

ALMA MEMO 591

Calibration Errors and Artifacts from Imperfect van Vleck Correction

Darrel Emerson

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Abstract

In modern radio astronomy systems, the signal from the receiver is usually converted to an IF band and then digitized. In order to keep cost and complexity within bounds, especially with broad bandwidth systems, a limited number of digitization bits is used, at the cost of a small reduction in sensitivity. The digitized signal may then be processed in a digital correlator, providing (e.g. with the X-F correlator) an average correlation function which after processing leads to a signal frequency power spectrum. Non-linearity present in the averaged correlation function, resulting from the inherent non-linearity of the digitizer, is compensated for in a way usually referred to as the van Vleck correction.

For a given but small number of quantization levels, the shape of the van Vleck correction function is itself dependent on the level of the input signal, compared to the size of the discrete steps of the digitizer. In typical radio astronomical observations, especially at mm- and submm-wavelengths, changing atmospheric conditions may produce relatively rapid change in the total noise power presented to the receiver from the antenna. For highest ultimate dynamic range of observations, either the level of the signal needs to be held constant in terms of the digitization steps, or else the specific van Vleck correction needs to be recalculated sufficiently often, based on the measured input level, so that the detailed correction function remains appropriate. Imperfect linearity correction will introduce errors in the derived amplitude and phase of an interferometer, which vary with frequency across the digitized pass-band. Harmonic distortion will appear within the pass-band, which may limit the spectral dynamic range.

Even with quite large mismatch of the signal to the ideal VV correction, calibration errors are relatively small and can probably be neglected if the signal is held constant within +/- 2 dB. The most stringent case is the spectral dynamic range requirement. For autocorrelation measurements, it may be necessary for the specific van Vleck correction curve to be recalculated whenever the signal level changes as much as ~+/-0.2 dB.

1 Introduction

This memo addresses specifically XF correlator schemes, where the digital hardware provides an average correlation function, with both negative and positive delays. The coarse digitization of the raw input signal causes this averaged function to suffer from non-linearity, but if the input signal level is known, then in principle the non-linearity can be corrected in software. This is known as the van Vleck (VV) correction. Only the XF correlator is considered here.

Potential errors in the initial correlation function are calculated here for 2-bit and 3-bit digitization schemes, but where the input signal at the digitizer is allowed to deviate by as much as +/-20 dB from the optimum; this well beyond what is likely to be encountered in ALMA. The VV correction is computed and applied with the false assumption that the input signal level had been held constant at the ideal value. Since the form of the VV correction depends on input level, this leads to an imperfect correction of the non-linearity present in the raw correlation function. This residual non-linearity leads, in general, to both amplitude and phase errors across the pass-band, and to spurious spectral features. The errors are calculated here, first (see Section 4) with the assumption that the input signal consists of broadband flat-spectrum noise. Narrow-band signals are then considered in Section 5.

In the ALMA context, either 2-bit or 3-bit data are used, depending on observing mode. Details are described in ALMA Memo #583, by Comoretto.

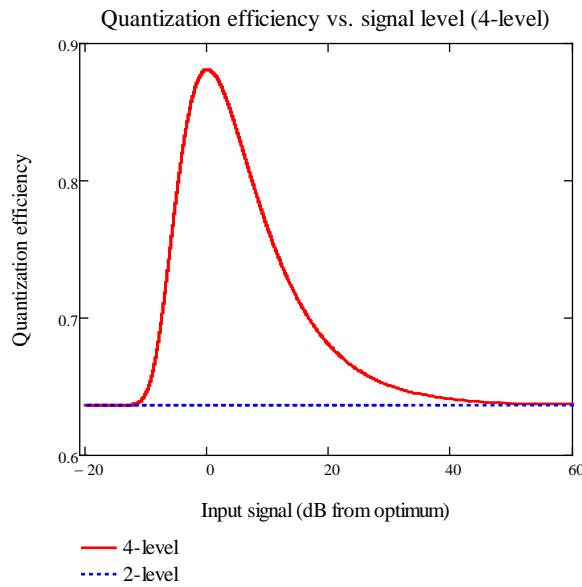
Note that for a 1-bit digitization scheme, where only the sign information of the analog signal is retained, the digitization is independent of the input level. The class of errors discussed here does not occur for 1-bit digitization.

2 Quantization Efficiency

As is well known, using a limited number of quantization levels leads to a loss of efficiency. For more than 1-bit digitizers, that loss of efficiency can be minimized by choosing the optimum quantization threshold level, in terms of the rms signal level. Thompson, Emerson & Schwab (“Convenient Formulas for Quantization Efficiency, ”, A.R. Thompson et al., Radio Science, 42, 7 RS3022, doi:10.1029/2006RS003585, 2007) give the following expression for the quantization efficiency: see the notation in that paper. For a 2-bit 4-level scheme, the optimum threshold step is 0.995686 of the rms signal level, at which point the efficiency is 0.8812. For a 3-bit 8-level scheme, the optimum threshold step of 0.586019 of the rms signal level, at which point the quantization efficiency is 0.96256.

Equation 1

$$\eta(v) := \frac{\frac{2}{\pi} \cdot \left[\frac{1}{2} + \sum_{m=1}^{N-1} e^{-\left(m\right)^2 \cdot \frac{v^2}{2}} \right]^2}{\left(N - \frac{1}{2}\right)^2 - 2 \cdot \sum_{m=1}^{N-1} \left[m \cdot \text{erf}\left[\left(\frac{m}{\sqrt{2}}\right) \cdot v\right]\right]}$$



**Figure 1 Quantization efficiency for 4-level, 3-bit digitization.
Note the dashed line showing the corresponding 1-bit value.**

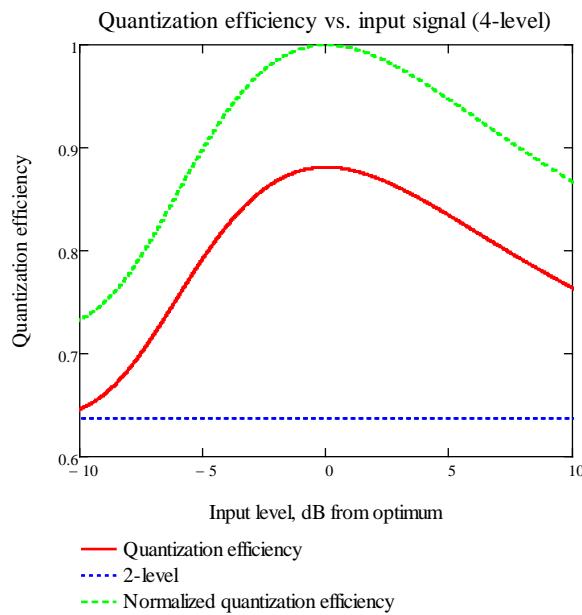


Figure 2 As above, but with the signal level scale expanded. The upper, dashed curve is after normalization to give comparative efficiencies compared to the optimum for this digitization scheme.

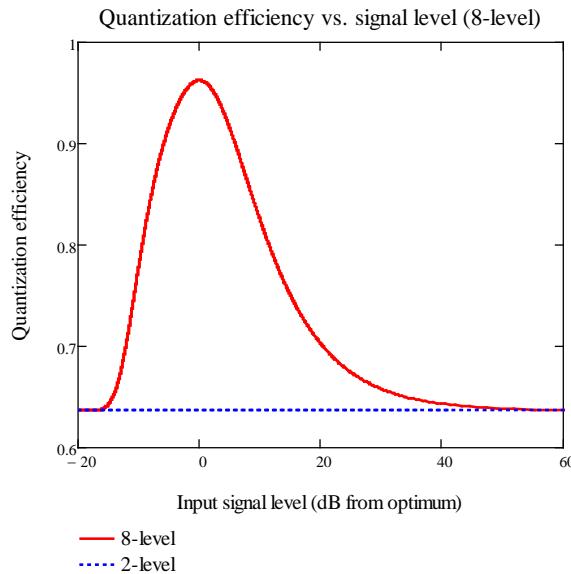


Figure 3 Quantization efficiency for 8-level, 3-bit data. Note the dashed line showing the corresponding 1-bit value.

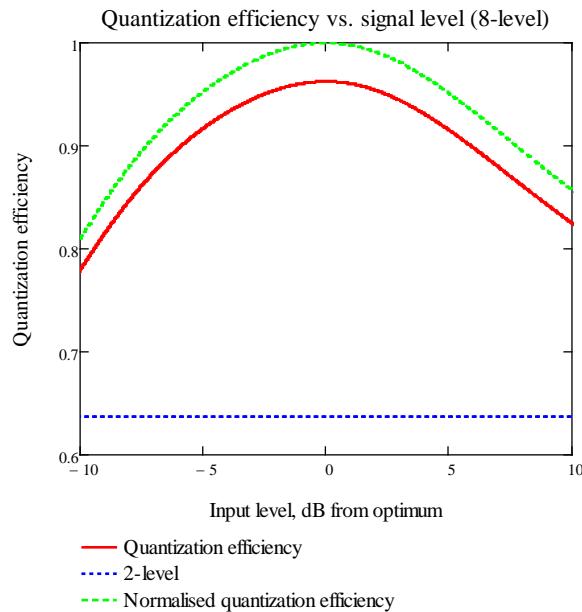


Figure 4 As above, with signal level scale expanded. The upper dashed curve shows the relative efficiencies normalized to the best possible with this quantization scheme.

Figure 1 and Figure 2 show the quantization efficiency as a function of signal level, expressed in dB from the optimum value, for 2-bit 4-level digitization. Figure 3 and Figure 4 show the same for 3-bit, 8-level quantization. In all figures, the efficiency for 1-bit digitizers is shown as the horizontal dashed line; at extreme high or extreme low input values, both 3-bit and 2-bit digitizers effectively degenerate into a 1-bit system. For the 2-bit 4-level scheme, if the input signal is as much as 10 dB below optimum, the system has already become nearly equivalent to 1-bit quantization.

From Figure 2, for a 2-bit 4-level system a 1% loss of sensitivity occurs if the input signal is +1.88 dB or -1.65 dB dB from the optimum value. From Figure 4, for a 3-bit, 8-level system a 1% loss of sensitivity occurs when the input signal is +2.21 or -2.16 dB from the optimum.

3 The van Vleck Correction

F. Schwab (GBT Memorandum No. 250, "Van Vleck Correction for the GBT Correlator" by F.R. Schwab (2007)) gives a convenient way of calculating the apparent correlation coefficient, given the digitization and signal parameters. Important references to the literature are included in that Memo.

Equation 3 from GBT Memo No. 250, setting $\text{sigmax}=\text{sigmay}=1$, and $\text{mux}=\text{muy}=0$ is:

Equation 2

$$r(\rho) := \sum_{i=1}^m \sum_{j=1}^n \left[(wx_i \cdot wy_j) \cdot \int_{vx_{i-1}}^{vx_i} \int_{vy_{j-1}}^{vy_j} \frac{1}{2\pi\sqrt{1-\rho^2}} \cdot e^{-\frac{(-1 \cdot (x^2 - 2 \cdot \rho \cdot x \cdot y + y^2))}{2(1-\rho^2)}} dx dy \right]$$

where $r(\rho)$ is the apparent, unnormalized correlation amplitude from the correlator, for a true correlation coefficient ρ . m and n are the number of input levels "wx" and "wy" are the values assigned to each input level, and ρ . "wx" and "wy" correspond to the digitizer threshold levels, expressed in units of the input signal rms. Here the two inputs to the correlator are assumed to be identical, so that the number of digitization steps are equal and $m = n$. An efficient way of deriving the true correlation coefficient ρ from the measured function $r(\rho)$ is to calculate and tabulate values of $r(\rho)$ for a sufficient number of values of true correlation coefficient ρ , and then to use an interpolation scheme to invert the equation to derive the true correlation ρ from a given value of r . In this memo a tabulation of 200 values of $r(\rho)$ and ρ is used. For 2-bit digitization there are 4 possible levels, so $m = n = 4$. For 3 bits there are 8 possible levels, so then $m = n = 8$. For 2-bit digitization, the values assigned to wx & wy are -3, -1, 1 & 3. For 3 bits, the values are -7, -5, -3, -1, 1, 3, 5 & 7. These parameters correspond to the ALMA correlator, but in any case the signal processing subsequently normalizes the correlated data, so the exact values are unimportant, provided that the intervals are equal.

The above Equation 2 exhibits a singularity as the correlation coefficient **p** approaches unity, so in generating the values in the table for **p=1**, corresponding to the important zero-lag autocorrelation function, alternative expressions are used. These are taken from, equations (3) and (4) of ALMA Memo #583, written by Comoretto, themselves derived from Schwab, and are repeated as **Equation 3** and **Equation 4** here.

Equation 3 (from ALMA Memo #583, by Comoretto)

$$9 - 8 \cdot \text{erf} \left[\frac{1}{\sqrt{2} \cdot \left(\frac{1}{dv} \right)} \right]$$

Equation 4 (from ALMA Memo #583, by Comoretto)

$$(N - 1)^2 - \sum_{k=1}^{\frac{N}{2}-1} \left[8 \cdot k \cdot \text{erf} \left[\frac{k}{\sqrt{2} \cdot \left(\frac{1}{dv} \right)} \right] \right]$$

Equation 3 can be used for a 2-bit quantization, while **Equation 4** can be used for multi-bit quantization. “**dv**” is the size of the quantization step at the digitizer, expressed in units of the rms of the input signal. **N** in **Equation 4** is 8 for a 3-bit digitizer and multiplicative weights $2 \cdot k + 1$ are assumed for level **k** of the digitizer - this is the scheme adopted in the ALMA correlator, leading to the values of **wx** & **wy** listed above.

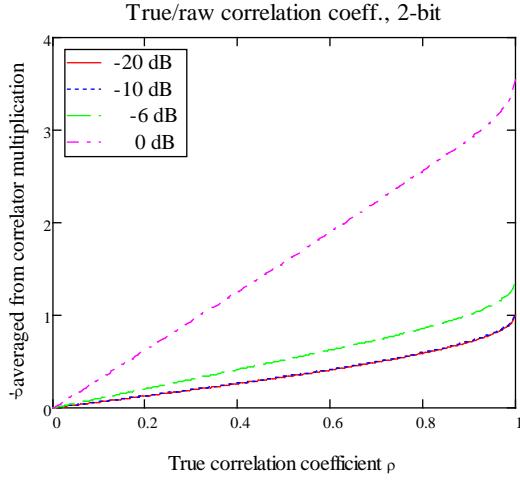


Figure 5 The unnormalised van Vleck correction for signal levels equal to or less than the optimum value

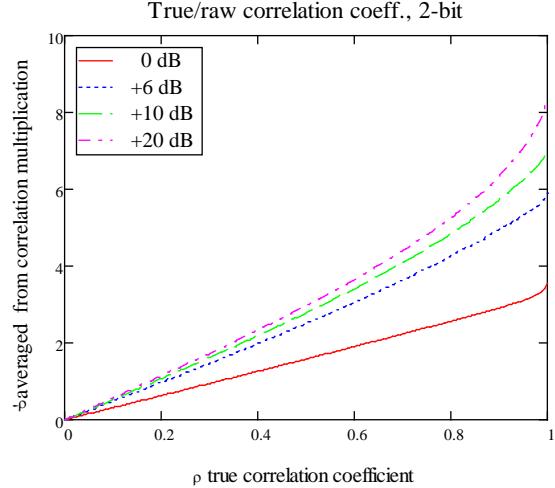


Figure 6 The unnormalised van Vleck correction for signal levels equal to or greater than the optimum value

Figure 5 and Figure 6 plot the expected correlated data values for 2-bit digitized data, where the input data ranges from 20 dB below to 20 dB above the optimum value. Figure 7 and Figure 8 show the same for 3-bit, 8-level data. All values are plotted as a function of the true correlation coefficient.

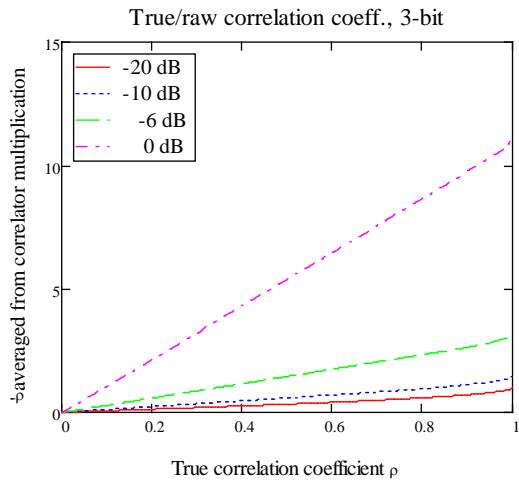


Figure 7 Correlator output values before normalization, for signals less than or equal to the optimum value

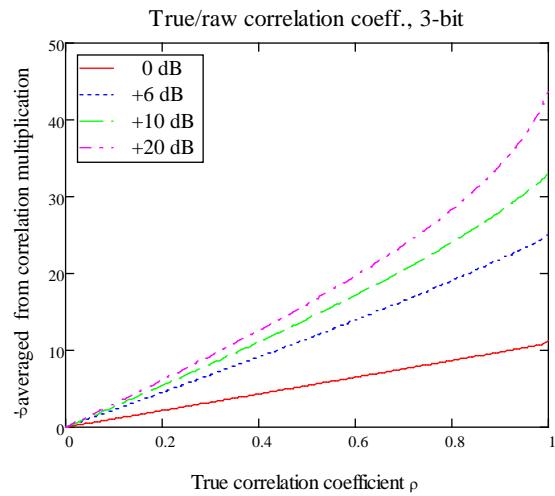


Figure 8 Correlator output values before normalization, for signals greater than or equal to the optimum value

In particular, in Figure 5, note that the curves for signal levels at -20 dB and -10 dB compared to the optimum are essentially superposed; this is a consequence (see Figure 1) of both -20 dB and -10 dB signal levels forcing the digitizer into effectively a 1-bit mode of operation.

Figure 9 to Figure 12 show the same curves, but normalized by scaling the 100% correlated value of the raw correlation function data to be 1.0. The plots show the true correlation coefficient as a function of the normalized raw data, for different levels of input signal, and for 4-level and for 8-level data. Also shown is a curve for 1-bit data, displaced above the other curves. As is seen, the more the signal level deviates from the optimum, for either higher or lower signal levels, and in both 4-level and 8-level data, the shape of the curve tends to that of 1-bit data. These curves represent the van Vleck corrections for different signal levels, for the different digitization schemes.

Note also that the curve with the signal level at optimum (0 dB) is closest to a straight line, especially in the 8-level case. If this optimum curve is used, wrongly, to correct data with a non-optimum signal level, then it is clear that the correction will be poor. In general, this will lead to both amplitude and phase errors across the digitized pass-band.

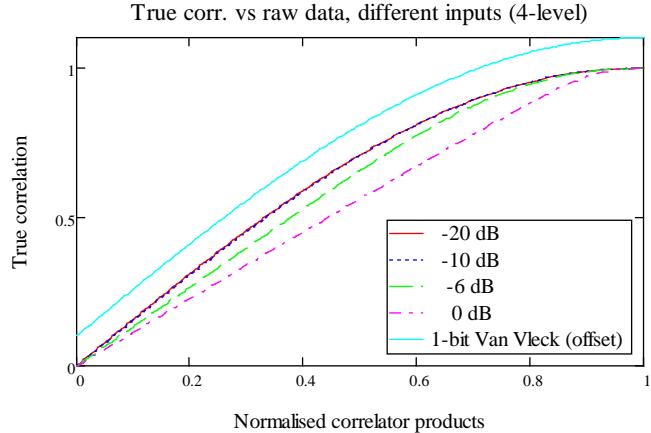


Figure 9 True correlation coefficient vs. normalised raw data, 4-level digitization

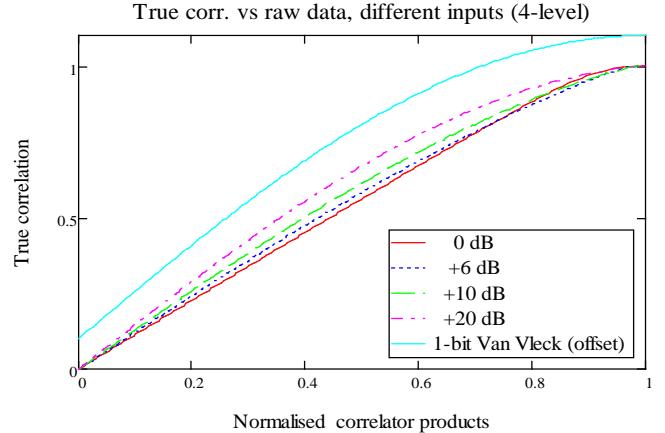


Figure 10 True correlation coefficient vs. normalised raw data, 4-level digitization.

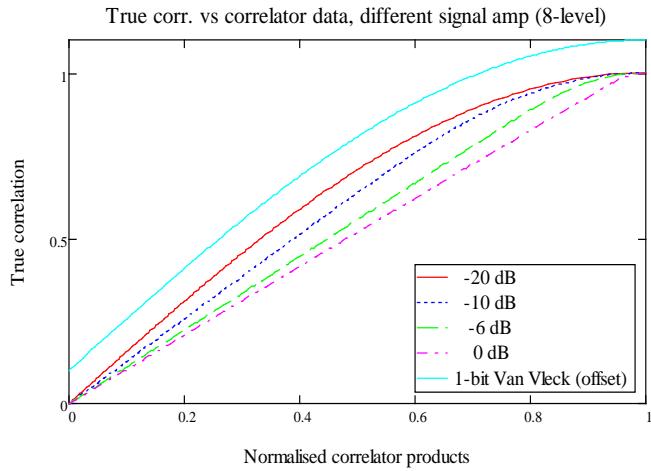


Figure 11 True correlation coefficient vs normalized raw data, 8-level digitization

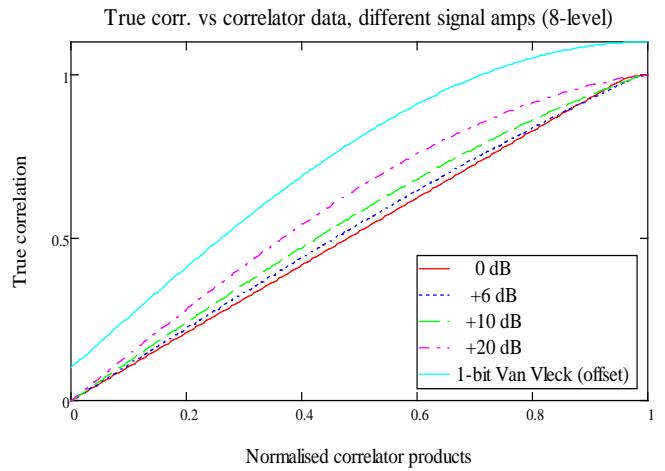


Figure 12 True correlation coefficient vs normalized raw data, 8-level digitization

Ideally, the signal level at the digitizer could be held constant by a real-time Automatic Level Control (ALC) system, but that may introduce other problems of calibration. In the absence of ALC, it is likely that the correction for the non-linearity of the raw correlation coefficients will be made with, to some extent at least, an incorrect VV curve. This means that the non-linearity of the raw correlation function may not be removed perfectly, or even that even greater non-linearity might be introduced. The magnitude of such potential errors is investigated here.

4 Calibration Errors

4.1 Gain calibration error

One way of calibrating the system gain through the correlator is to normalize to the zero-lag autocorrelation value. The van Vleck curve then relates that zero-lag amplitude to the small-signal correlator gain, for a given quantizer threshold. However, even after normalizing a given integration according to its own zero-lag autocorrelation value, if the signal level has changed with respect to the to the quantization threshold steps, then the wrong van Vleck correction may be invoked, and which at the very least may have the wrong small-signal slope.

Figure 13 shows the relative system gain factor, for 4-level and 8-level quantization, as a function of signal level, if the van Vleck correction is not recalculated. The signal is relative to the optimum value for the respective quantization schemes. In both cases, the signal has first been normalized according to the zero-lag value of the autocorrelation function. The gain variation follows from the different small-signal slopes of the respective van Vleck correction that should be applied for a given signal/quantization step value.

Note that, for the 4-level 2-bit digitization, even after normalization, if the van Vleck correction function is not recalculated, then a 6 dB reduction in signal input level will lead to a calibration amplitude error of 15%. For 8-level, 3-bit data, either a 6 dB reduction or increase in signal level leads, after normalization to an error of -7% in amplitude calibration. Alternatively, if a calibration error less than 1% can be tolerated, then for a 4-level system the input signal should be held between -1.65 to +1.88 dB of the optimum, and for the 8-level system between -2.16 and 2.21 dB of the optimum.

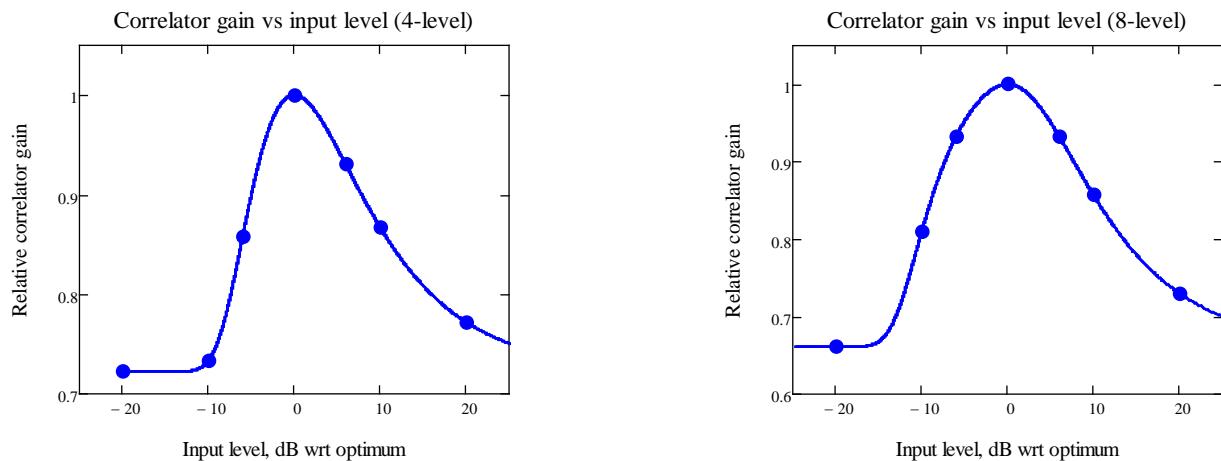


Figure 13 The correlator gain, after normalization to the zero-lag autocorrelation value, as a function of signal input level referred to the optimum quantization threshold. 4-level quantization. The symbols on the line correspond to input levels of -20, -10, -6, 0 +6,+10 & +20 dB.

When correcting data using the zero-lag auto-correlation value, the data should also be scaled according to one of the gain curves shown in Figure 13. This is equivalent to adopting a linear approximation to the true van Vleck corrections shown in Figure 9 to Figure 12, giving the correct small-signal slope. Doing so will ensure that signals with small correlation coefficients are precisely calibrated no matter how big a deviation from the optimum input signal magnitude, although large correlated signals – and in particular the auto-correlation function – may not then be scaled correctly. However, if the correct VV curve is used for the given input signal value, all these problems disappear.

4.2 Pass-band amplitude and phase errors

Apart from the global gain errors discussed above, the use of an incorrect van Vleck correction curve will introduce amplitude and phase errors as a function of frequency within the IF pass-band. This is just a consequence of the imperfectly corrected shape of the digitized correlation function.

Errors of relative amplitude and phase introduced into the digitized receiver pass-band were investigated as follows:

- 1) Starting with a flat broadband signal across the receiver pass-band, with a constant cross-correlation phase, a lag correlation function, as would appear in an XF correlator, was generated. The function was generated for several different values of peak (zero lag) correlation coefficient.
- 2) This correlation function was intentionally distorted, using a curve such as that shown in Figure 5 to Figure 8, to simulate digitization with input signal deviating from the optimum level at the digitizer by +/-6, 10 or 20 dB.
- 3) This distorted function was then corrected, but for the purposes of this simulation intentionally using the wrong van Vleck function – specifically, correcting with the 0 dB curve shown in Figure 9 to Figure 12, instead of using the appropriate +/-6, 10 or 20-dB curves.
- 4) With a Fourier transform, this distorted lag function was then converted to amplitude and phase across the receiver-pass-band. In general, unwanted pass-band amplitude and phase errors have been introduced. The amplitude of these errors has been plotted as a function of signal level, of correlation coefficient, and of phase difference of the original signal.

Looking at Figure 9 to Figure 12, it is clear that at small correlation coefficient values (e.g. $\rho \leq 0.1$), all of the van Vleck correction curves approximate well to a straight line. So, the uncorrected correlation function distortion examined here is only likely to become important with relatively high (e.g. $\rho > \sim 0.3$) correlation coefficients.

4.3 The effect of phase

The form and magnitude of pass-band amplitude and phase errors depends on the relative phase of the two input signals being correlated. In particular, if the phase difference is zero, or any multiple of $\pi/2$,

no phase error is introduced. This is easy to see by considering the lag correlation function. The phase is derived from positive and negative lag terms. For zero phase difference or any even multiple of $\pi/2$ radians, the lag function is entirely symmetric; positive lag terms equal negative lag terms. For odd multiples of $\pi/2$, the lag function is completely asymmetric – negative delay terms are the negative of the corresponding positive delay terms.

If the lag function is initially 100% symmetric about the zero lag point, then no amount of amplitude distortion will introduce any asymmetry, since negative lag terms are of magnitudes equal to the positive terms, and will be distorted identically. Similarly, if the lag function is initially wholly asymmetric, then no amount of amplitude distortion will introduce any spurious symmetric term. So, for wholly symmetric, or wholly antisymmetric lag functions, the degree of symmetry or of asymmetry is not changed, and therefore no phase error is introduced into the pass-band. The worst phase errors would be expected with a signal input phase difference of around $\pi/4$, or any odd multiple of that. This is indeed found to be the case, as shown below.

4.4 Examples of pass-band errors

The magnitude of pass-band error depends strongly on the relative correlation amplitude. In the following examples, an arbitrary but relatively high correlation coefficient of 0.75 has been adopted. A broadband input signal with constant amplitude and phase across the pass-band has been assumed.

Figure 14 and Figure 15 show, for 4-level 2-bit and for 8-level 3-bit data, the pass-band error amplitude and phase, resulting from incorrect van Vleck correction. The input signal levels were set to -20, -10, -6, 0, +6, +10 and +20 dB with respect to the optimum level, but the VV correction applied in all cases was that appropriate to the optimum signal level (0 dB). In both sets of figures, the left hand column shows the relative amplitude error across the pass-band, and the right hand column the phase error. The top row is for a cross-correlation phase, constant across the pass-band, of 45 degrees, followed by a phase of 0 and then of 90 degrees.

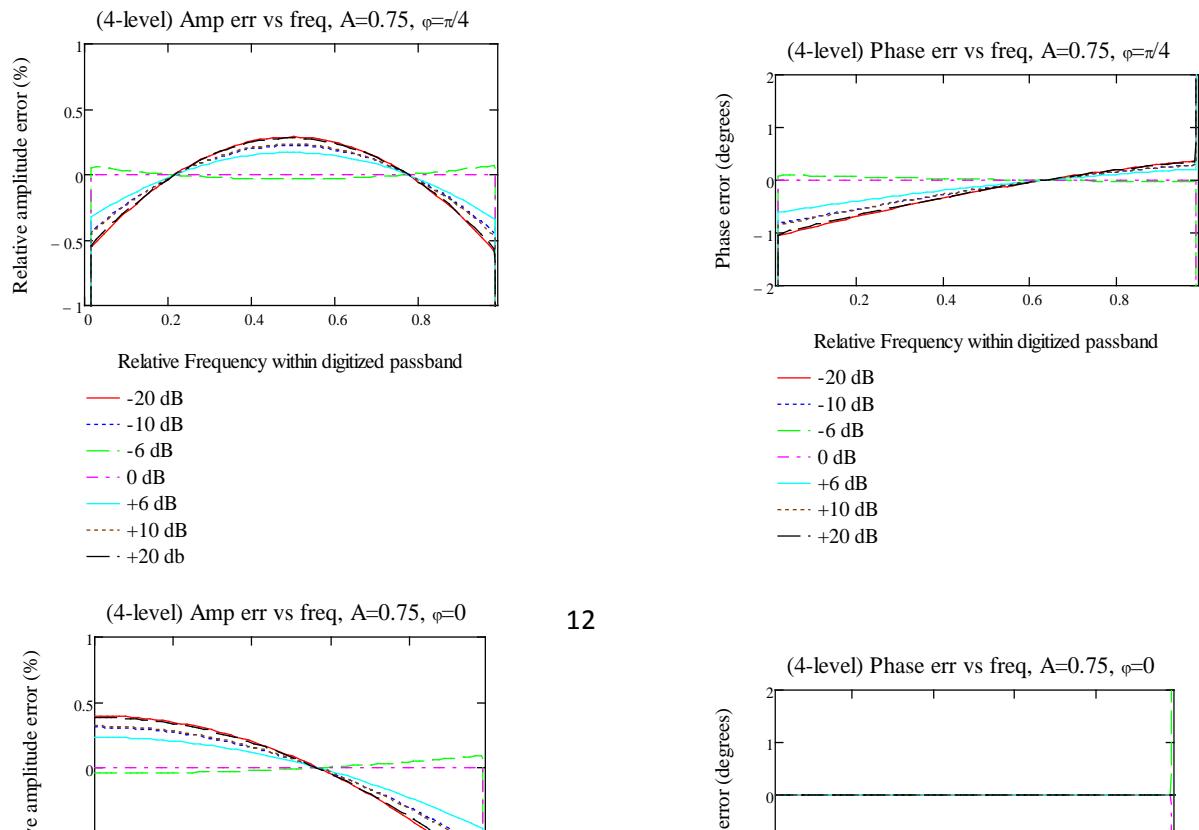


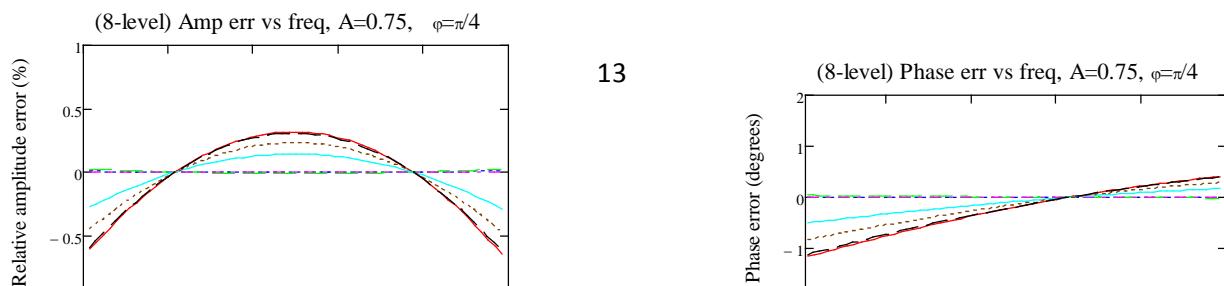
Figure 14 4-level passband errors

Figure 15 8-level passband errors

Several features of the pass-band distortion are apparent:

- 1) The characteristic distortions are remarkably similar between 4-level, 2-bit and 8-level, 3-bit data, both in form and in magnitude, especially for the extreme signal levels.
- 2) For a cross-correlation phase difference of 0 or $\pi/2$, there is no phase error introduced
- 3) The form of the amplitude distortion depends on the cross-correlation phase.
- 4) The magnitude of error introduced is relatively modest. Even with 0.75 correlation coefficient, and with signal inputs +/-20 dB mismatched to the VV correction, the peak amplitude error is only $\sim 2\%$, and the peak phase error only about 1 degree.
- 5) Not surprisingly, the onset of distortion is sharper with 2-bit than with 3-bit data. This is fairly obvious in the top left plot of Figure 14 and Figure 15. For 4-level, 2-bit data the curve for +6 dB signal level is already approaching the extreme +20 dB error line. For the 8-level data, the +6 dB curve is only about halfway towards the +20 dB error line.

4.5 Amplitude and phase errors vs cross-correlation phase

Figure 16 and Figure 17 show the peak-to-peak amplitude (%) and phase (degrees) error across the passband as a function of cross-correlation phase; these were computed from Figure 14 and Figure 15, calculated for a cross-correlation amplitude of 0.75. Note that, as predicted, the phase error is zero for cross-correlation phases of 0 or $\pi/2$, and peaks at around a $\pi/4$ phase difference. The amplitude distortion has only a very weak dependence on phase difference.

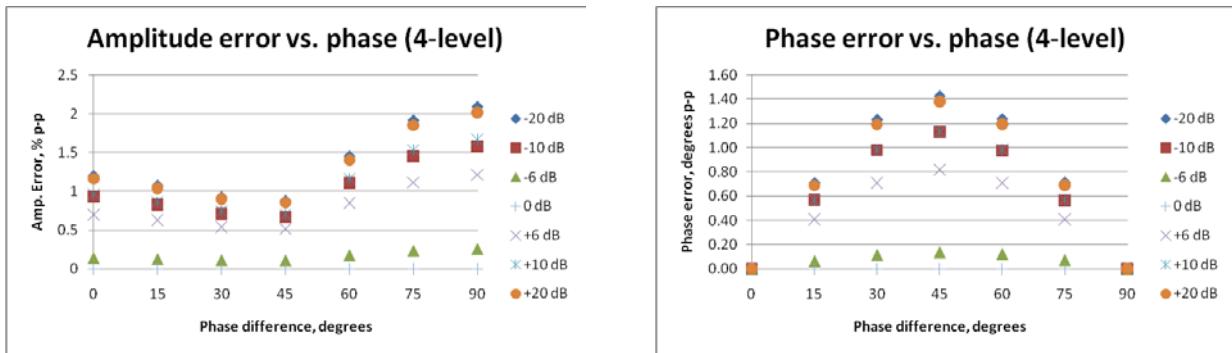


Figure 16 Amplitude and phase peak-to-peak error across the pass-band. 4-level, 2-bit data. Amplitude is shown in % change, and phase in degrees. This was calculated for a correlation coefficient of 0.75.

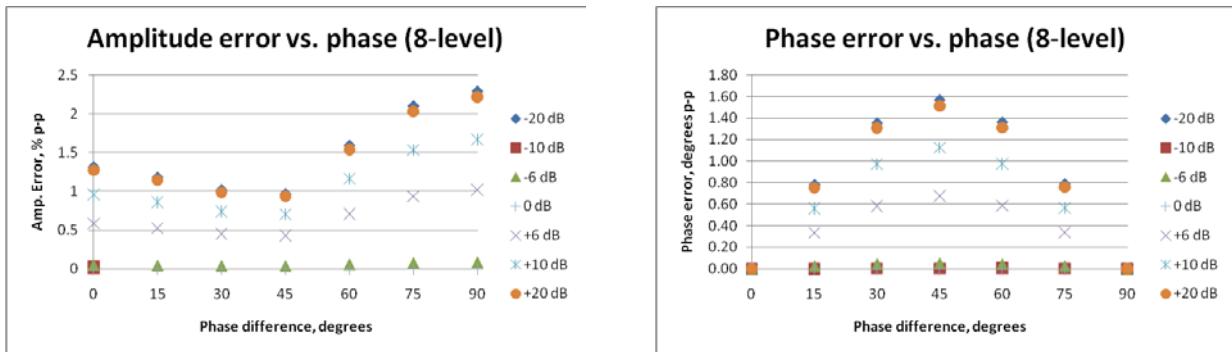


Figure 17 Amplitude and phase peak-to-peak error across the pass-band. 8-level, 3-bit data. Amplitude is shown in % change, and phase in degrees. This was calculated for a correlation coefficient of 0.75.

4.6 Errors vs correlation coefficient

Figure 18 and Figure 19 illustrate the increase in pass-band amplitude and phase error with increasing correlation coefficient, for input signal levels ranging from -20 dB to +20 dB from the optimum value. The cross-correlation phase was set at 45 degrees; see Figure 16 and Figure 17 for the dependence of errors on this phase. As is seen, the errors increase quite rapidly for correlation coefficients above about 0.5. Below that, even for input levels +/-20 dB away from optimum, pass-band errors are well below 1% in amplitude and below 1 degree of phase.

As expected, for both 4-level and 8-level digitization, the errors for +20 dB signal level are nearly identical to those at -20 dB signal level; in both cases, the sampler degenerates towards 1-bit system. Also as expected, the degeneration into that regime is more gradual with an 8-level scheme than with the 4-level. With both 4-level and 8-level quantization, the extreme peak-to-peak pass-band error, with $\sim \pm 20$ dB input level setting and high (>0.9) correlation coefficient, becomes about 3.5% in amplitude, and about 7 degrees of phase.

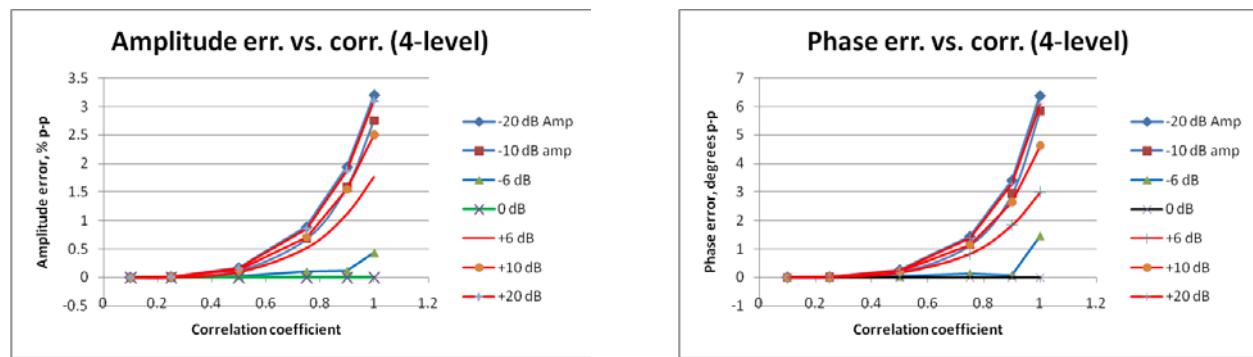


Figure 18 Pass-band peak amplitude and phase errors vs. correlation coefficient, for different input signal levels. 4-level, 2-bit data. Cross-correlation phase set to 45 degrees.

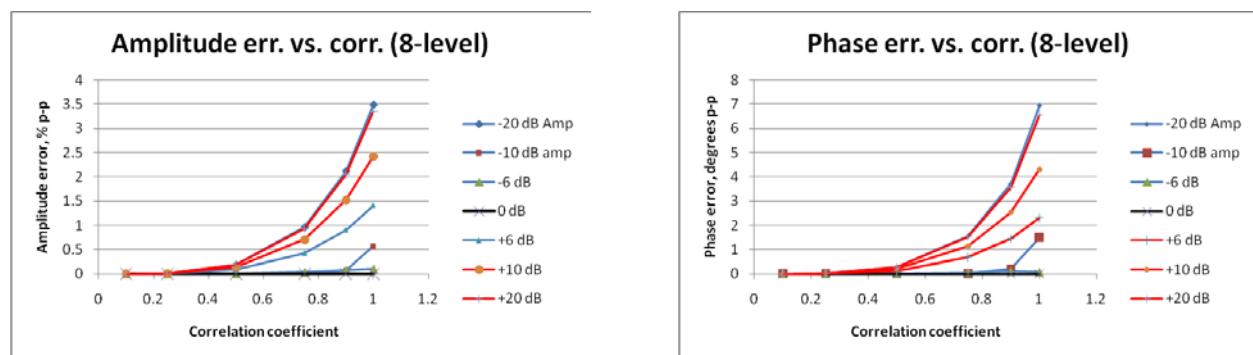


Figure 19 Pass-band peak amplitude and phase errors vs. correlation coefficient, for different input signal levels. 8-level, 3-bit data. Cross-correlation phase set to 45 degrees.

Even for 100% correlated signals – i.e. autocorrelation – as much as a 6 dB mismatch between the signal level and the VV correction only introduces <2% of peak-to-peak pass-band gain error. However, such a change in signal level is likely with some receivers, between the hot load calibrator and cold sky.

5 Narrow-band Harmonic Distortion within the Spectrum

The above has considered the amplitude and phase calibration errors likely to be introduced across the signal pass-band. This section considers the narrow-band spectral distortion produced by using an incorrect van Vleck correction, resulting in spurious spectral features.

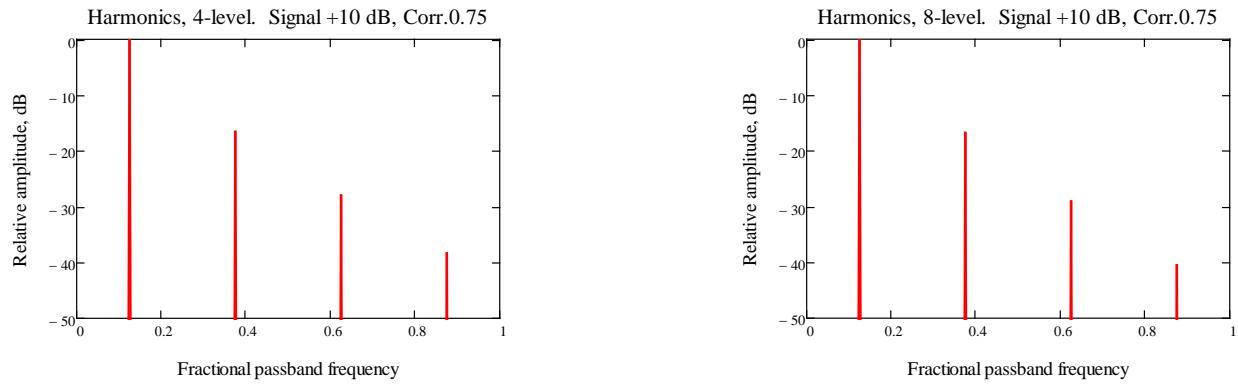


Figure 20 Example of harmonic distortion from a narrow-band signal in the passband. The signal is 10 dB stronger than the optimum, but the van Vleck correction wrongly has been applied for an optimum signal level. The left image is for a 4-level, 2-bit digitization, and on the right is the 8-level 3-bit spectrum. The signal was 75% correlated. The 3rd, 5th and 7th harmonics are obvious.

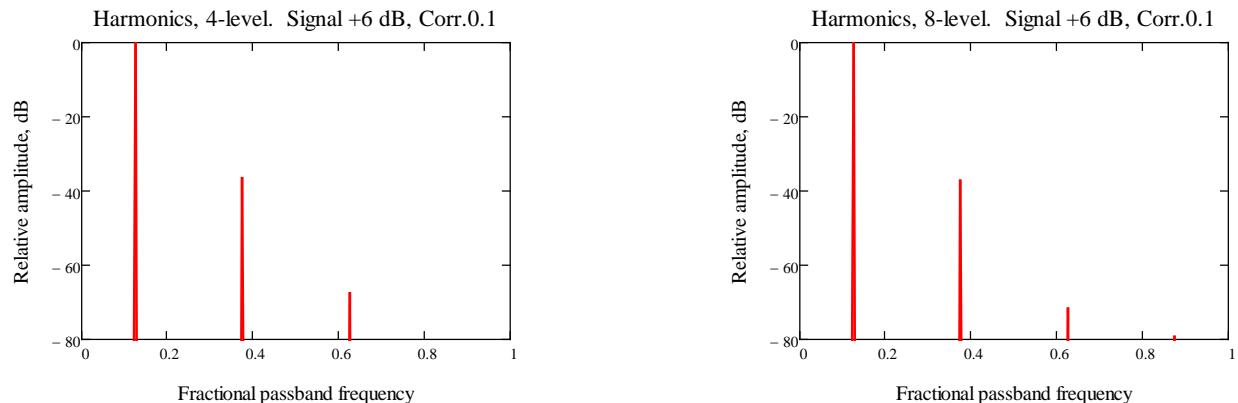


Figure 21 As above, but with signal +6 dB above optimum, and correlation coefficient 0.1. Note the change of vertical scale compared to Figure 20.

Figure 20 shows the harmonic distortion present when a signal 10 dB above the optimum level is digitized, but where a van Vleck correction appropriate for the optimum signal level has wrongly been applied. The input signal correlation coefficient was 0.75. The original signal is shown with amplitude 0 dB, and the 3rd, 5th and 7th harmonics stand out. This simulation is for a baseband signal. With the ALMA configuration, where the digitized signal is offset from DC, the same harmonics will be aliased back into the pass-band, and will appear with a less obvious harmonic relationship. Figure 21 shows the same as Figure 20, but where the signal input is now +6 dB wrt the optimum, and the correlation coefficient is only 0.1. Note the change in vertical scales between Figure 20 and Figure 21.

Figure 22 shows the total harmonic distortion power as a function of correlation coefficient, for signal levels from -20 dB to +20 dB compared to the optimum. As before, the van Vleck correction was applied as if the signal were still at the optimum value. Total Harmonic Distortion (THD) is defined here to be the sum of all harmonics up to and including the 7th, although the value is dominated by the 3rd harmonic.

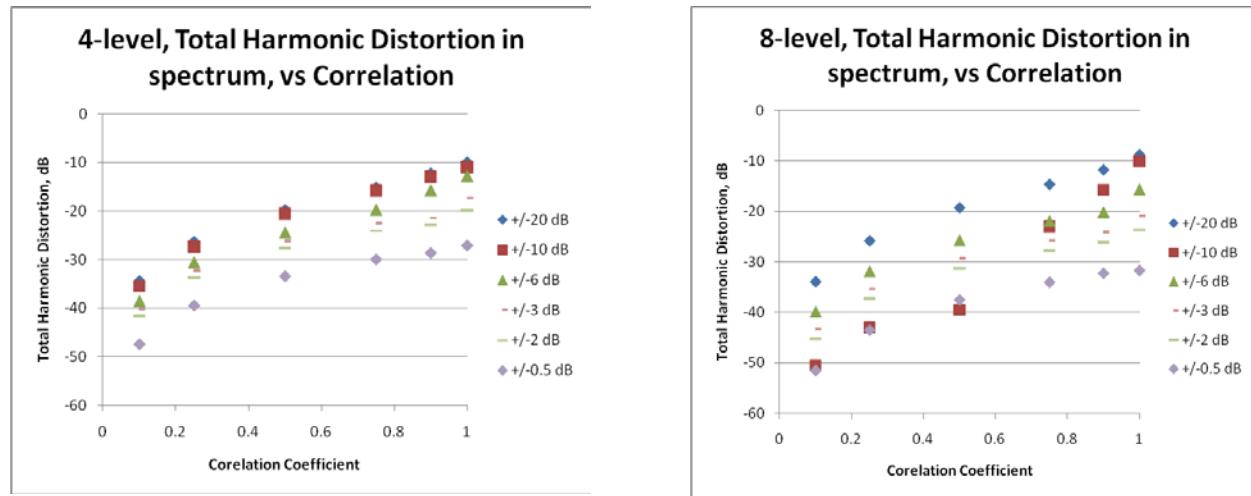


Figure 22 Total harmonic distortion vs. correlation coefficient, for signal levels from -20 dB to +dB wrt optimum

Since the distortion of the lag function is symmetric +/- of zero, only odd harmonics would be expected. Note that, if there is a significant DC offset at the samplers, then that symmetry is destroyed and so even harmonics will begin to appear. That is not considered further here.

Figure 23 shows the total harmonic distortion, adding the power in the 3rd, 5th and 7th harmonics, as a function of signal levels from -20 to +20 dB, for correlation coefficients of 0.1, 0.5, 0.9 and 1.0. Corresponding plots for 4-level, 2-bit and 8-level, 3-bit quantized data are shown.

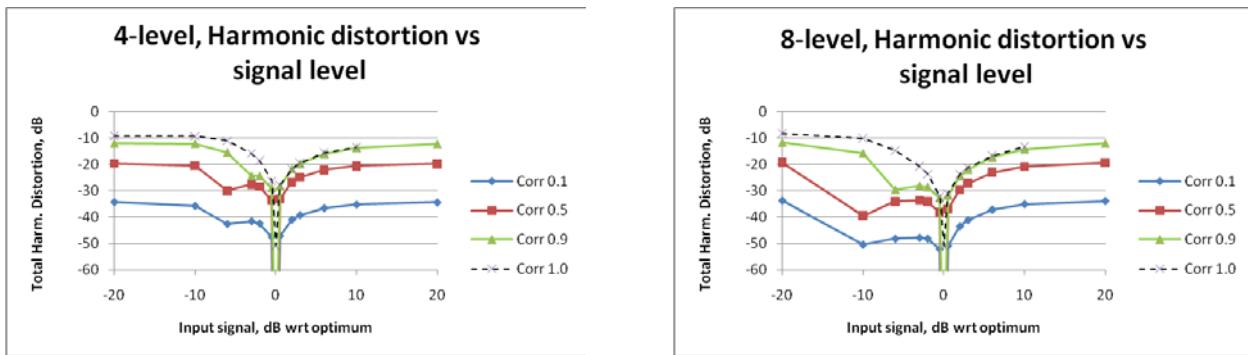


Figure 23 Harmonic distortion vs input signal

The ALMA requirement for spectral dynamic range is 10^4 , or 40 dB. To answer the question

“How close does the signal have to be from its matching van Vleck correction, in order that the spurious line features are sufficiently low?”

values were taken from Figure 23, and the permitted range of input signal level was derived, for a given level of THD. This is plotted in Figure 24. Values for matching positive and negative dB signal levels have been averaged, and the allowable change of input signal level for a THD of -30 dB and of -40 dB is plotted against correlation coefficient.

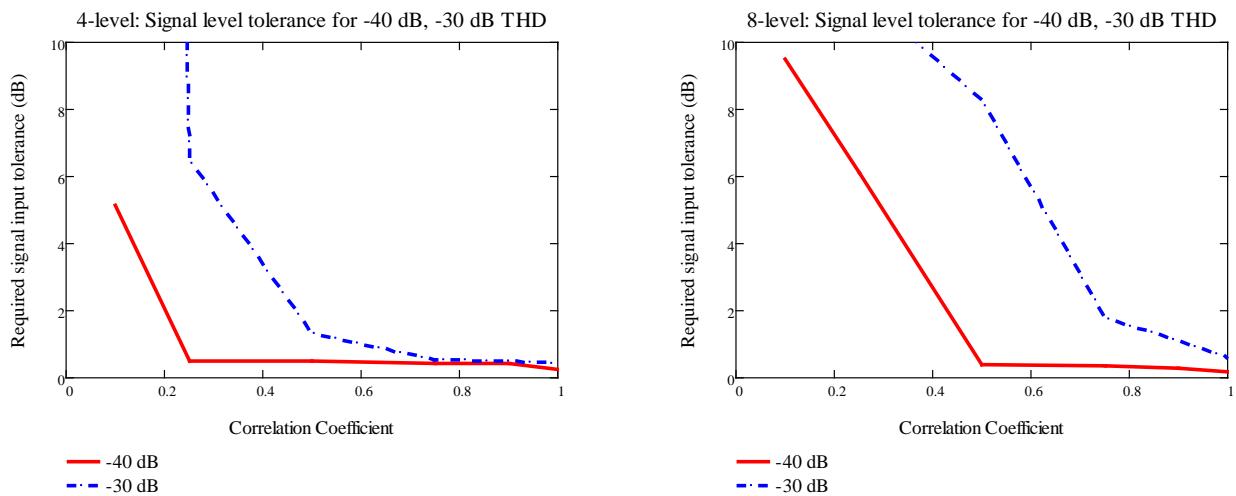


Figure 24 The input signal level tolerance for -30 dB and -40 dB THD, as a function of correlation coefficient.

Not surprisingly, for very low correlation coefficients, there is very little harmonic distortion – at sufficiently low correlation values, all van Vleck curves approximate well to linear. For 4-level data, at correlation coefficient of 0.1, a 5-dB mismatch between van Vleck correction and signal input level can be tolerated, without exceeding -40 dB THD. For 8-level data, nearly 10 dB mismatch can be tolerated.

However, for autocorrelation the situation is much more demanding. For 4-level data, in order that THD does not exceed -40 dB, the signal input level should match the van Vleck correction to within better than about +/-0.24 dB. For 8-level data the match should be within about +/-0.19 dB. If the requirement on spectral dynamic range is relaxed from 40 dB to 30 dB, then the autocorrelation signal input level would only need to be within about +/-0.43 dB (4-level) or +/-0.62 dB (8-level) of its matching VV correction. Note that other factors, such as bandpass calibration and stability, also affect spectral dynamic range.

These distortion terms, as well as the calibration factors described in Section 4, result from the variation in the shape of the VV correction, as a function of signal level. Note that for a 1-bit quantization, there is **no** change in shape of the VV correction for any input level, and so these calibration errors and distortion terms do not exist with purely 1-bit digitization.

6 Summary and Conclusions

As is well known, if the input signal to a digitizer having a limited number of quantization levels is not at its optimum value, then there is a loss of signal-to-noise ratio. This loss is shown in Figure 1 through Figure 4. However, if the correct VV correction is used to retabulate the measured lag function in an XF correlator, then this loss of signal-to-noise is the **only** consequence.

In principle, the form of the VV correction is different for every different level of input signal, so that ideally the specific VV correction curve should be matched to every change in signal level. For practical reasons, the VV correction may not always be updated sufficiently frequently to take account of small changes in input level. This implies calibration errors and the potential for spurious spectral features.

If an imperfectly matched VV correction is used, then:

- 1 There is a possibility of error in overall gain calibration. After normalization using the autocorrelation zero-lag factor, this can be mitigated provided the appropriate gain correction factor (see Figure 13) is taken into account, and provided the correlation coefficient between the cross-correlation input signals is sufficiently small. Even if this simple correction is not applied, the gain error may be relatively small. In order to keep the gain error below 1%, then a signal variation of about +/-2 dB can be tolerated (-1.65 dB to +1.88 dB for 4-level systems, and from -2.16 to +2.21 dB for 8-level).

If the correlation coefficient is relatively large, then additional errors appear, resulting from the different degrees of non-linearity inherent in the digitization process and in the VV corrections. These errors only appear if the specific VV correction is not matched perfectly to the specific input signal level.

- 2 There is an error introduced, in both amplitude and phase, as a function of frequency within the pass-band. The detail and magnitude of these errors depends on the correlation coefficient, the

degree of mismatch between signal level and VV correction, and on the relative phase of 2 inputs of a cross-correlator.

Even for 100% correlated signal, as much as a 6 dB signal level mismatch introduces a pass-band gain error of less than 2%. Similarly, even for 100% correlation and a 6 dB signal-VV correction mismatch, the peak cross-correlation phase error across the pass-band is only ~3 degrees or less. For a correlation coefficient no more than 0.6, even with a 20-dB signal level mismatch the peak-to-peak pass-band gain error is never more than 0.5% with either digitization scheme, and the peak-to-peak phase error is less than 1 degree.

- 3 There is harmonic distortion introduced into narrow-band signals within the pass-band. For signals sampled at baseband the precise harmonic frequencies are obvious, but for the offset-frequency ALMA sampling the harmonics are aliased back into the pass-band and the precise frequencies of the spurious features may not be so obvious. The magnitude of harmonic distortion depends on the degree of signal-to-VV correction mismatch, and on the correlation coefficient. For autocorrelation data, in order that THD does not exceed -40 dB, the signal input level should match the van Vleck correction to within about +/-0.2 dB for either 4-level or 8-level data.

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