ALMA MEMO 575

FLUX CONCENTRATION DURING SOLAR OBSERVATION FOR ALMA ANTENNAS

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ABSTRACT

This memo describes the calculation of the incident heat flux on the subreflector surface of an ALMA antenna when the bidirectional reflectance distribution function (BRDF) of the main reflector panels is known. Theory and related formulae are provided for the calculation. A convolutional approach is used in this memo. When the FWHM of the bidirectional scattering distribution function (BSDF) of the primary panel surface is large, the flux on the subreflector during solar observation is relatively small. The memo provides the flux numbers and the flux intensity on the subreflector.

1. INTRODUCTION

In the ALMA subreflector and nutator design, one important problem is the heat flux input into the subreflector surface when the antenna is in the solar observation mode. This flux value has an important effect on the subreflector and nutator structure design and analysis. In this memo, the flux input into the subreflector surface is calculated for reference.

The sun is a heat source when a solar observation is performed. It is measured by satellite to be roughly 1366 watts per square meter, though it fluctuates by about 6.9% during a year - from 1412 W/m^2 in early January to 1321 W/m^2 in early July, due to the earth's varying distance from the sun, and by a few parts per thousand from day to day. The flux of the sun on a high mountain site we assumed is about 1290 W/m^2. If the primary dish has a mirror-like surface, the flux focused by the primary reflector will burn the subreflector in seconds. Fortunately, all the panels of ALMA antennas have their surface specially treated through chemical etching so that they diffuse the incident flux into a wide angular range.

2. BRDF AND SURFACE REFLECTANCE

The directional reflectivity of a surface is described by a Bidirectional Reflectance Distribution Function (BRDF)[2],[3]. The expression for the BRDF is:

$$BRDF = \frac{dE_s / d\Omega_s}{E_i \cos\theta_s} \approx \frac{E_s / \Omega_s}{E_i \cos\theta_s} \tag{1}$$

where E_s is the radiation over an angular area Ω_s with a reflecting angle of θ_s and E_i is the surface total irradiation at a point. BRDF is a ratio between radiation of a unit angular area and irradiation weighted with the cosine of the radiation defusing angle. The BRDF of the panel is measured from transmitted radiant flux by the incident flux at a surface. The incident flux is at positive normal direction with an angle of +0º and the reflecting radiation received in front of the specular direction is called front scattering, having a positive angular sign and that with a minus angular sign is on the back side and is called back scattering. The scattering BRDF at 0º angle corresponds to the BRDF at the specular reflection angle if the incident irradiation is not at the normal direction. The Bidirectional Scattering distribution function is the product of the BRDF and the cosine of the polar angle:

$$BSDF(\theta,\varphi) = BRDF(\theta,\varphi)\cos\theta \tag{2}$$

An important property of the BSDF is that the surface integral of all outgoing BSDF must be less than or equal to unity:

$$\int_\Omega BSDF \cdot d\Omega = \int_\Omega BRDF \cos\theta \cdot d\Omega = \iint BRDF \cos\theta \sin\theta \cdot d\theta \cdot d\phi \le 1 \tag{3}$$

The value of this integral represents the surface total reflectance ratio.

3. FROM BRDF TO TOTAL REFLECTANCE

A 2006 report [1] provides BRDF data of one type of the ALMA antenna panels. The measured BRDF for 500 nm wavelength in the meridian plane is shown is Figure 1(a). From the figure it is seen that the BRDF is asymmetric about the y axis. The front scattering at positive incidence angle is a little larger than the back scattering. Since the measured BRDF function $BRDF_{slice}$ is on the meridian plane only, we reconstruct the 2-D BRDF function according to the following formula:

$$BRDF(\theta,\varphi) = \cos^2(\frac{\varphi}{2}) BRDF_{slice}(\theta) + \sin^2(\frac{\varphi}{2}) BRDF_{slice}(-\theta) \tag{4}$$

From the 2-D BRDF function, the BRDF on the perpendicular plane is shown in Figure 1(b). The product of BRDF and the cosine of polar angle is the bidirectional scattering directional function (BSDF).

$$BSDF(\theta,\varphi) = BRDF(\theta,\varphi)\cos\theta \tag{5}$$

Figure 2 shows the two dimensional BSDF function of the panel surface. The FWHM of the BSDF is 44.56º in meridian plane and 44.916 º in other direction for 500 nm wavelength. The peak value of the BSDF function is 1.1035.

The FWHM of the BSDF is 42.31º in meridian plane and 42.76º in other direction for 900 nm wavelength. The peak value of the BSDF is 1.1933. The integral of the BSDF function in the half sphere area produces the total reflectance of the panel surface:

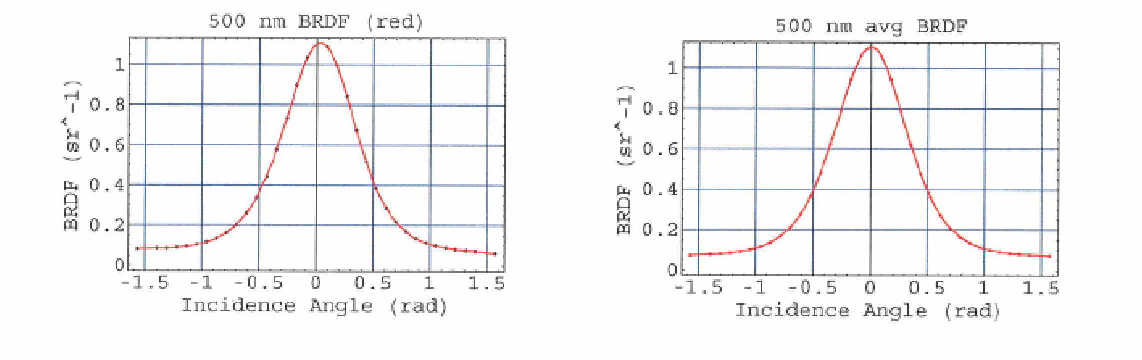

Figure1. (a) Measured BRDF function in the meridian plane and (b) the average BRDF value for the perpendicular plane

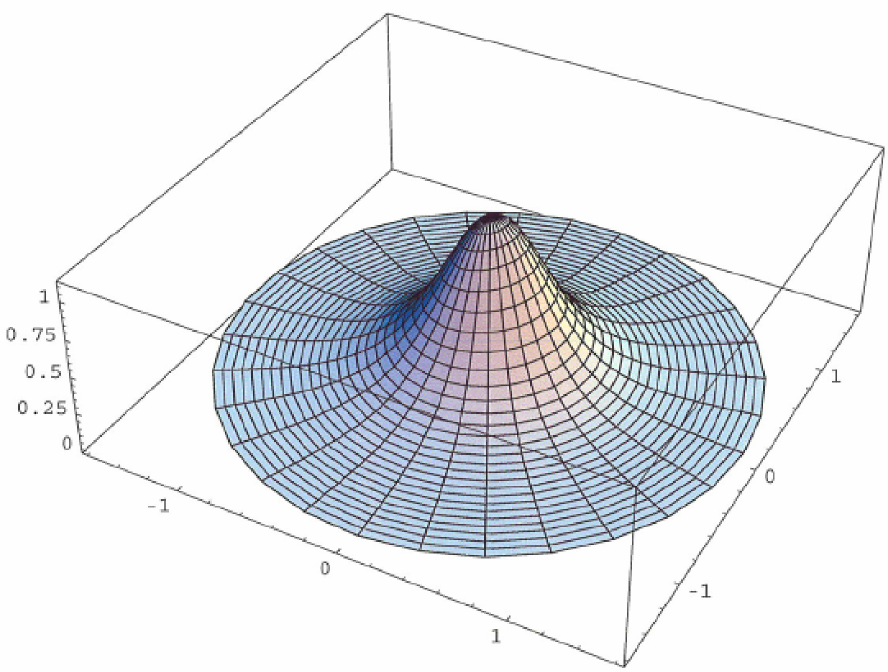

Figure 2. Assumed 2-D form of the BSDF function of the primary panels.

$$\int_{\Omega} BSDF \cdot d\Omega = \int_{\Omega} BRDF \cos\theta \cdot d\Omega = \iint BRDF \cos\theta \sin\theta \cdot d\theta \cdot d\phi \qquad (6)$$

The total reflectance ratio of the dish surface for the 500nm wavelength is 0.871151. The 12 m ALMA antenna has a secondary mirror of 0.75 m diameter. The solar flux is 1290 W/m^2.

The projected area of the primary dish is 112.655 m^2. The incident flux on the dish panels is 145,325 W. The total reflected flux is then 126,600 W for wavelength of 500nm. The total reflectance calculated for the 900 nm wavelength is 0.874055. The total reflected flux at this wavelength is 127,022 W.

4. ANGULAR AREA OF THE SUBREFLECTOR

Figure 3 shows the geometry of the antenna reflectors. The unit used is meters. From different reflecting positions, the subreflector will occupy a different angular area as shown in Figure 4. The unit used in the figure is degrees. The coordinate center corresponds to the specular direction in the view projection. The largest circle represents the cone view projection at the center vertex point of the primary dish. Since the vertex point and the edge of the subreflector form a symmetrical conical shape, the area projection is an exact circle. As the view point moves away from the center, the specular reflection point shifts to one side of the subreflector. The area of the front scattering side becomes larger than that of back scattering side. Because both the distance, between the view point on the parabola and the edge of the subreflector, and the angle, between the subreflector and specular reflection direction, becomes larger, the major and minor radii of the projection ellipse also gradually become smaller. The smallest ellipse represents the cone view projection from the edge point of the primary dish. The asymmetry of this ellipse includes a little bit shading effect from the convex surface. At this view point, there is no back scattering on the subreflector surface.

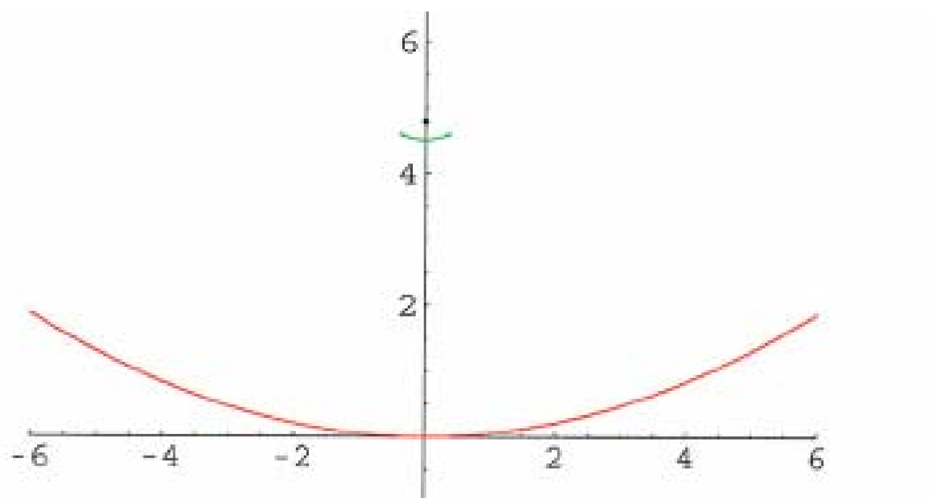

Figure 3. The geometry of the reflectors of ALMA telescopes

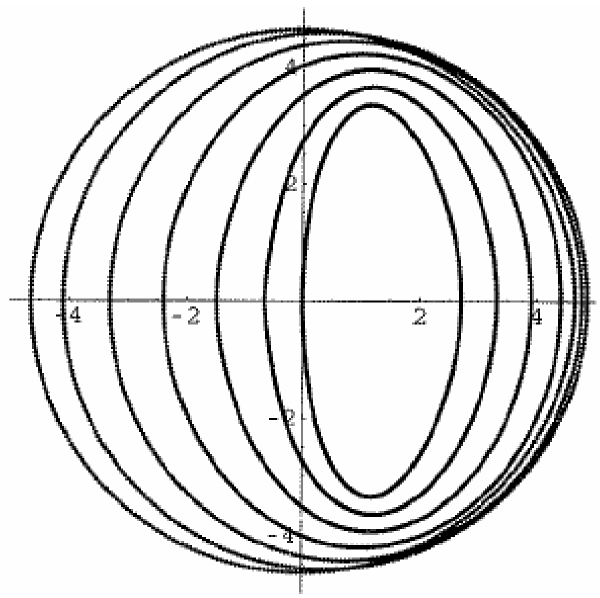

Figure 4. The projected cone views of the subreflector from the different positions on the primary reflector.

5. CONVOLUTION OF ANGULAR AREA AND THE BRDF

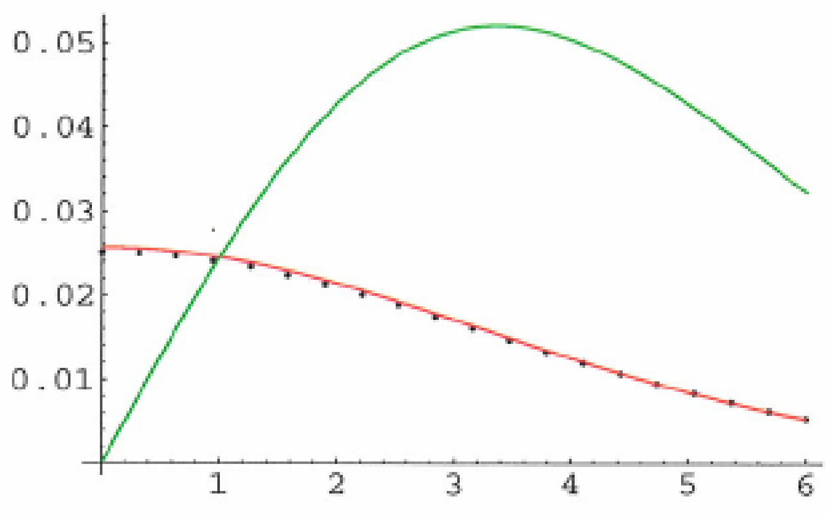

Figure 5. The convolution between the projected cone area and the BSDF function (red line) and the area weighted convolution (green line) for 500 nm wavelength.

For deriving the scattered flux ratio on the subreflector surface, both ray tracing and convolution methods can used. The ray tracing method requires dense rays around the specular direction to fill the projected subreflector area. The results calculated provide the flux density on the subreflector but difficulties exist in interpreting the rays' influence area when the incidence angle is large on the subreflector. In this memo, a convolution is made between the projected cone area and the BSDF function to provide the total flux on the subreflector. This method does not provide the flux distribution contour. The convolutional method does not require any approximation. The diffusivity function along the primary radius is then:

$$DIFF(r) = \frac{BSDF(\theta,\varphi) ** cone(r,\theta,\varphi)}{\int_0^{2\pi}\int_0^{\pi/2} BSDF(\theta,\varphi)\sin\theta d\theta d\varphi} = \frac{\iint_{cone(r,\theta,\varphi)} BRDF(\theta,\varphi)\cos\theta\sin\theta d\theta d\varphi}{\int_0^{2\pi}\int_0^{\pi/2} BRDF(\theta,\varphi)\cos\theta\sin\theta d\theta d\varphi} \qquad (7)$$

This diffussivity function DIFF (r) is shown as red line in Figure 5. It is the diffusivity as a function of radius on the primary dish. However, the solar flux area on the radius of the primary reflector is not a constant. To get the diffusivity for the subreflector, an area weighted diffusivity is necessary:

$$\frac{\int_0^{2\pi}\int_{3/8}^{6} DIFF(r) r dr d\psi}{\int_0^{2\pi}\int_{3/8}^{6} r dr d\psi} = 0.012793 \qquad (8)$$

The area weighted diffusivity is shown in a green line in Figure 5. The area weighted diffusivity for 500 nm wavelength is 0.01293. From the total reflected flux of the primary reflector, the total flux on the subreflector is 1,620 W from the area weighted diffusivity for 500 nm wavelength. The subreflector projected area is 0.441786 m^2, so that the flux intensity on the subreflector is 3,666.92 W/m^2 for 500 nm wavelength.

The red and green lines of Figure 6 are the diffusivity function with the radius on the primary reflector and the area weighted diffusivity for the 900 nm wavelength. The average area weighted diffusivity is 0.0138034 and the total flux on the subreflector is 1753.33 W and the flux intensity is 3968.74 W/m^2.

The solar spectrum covers a wider range from 250 nm to almost 2,500 nm. The spectrum peak is at about 550 nm. The 500 nm height is about 95 % from the peak of the spectrum. The 900 nm height is about the half of the spectrum peak. Most of the solar heat can be represented by the average flux of the 500 nm and 900 nm.

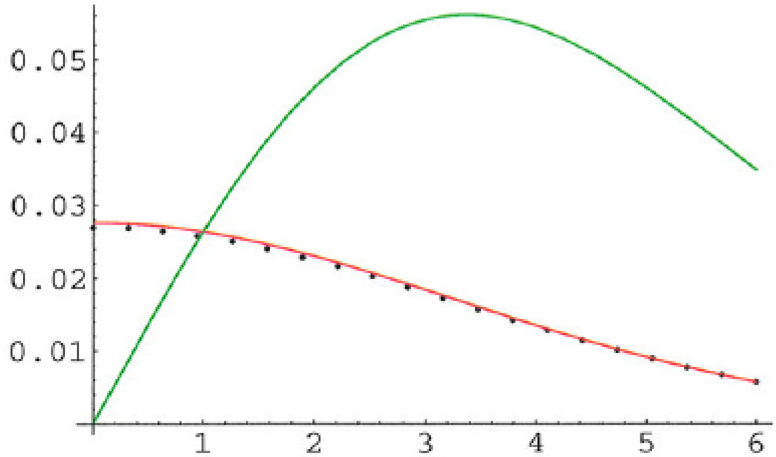

Figure 6. The convolution between the projected cone area and the BSDF function (red line) and the area weighted convolution (green line) for 900 nm wavelength.

6. CONCLUSION

For millimeter-wavelength solar observations the diffusivity of the primary panel surface is a very important concern. For without special treatment, the near specular reflection from the primary surface would focus almost all the solar heat onto the subreflector surface, which is small in area. Near specular reflection could produce a flux intensity approaching 300,000 W/m^2. This high heat flow could burn out the subreflector, as well as its support structure. However, with surface roughening achieved via chemical etching or other special treatment, the flux density on the subreflector surface is much lower, perhaps as low as one-percent of the untreated value, as shown in Section 5, hence structural safety is assured. What motivated the detailed analysis presented here was an evident discrepancy in parts of the contractor's engineering calculations, which we sought to correct.

The detailed computational analysis described in this memo was carried out within the Mathematica programming environment. Mathematica is a product of Wolfram Research, Inc.

Reference:

[1] NASA report: bidirectional reflective distribution function of ALMA project coupon, Aug 30, 2006.

[2]Nicodemus, F. E. et al., Geometrical considerations and nomenclature for reflectance, US Dept of Comm., NBS, 1977.

[3]Nicodemus, F. E., Directional reflectance and emissivity of an opaque surface, Applied optics Vol 4, p767, 1965.

[3] Bennet, J. M. and Mattsson, L., Introduction to surface roughness and scattering, 2nd edition, Optical Society of America, Washington DC, 1999.