05	PAL(21,T)	$\frac{T^3}{4096}\cdot\frac{d^3D}{dt^3}$	1.3E-08
PAL(6,T)	$\frac{T^2}{128}\cdot\frac{d^2D}{dt^2}$	1.1E-05	PAL(22,T)	$\frac{T^3}{8192}\cdot\frac{d^3D}{dt^3}$	6.5E-09
PAL(7,T)	$\frac{T^3}{512}\cdot\frac{d^3D}{dt^3}$	1.0E-07	PAL(23,T)	$\frac{T^4}{32768}\cdot\frac{d^4D}{dt^4}$	6.2E-11
PAL(8,T)	$\frac{T}{32}\cdot\frac{dD}{dt}$	1.2E-03	PAL(24,T)	$\frac{T^2}{2048}\cdot\frac{d^2D}{dt^2}$	6.9E-07
PAL(9,T)	$\frac{T^2}{128}\cdot\frac{d^2D}{dt^2}$	1.1E-05	PAL(25,T)	$\frac{T^3}{8192}\cdot\frac{d^3D}{dt^3}$	6.5E-09
PAL(10,T)	$\frac{T^2}{256}\cdot\frac{d^2D}{dt^2}$	5.6E-06	PAL(26,T)	$\frac{T^3}{16384}\cdot\frac{d^3D}{dt^3}$	3.3E-09
PAL(11,T)	$\frac{T^3}{1024}\cdot\frac{d^3D}{dt^3}$	5.2E-08	PAL(27,T)	$\frac{T^4}{65536}\cdot\frac{d^4D}{dt^4}$	3.1E-11
PAL(12,T)	$\frac{T^2}{512}\cdot\frac{d^2D}{dt^2}$	2.8E-06	PAL(28,T)	$\frac{T^3}{32768}\cdot\frac{d^3D}{dt^3}$	1.6E-09
PAL(13,T)	$\frac{T^3}{2048}\cdot\frac{d^3D}{dt^3}$	2.6E-08	PAL(29,T)	$\frac{T^4}{131072}\cdot\frac{d^4D}{dt^4}$	1.5E-11
PAL(14,T)	$\frac{T^3}{4096}\cdot\frac{d^3D}{dt^3}$	1.3E-08	PAL(30,T)	$\frac{T^4}{262144}\cdot\frac{d^4D}{dt^4}$	7.7E-12
PAL(15,T)	$\frac{T^4}{16384}\cdot\frac{d^4D}{dt^4}$	1.2E-10	PAL(31,T)	$\frac{T^5}{1048576}\cdot\frac{d^5D}{dt^5}$	7.3E-14

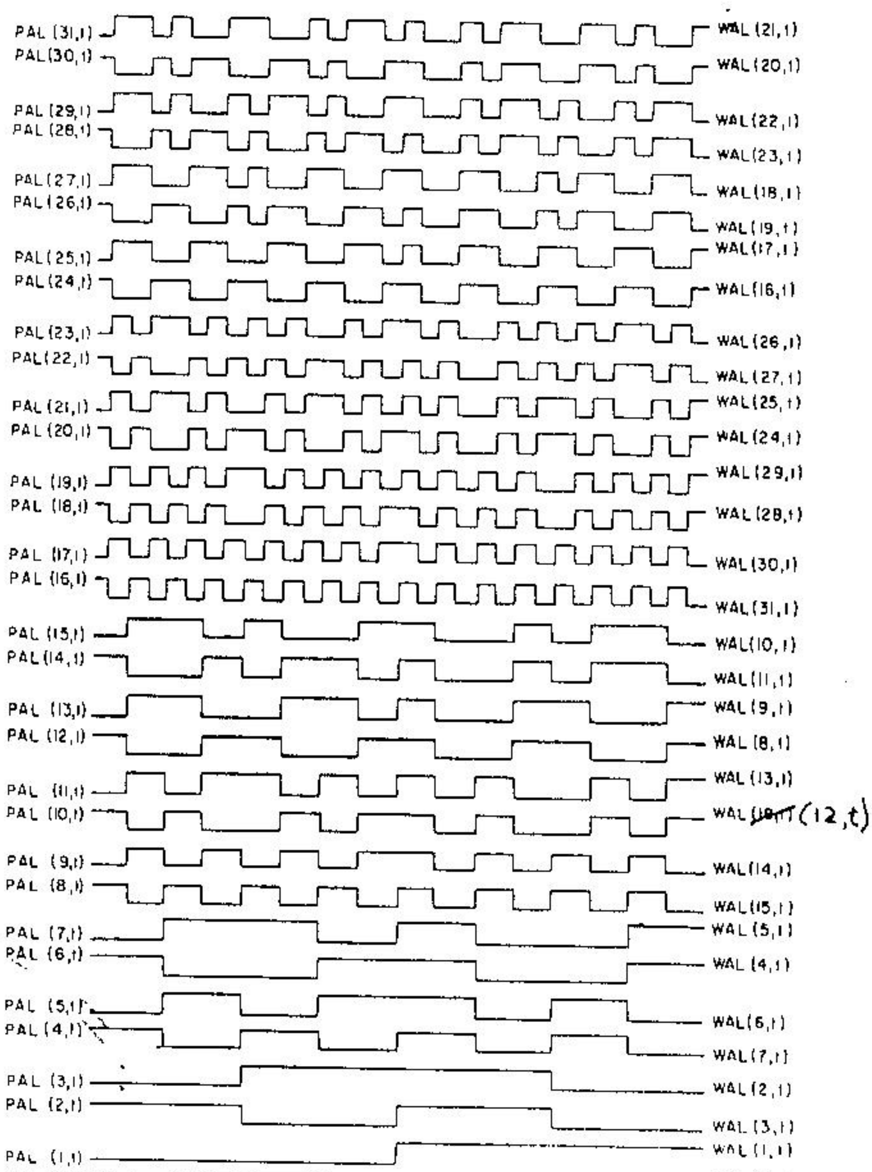

FIG. 2.2. A set of Walsh functions arranged in Natural order.