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Frequency profile difference between ACA Correlator and 64-Antenna Correlator

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Abstract

ACA Correlator and 64-Antenna Correlator adopt FX and XF architecture, respectively. This difference introduces different frequency responses between them. However, for combined imaging of the ACA and the 12m-Array data, their frequency profile compatibility is required, and one possible method is the convolution of weighting function in the frequency domain.

We have studied this method and found that “a XF frequency response convolved with Hanning window function” is well synthesized by linear combination of “twelve FX frequency responses”, when one frequency channel width of the 64-Antenna Correlator is two times as large as that of the ACA Correlator. Then, we have calculated frequency profiles by convolving synthesized and original frequency responses with several object profiles. The maximum profile difference between them is estimated to be small by 0.6 % or less except for the case of δ function as an object profile.

1. Frequency response difference between FX and XF correlators

The shape of frequency response is different between ACA Correlator and 64-Antenna Correlator because of their architectures. The ACA Correlator adopts FX method and, on the other hand, XF method is adopted by the 64-Antenna Correlator. The calculation order of “F” (FFT) and “X” (multiplication for the FX method, correlation for the XF method) is opposite between the FX and XF methods as shown in Figure 1. A FX correlator calculates FFT of data from each antenna, and then, performs multiplications between their FFT results. A XF correlator calculates correlation between data from each antenna, and then, performs FFT of their correlation results. In addition, we must pick up data with finite length at the FFT operation. This indicates multiplying the data by rectangular window function, which corresponds to convolving sinc function in the frequency domain. Thus, the frequency response of a XF correlator becomes the shape of sinc function profile. On the other hand, that of a FX correlator is sinc squared function profile, because FFT results are multiplied in “X” part. Hereafter, we use words “FX correlator” and “XF correlator” instead of

"ACA Correlator" and "64-Antenna Correlator", because their response difference originates from their different architectures.

This difference of the frequency responses might cause the difference of frequency profiles and spectral images between ACA and 12m-Array for the same spatial frequency data. Following options are considered to minimize the difference between the two frequency main-lobe/side-lobe response curves of individual frequency channels:

(a) Multiplication of window function before FFT in the time domain.

(b) Multiplication of window function after correlation, temporal integration and inverse FFT in the lag domain.

(c) Convolution of weighting function after correlation in the frequency domain.

For the option (a), this option loses half portion of signal data if the frequency resolution is not the highest resolution. The equivalent of box-car windowing in the lag domain is not realized in the option (a). The option (b) is always possible with adequate computation power, provided that required frequency resolution is sufficiently lower than the inverse of FFT segment length. The option (c) is also supported by the present ACA Correlator hardware design by weighted binning function. This option is valid for the case that required frequency resolution is sufficiently lower than the inverse of FFT segment length, but it does not require high data transfer rate from the correlator to the computing and does not require computation resources compared to the option (b) in the computing.

Therefore, we have studied the case (c) and report the results in this memo.

2. Estimation method of the frequency profile differences

The estimation of the frequency profile difference between FX and XF correlators consists of three steps. At the first step, we synthesized a XF frequency response (hereafter, XF freq-response) from linear combinations of FX freq-responses. Then, we calculated frequency profiles (hereafter, freq-profiles) by convolving original and synthesized XF freq-responses with several object profiles (hereafter, obj-profiles). At the last step, we estimated the difference between the freq-profiles from synthesized and original XF freq-responses in each object.

2.1. Synthesis of a XF frequency response

We synthesized a XF freq-response from linear combinations of FX freq-responses using least square fit. Figure 2 shows an example of this procedure, where twelve FX freq-responses are used to synthesize a XF freq-response in the range between its 2nd null points. Hanning window function is applied to the XF freq-response in the frequency domain, but no window function is applied to the FX freq-responses.

The binning of multiple FX freq-responses is necessary in order to synthesize a certain range of XF freq-response, because the width of sinc function is larger than that of sinc squared function. This difference is enhanced by window functions usually adopted in XF correlator. In addition, FX correlator internally calculates power spectra at the highest frequency resolution, although the resolution depends on correlator configuration in the case of XF correlator. For example, the full width half maximum of XF freq-response (green line) is four times as large as that of FX freq-response (one of red lines) in the case of Figure 3. Weighting at the binning is also necessary. If non-weighted spectral averaging is used within FX correlator, its freq-response at the output becomes the sum of sinc squared functions regularly aligned in the frequency domain (red lines in Figure 3). Its shape (the sum of red line in Figure 3) is completely different from that of XF correlator (green line in Figure 3). Their weight values are determined by least square fit in order to minimize the freq-response difference (see blue lines in Figure 3)

2.2. Object profiles

We have assumed four profiles as object profiles. They are Gaussian, two-component Gaussian, rectangular and δ function profiles.

- Gaussian profile

A Gaussian like profile is a popular one. Such a profile is often observed toward dark clouds. The tested Gaussian profile has a full width half maximum (FWHM) of 8 times the width of one XF frequency channel (hereafter, ΔXF freq-ch) as shown in Figure 4.

- Two-component Gaussian profile

Some astronomical objects show a line profile composed of several components. We have examined the case that two Gaussian components, whose FWHM values are 4 and 2 ΔXF freq-ch, are located by a separation of 6 ΔXF freq-ch. This profile is shown in Figure 5.

- Rectangular profile

Such a steep profile as a rectangular profile is observed toward expanding envelopes of late type stars. The full width (FW) of the adopted rectangular profile is 16 ΔXF freq-ch as shown in Figure 6.

- δ function profile

It is unlikely that astronomical objects emit δ function like profile. However, we adopted this profile to test very severe cases. For this case only, we have estimated the profile differences of two configurations, because obtained freq-profile depends on δ function's position within one XF freq-ch. One configuration is that a δ function is located at the center of a XF freq-ch, and the other is the configuration that a δ function is offset from the center of a XF freq-ch by a quarter of a XF freq-ch width. These profiles are shown in Figure 7.

2.3. Frequency profile difference

For the estimation of the frequency profile differences, we define a parameter "Profile Difference" in the following equation.

$$\text{Profile Difference}[\%] \equiv \frac{\left|[\text{synthesized XF}] - [\text{original XF}]\right|}{\max([\text{original XF}])} \times 100$$

$$\text{"max()" is a maximum of inputted values.}$$

This value indicates the ratio of the differences between synthesized and original XF results at each frequency channel to the maximum value of original XF results.

3. Results

3.1. Synthesis of a XF frequency response

Figure 8 show the result of the synthesis of a XF freq-response. The synthesis parameters are summarized in Table 1. We adopt that XF freq-ch width is twice as large as FX freq-ch width. The differences between synthesized and original XF freq-responses will become smaller in the case that XF freq-channel width is larger than twice FX freq-ch width. Twelve FX freq-responses are used to synthesis the XF freq-responses in the range between its 2nd null points. Since the number of used FX freq-responses is even, the peak positions of the synthesized and original responses are not coincident. It will be difficult to adopt odd number, because observation frequency of the ACA must be shifted by a half of a FX freq-ch width. A frequency range is divided by factorial of 2 frequency channels in both the ACA Correlator and the 64-Antenna Correlator because of FFT operations. In addition, a XF freq-ch width depends on the configuration of the 64-Antenna Correlator, and a FX freq-ch width is always the highest frequency resolution of 3.8 kHz. As a result, a XF freq-ch width is always factorial of 2 multiple of a FX freq-ch width. Thus, the center position of a XF freq-ch is always shifted from that of a FX freq-ch by a half of a FX freq-ch width, except for the highest resolution of 3.8 kHz in the two correlators. Hanning window function is applied to the XF freq-response in the frequency domain. XF freq-response without window function has large sidelobe (1st sidelobe reaches 22% level of the peak in the mainlobe) and Hanning window function is commonly used in radio interferometers. No window function is applied to the FX freq-responses.

3.2. Gaussian profile

Figure 9 shows the results of the Gaussian profile (see Figure 4) as an object profile. The upper panel shows the object profile and the freq-profiles. Three lines are almost consistent with one another, suggesting that the differences between them are small. The red line in the lower panel shows Profile Difference calculated from the freq-profiles of the upper panel. The maximum Profile Difference is 0.37%, and good profile compatibility is available between the synthesized and original XF freq-responses in the case of the Gaussian profile.

3.3. Two-component Gaussian profile

Figure 10 show the results of the two-component Gaussian profile (see Figure 5). Each panel is the same as Figure 9 except that the object profile is the two-component Gaussian profile. The difference between the object profile and the freq-profiles is large around the narrower Gaussian component, because the frequency resolution of the freq-responses is not enough. However, the difference between the two freq-profiles is small, and the maximum Profile Difference is only 0.40 % at the border of two Gaussian profiles.

3.4. Rectangular profile

Figure 11 shows the results of the rectangular profile (see Figure 6). Each panel is the same as Figure 9 except that the object profile is the rectangular profile. Figure 11 shows the maximum Profile Difference is 0.60%, and good compatibility is also available in the case of object profiles with such a steep profile as a rectangle.

3.5. δ function profile

3.5.1. δ function located at the center of a XF freq-ch

Figure 12 shows results of the δ function profile at the center of a XF freq-ch (see Figure 7 left). Each panel is the same as Figure 9 except that the object profile is the δ function profile located at the center of a XF freq-ch. Figure 12 shows that the maximum Profile Difference is 3.9%. The δ function and rectangular profiles are similar in steepness, however, their maximum values of Profile Difference are largely different. This is because the freq-profiles are smoothed by a flat and wide profile of a rectangle in the case of the rectangular profile.

3.5.2. δ function offset from the center of a XF freq-ch by a quarter of a XF freq-ch width

Figure 13 shows the results of the δ function profile offset from the center of a XF freq-ch (see Figure 7 right). Each panel is the same as Figure 9 except that the object profile is the δ function profile offset from the center of a XF freq-ch by a quarter of its width. Since δ function is shifted from a center of a frequency channel, Profile Difference is not symmetrical. Maximum Profile Difference is 2.9%, which is smaller than that of δ function located at the center of a XF freq-ch.

3.6. Maximum Profile Difference vs. FWHM of Gaussian profile

We have checked the relation between the maximum Profile Difference and FWHM of a Gaussian profile as an object profile. Figure 14 shows the change of the maximum Profile Difference as increasing FWHM of a Gaussian profile. It is expected that the maximum Profile Difference decreases as the FWHM becomes large, because FX and XF freq-responses are more smoothed by more slowly changed Gaussian profile in the frequency domain. Figure 14 generally shows the decrement of the maximum Profile Difference as the FWHM increases, however, it also shows a local peak around FWHM of 6 ΔXF freq-ch. This behavior can be explained by the relation between FWHM of a Gaussian profile and a range of the frequency profile synthesis, which is within 2nd null points of original XF freq-response in this sample. When a Gaussian profile is narrow enough to a XF freq-response, its freq-profile approaches the response to δ function. If FWHM of a Gaussian profile exceed a half of a synthesized range of the frequency response (3 ΔXF freq-ch in this sample), the Gaussian profile comes to involve both of inside and outside of the synthesized range (see FWHM=3 in Figure 15). Since no profile synthesis is performed at the outside, freq-response difference becomes slightly large there and the difference contributes to Profile Difference. As a result of this contribution, maximum Profile Difference temporarily increases as shown in Figure 14. When a Gaussian profile becomes wide enough to the synthesized range, the freq-response differences outside the synthesized range are well smoothed and become less significant, and Profile Difference decreases again.

This result also suggests that frequency channel resolution comparable to FWHM of a Gaussian profile is at least required for good compatibility in frequency profile. In the case of this memo, that is about 4 ΔXF freq-ch, because Hanning window function is applied to an original XF freq-response.

4. Summary

We have estimated the frequency profile differences between ACA Correlator and 64-Antenna Correlator in the case of convolution of weighting function after the correlation in the frequency domain. When one frequency channel width of a XF correlator is twice as large as that of a FX correlator, it is possible to synthesize “a XF freq-response convolved with Hanning window function” from linear combinations of “FX freq-responses”. The Profile Difference in the object profiles tested in this memo can become small by 0.6 % or less except for the δ function cases. It is severe case that one frequency channel width of a XF correlator is only twice as large as that of a FX correlator. Larger frequency channel width of a XF correlator will provide better compatibility between frequency profiles of FX and XF correlators.

We have also studied the relationship between the maximum Profile Difference and FWHM of a Gaussian profile. The result shows that the maximum Profile Difference decreases as FWHM of a Gaussian profile increases if the FWHM is wide enough to the synthesized range. This also suggests that frequency channel resolution comparable to the FWHM of a Gaussian profile is at least required for good compatibility in frequency profile.

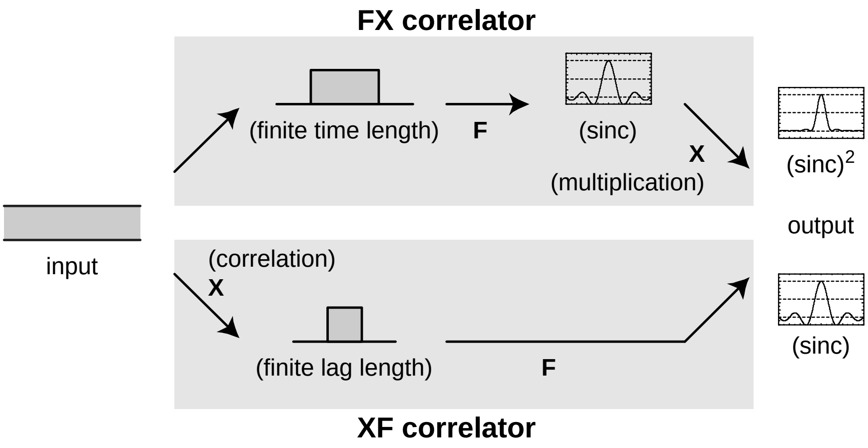
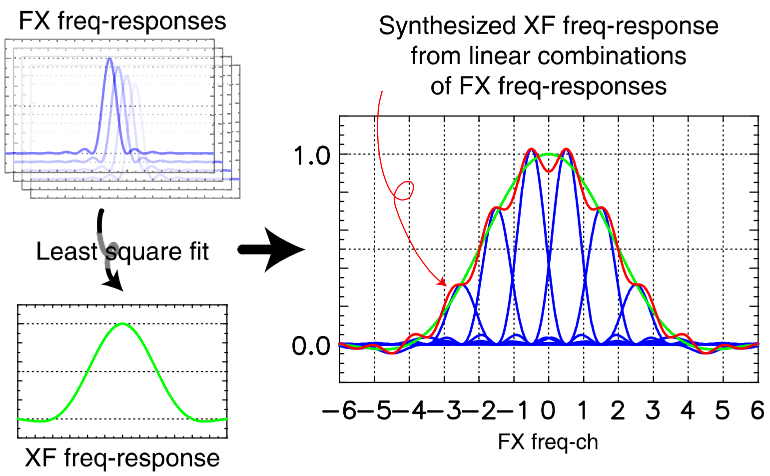


Figure 1 Calculation order of "F" and "X" in FX and XF correlators

"F" means Fourier transform. "X" is multiplication and correlation in FX and XF correlators, respectively.



"FX freq-ch" is one frequency channel of a FX correlator. Its width is always 3.8 kHz for the ACA Correlator, corresponding to 0.005 km s^{-1} at 230 GHz. "XF freq-ch" is one frequency channel of a XF correlator. Its width depends on the configuration of the 64-Antenna Correlator. In the case of this figure, the "XF freq-ch" width is 7.6 kHz, corresponding to 0.01 km s^{-1} at 230 GHz.

Figure 2 Synthesis of a XF freq-response from linear combinations of FX freq-responses

Blue lines show weighted FX freq-responses. Green line shows original XF freq-response. A red line is synthesized XF freq-response from linear combinations of FX freq-responses. Hanning window function is applied to the XF freq-response in the frequency domain, but no window function is applied to the FX freq-responses.

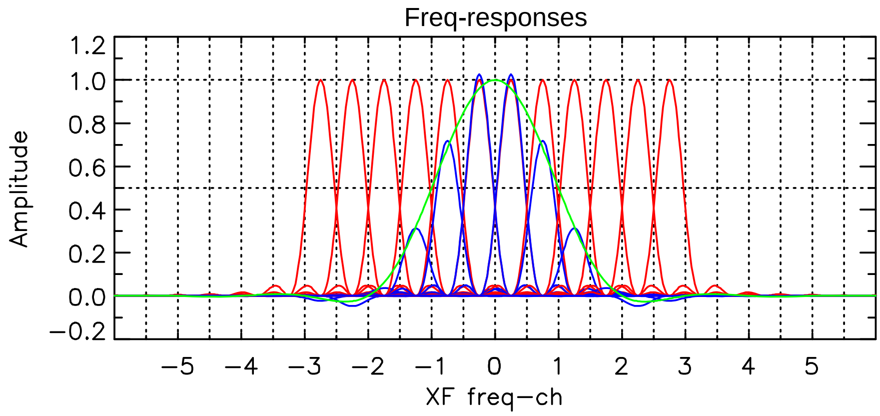


Figure 3 Frequency responses of FX and XF correlators

The plot shows XF freq-response with green line, FX freq-responses with red lines and weighted FX freq-responses with blue lines in the case of Figure 2.

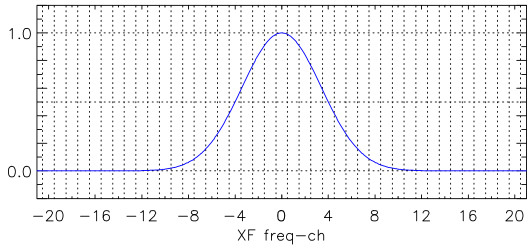


Figure 4 Gaussian profile

Blue line shows a Gaussian profile whose FWHM is 8 ΔXF freq-ch.

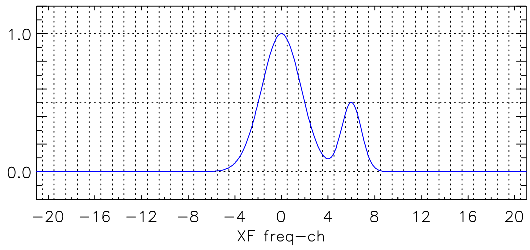


Figure 5 Two-component Gaussian profile

Blue line shows that two Gaussian components, whose FWHM are 4 and 2 ΔXF freq-ch, are located by a separation of 6 ΔXF freq-ch.

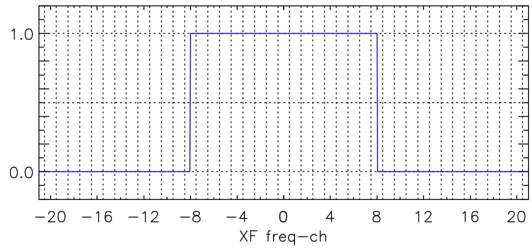


Figure 6 Rectangular profile

Blue line shows a rectangular profile, whose full width is 16 ΔXF freq-ch.

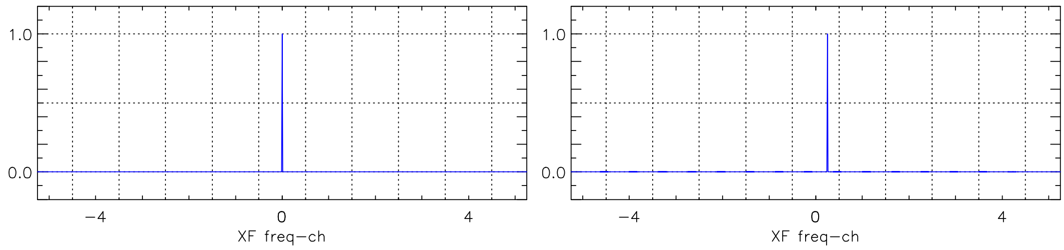


Figure 7 δ function profiles

(Left) Blue line is δ function profiles located at the center of a XF freq-ch. (Right) Blue lie is δ function profiles offset from the center of a XF freq-ch by a quarter XF freq-ch width.

Table 1 Synthesis parameters of a XF frequency response

FX	Window function	None
	ΔFX freq-ch	3.8 kHz
	Number of profiles for synthesis	12
XF	Window function	Hanning
	ΔXF-ch	2×ΔFX freq-ch

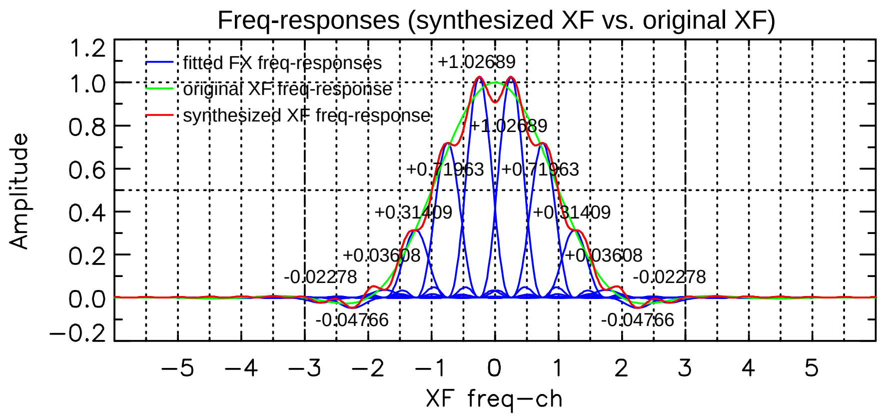


Figure 8 Synthesis of a XF freq-response

Red line is synthesized XF freq-response from linear combinations of FX freq-responses (blue lines) using least square fit to original XF freq-response (green line). Numerical values in the plot are resultant fitting parameters. Dash-dotted lines indicate the synthesized range of XF freq-response.

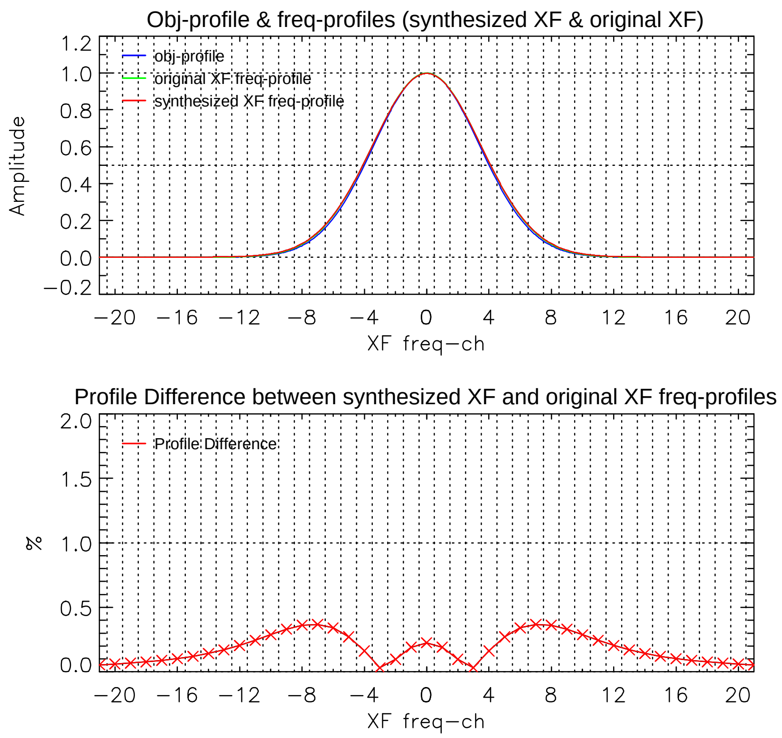


Figure 9 Profile Difference in the Gaussian profile

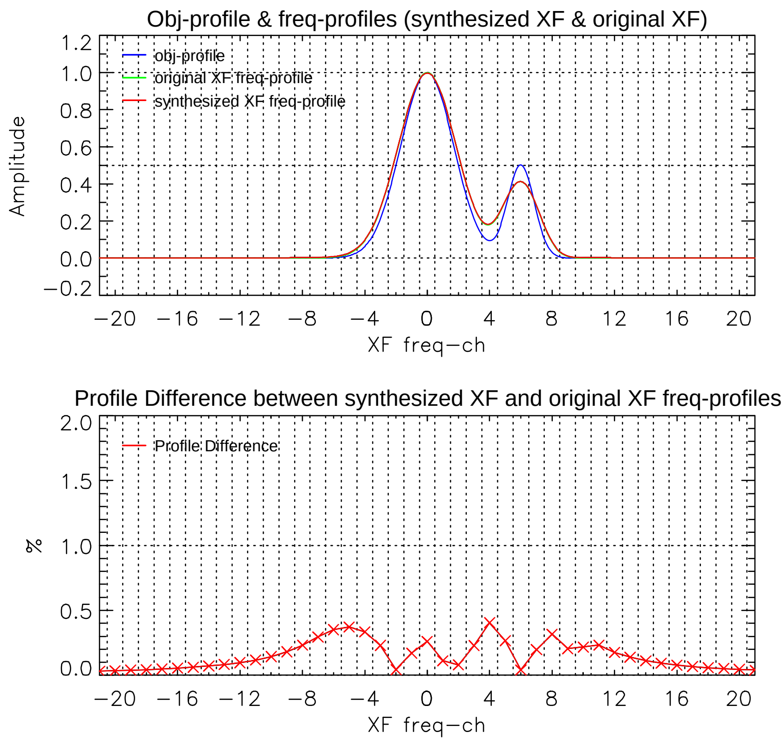


Figure 10 Profile Difference in the two-component Gaussian profile

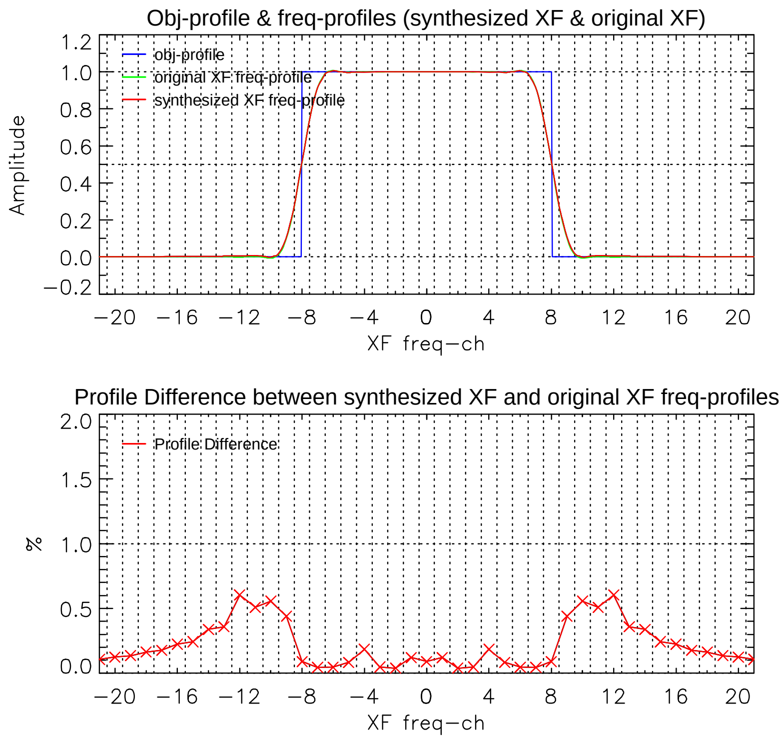


Figure 11 Profile Difference in the rectangular profile

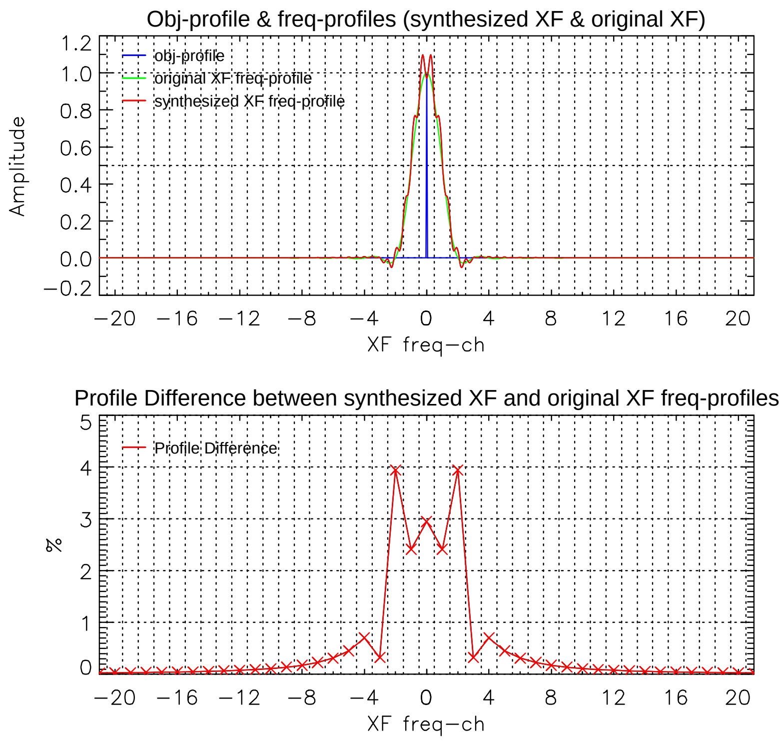


Figure 12 Profile difference in the δ function profile located at the center

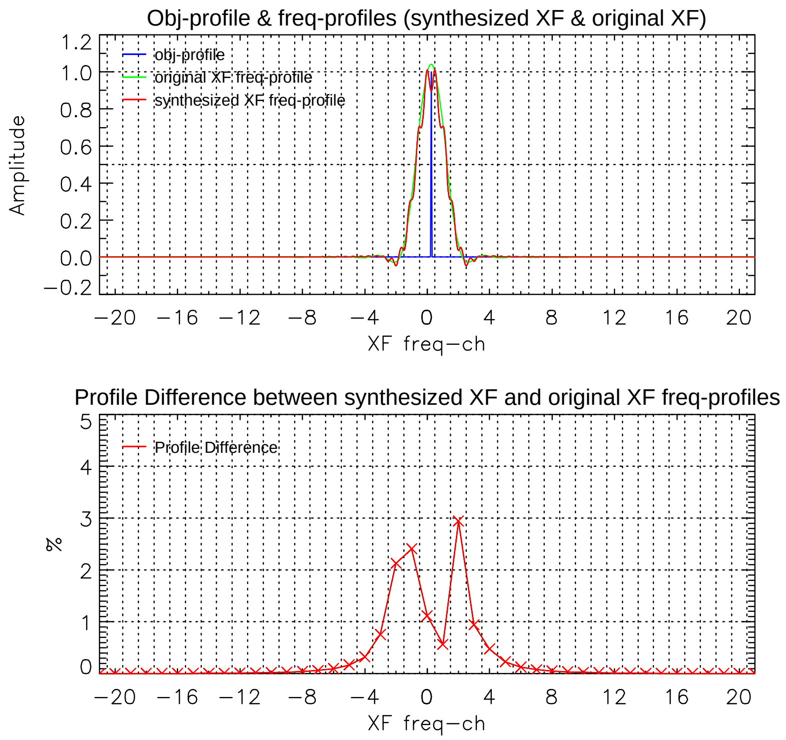


Figure 13 Profile difference in the δ function profile offset from the center

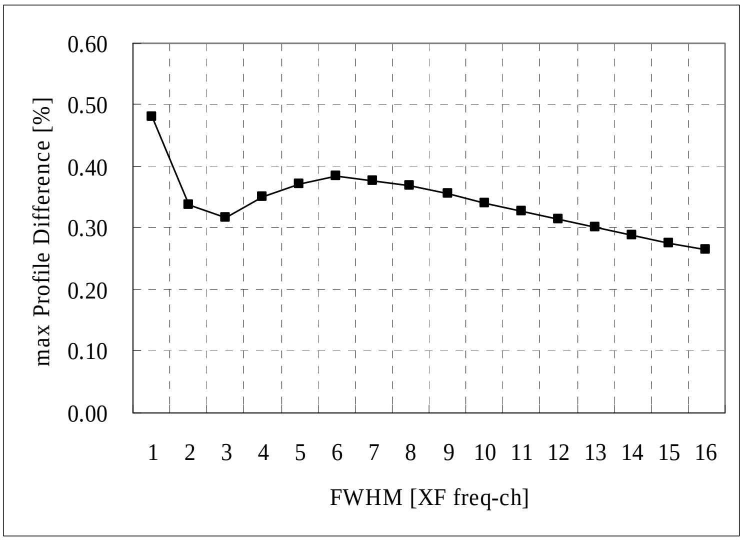


Figure 14 Maximum Profile Difference vs. FWHM of a Gaussian profile

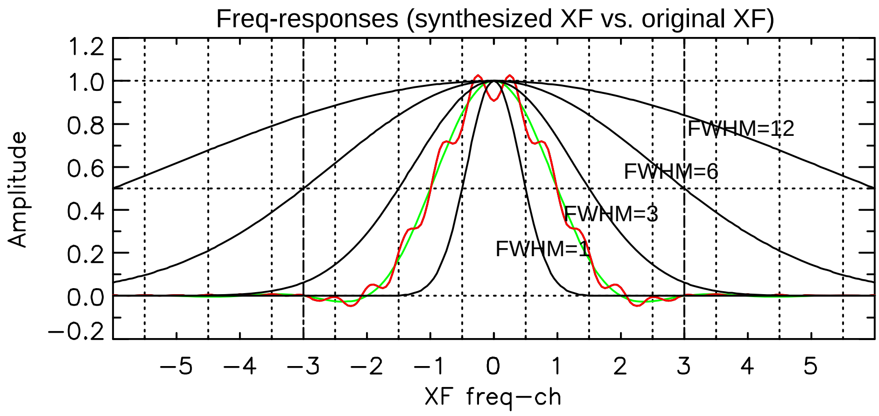


Figure 15 Frequency responses vs. FWHM of a Gaussian profile

Gaussian profiles with various FWHM, which are shown by black lines, are overlaid on Figure 8. Dash-dotted lines indicate the synthesized range of XF freq-response.