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A DIGITAL CROSS-CORRELATION INTERFEROMETER

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I. Abstract

This report supersedes Electronics Division Internal Report No. 19, "A Prototype Digital Cross-Correlator for the NRAO Interferometer".

There is a growing need for high spatial resolution, extragalactic hydrogen line observations. The autocorrelation receiver may be used for such high resolution work in conjunction with the interferometer. This report considers the theoretical and practical requirements for a digital cross-correlation interferometer, and suggests a method of modifying the present NRAO autocorrelation receiver.

II. Theoretical Considerations

Weinreb [1] has suggested a method of cross-correlating gaussian noise voltages, which we shall consider for the correlation interferometer. If x(t) is a sample function of a gaussian random process with zero mean, and y(t) is the function formed by infinitely clipping x(t), we have

$$\begin{aligned} y(t) &= A \quad \text{when} \quad x(t) > 0 \\ y(t) &= 0 \quad \text{when} \quad x(t) = 0 \\ y(t) &= -A \quad \text{when} \quad x(t) < 0 \end{aligned}$$

where A is a constant. For infinite clipping we can ignore y(t) = 0. According to Van Vleck the normalized (to mean square) autocorrelation functions of x(t) and y(t) are related by

$$\rho_x(\tau) = \sin \left[\frac{\pi}{2} \rho_y(\tau)\right] \tag{1}$$

This is also true in our case for cross-correlation functions.

In order to cross-correlate the two noise voltages, each noise voltage is filtered (in "rectangular" filters from zero to B), infinitely clipped, sampled at 2B, and corresponding samples multiplier and summed (integrated). Since each sample can only have values +1 and -1, the product can only have these values. In fact, due to the nature of the digital equipment we let -1 be represented by zero, which will give $\rho_x(\tau) = \frac{1}{2}$ for zero correlation.

First we consider the hypothetical case of signals whose rms is equal to the total (system + signal) rms (i.e., zero system contribution). Considering the signals arriving from the IF amplifiers of an interferometer (from zero frequency upward), the correlated component varies cosinusoidally as a point source traverses the interferometer fringes on the sky. For the correlation interferometer in the region of zero delay (i.e., neglecting fringe amplitude modulation [2]) we have

$$\rho_x(\tau) = \cos 2\pi \frac{\tau}{T}$$

where

$$T = \text{fringe period.}$$

Also we have

$$\rho_x(\tau) = \sin \frac{\pi}{2} \rho_y(\tau)$$

from $\rho_x = 1$ to $\rho_x = 0$. Hence

$$\cos 2\pi \frac{\tau}{T} = \sin \pm [\frac{\pi}{2} \rho_y(\tau)]$$

$$= \cos \pm [\frac{\pi}{2} \rho_y(\tau) - \frac{\pi}{2}]$$

from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$. Thus we have

$$\rho_y(\tau) = \frac{4\tau}{T} - 1 \quad \text{from} \quad \tau = 0 \text{ to } \tau = \frac{T}{4}$$

$$\rho_y(\tau) = \frac{4\tau}{T} + 1 \quad \text{from} \quad \tau = \frac{-T}{4} \text{ to } \tau = 0.$$

This is a consequence of the fact that the Van Vleck equation is only concerned with magnitudes. Hence we should have the record shown in Figure I. This form of response is easily checked experimentally by varying τ in small steps.

Remembering that $\rho_y(\tau)$ is the observed correlation function, a shift in zero level and dynamic range of ρ_y does not affect the form of ρ_y. Hence, for a digital system where + 1 gives 1 and - 1 gives 0, ρ_y is as shown in Figure II provided that the interferometer signals are completely correlated at $\tau = 0$.

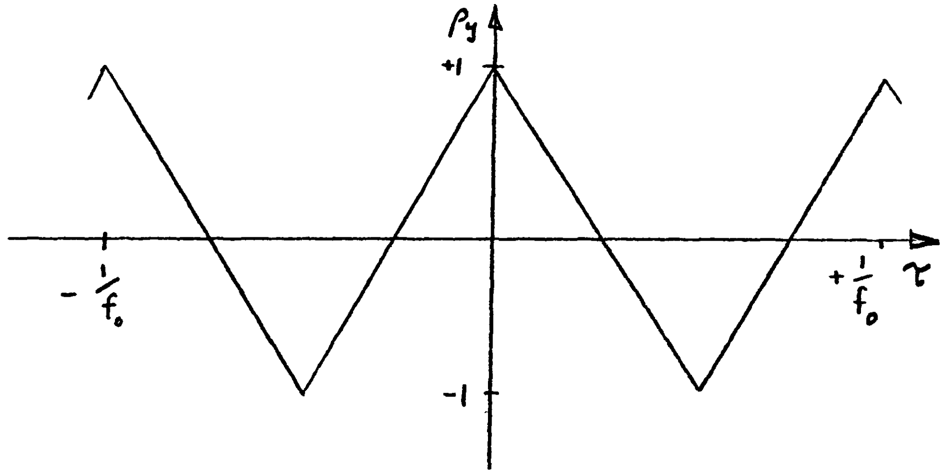


FIG. I

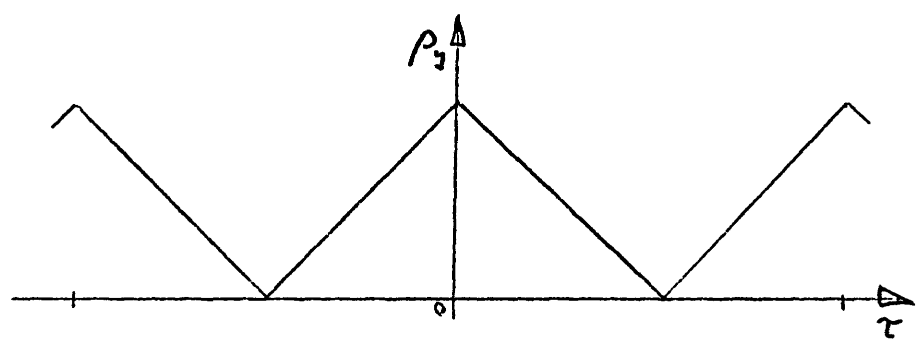


FIG. II

The above conditions are not fulfilled when the system noise temperature, T_S, exceeds zero. Let us consider signal-to-noise ratio in more detail.

First we must consider the degree of correlation of the clipped noise. Let a source be observed by an interferometer with spacing D/λ (Figure III). The correlated component at spatial frequency D/λ is T_U

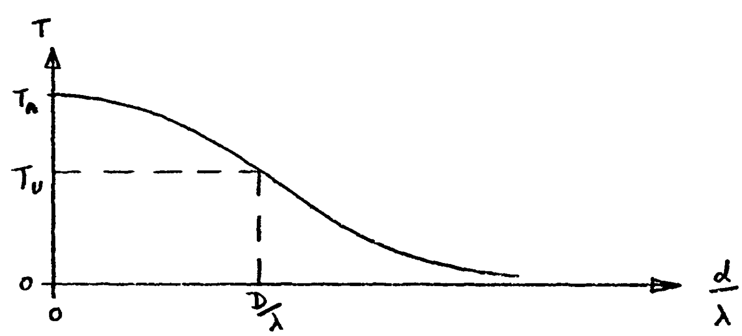


Figure III

From one antenna $$y_1 = a_1\left(t, \frac{d}{\lambda}\right) + n_1(t)$$

From another antenna $$y_2 = a_2\left(t - \tau, d + \frac{D}{\lambda}\right) + n_2(t - \tau)$$

Time average $$\overline{y_1 y_2}\ \frac{D}{\lambda} = \overline{a_1\left(t, \frac{d}{\lambda}\right) a_2\left(t - \tau, \frac{d + D}{\lambda}\right)}$$

And for signals $$a_1\left(t, \frac{d}{\lambda}\right) = a_2\left(t, \frac{d}{\lambda}\right)$$

$$\rho_y(\tau, D) = \frac{\overline{a\left(t, \frac{d}{\lambda}\right) a\left(t - \tau, \frac{d + D}{\lambda}\right)}}{\sqrt{\overline{y_1^2}\ \overline{y_2^2}}}$$

$$\rho_x(\tau, D) = \frac{T_U}{T_A + T_S}$$

Let n be the number of correlated counts, and N the total number of possible counts. Hence, for possible counts between 0 and 1 (zero correlation count = $\frac{1}{2}$)

$$\rho_y = \left(\frac{2n}{N} - 1\right)$$

remembering that a negative correlation coefficient can occur on the interferometer.

i.e.,
$$\rho_x = \sin \frac{\pi}{2}\left(\frac{2n}{N} - 1\right)$$

$$= \frac{T_U}{T_A + T_S} \cdot \cos 2\pi \frac{\tau}{T}$$

Thus $$\frac{n}{N} = \frac{1}{\pi} \sin^{-1}\left(\frac{T_U}{T_A + T_S} \cos 2\pi \frac{\tau}{T}\right) + \frac{1}{2}$$

The values of $\frac{n}{N}$ are shown in Figure IV for $T_A + T_S = 200$ °K taking various values of T_U.

The standard deviation of n is $\sqrt{N}$, and hence ϵ_{rms} is $\frac{1}{\sqrt{N}}$. Hence

$$\rho_x' = \sin\left[Z + \frac{\pi}{\sqrt{N}}\right]$$

where

$$Z = \frac{\pi}{2}\left(\frac{2n}{N} - 1\right)$$

But in all practical cases ($T_S \simeq 200$ °K, $T_U < 2$ °K) $n \rightarrow \frac{N}{2}$, and $\frac{\pi}{2} << \sqrt{N}$. Hence

$$\rho_x' \rightarrow Z + \frac{\pi}{2\sqrt{N}}$$

$$\epsilon_x = \frac{\pi}{2\sqrt{N}}$$

$$= \text{rms error in } \rho_x$$

For a possible count of 2^{20}, the maximum ϵ_x is $\frac{\pi}{2^{11}}$, i.e., approximately 0.0015. Hence the signal-to-noise ratio for $T_U = 5$ °K is approximately taking the fringe maximum as "signal". Hence for fringe amplitudes representing < 5 °K, a larger value of N should be used.

Two important facts emerge from the above considerations:

(a) Fringe phase is determined by the zero crossover of the sawtooth waveform (Figure IV). The accuracy of this crossover depends strongly on the stability of the clipping zero.

(b) Since the signal has been clipped, amplitude information can only be obtained by comparing the correlated (interferometer) component with the uncorrelated (system noise) component.

The method of obtaining amplitude information is to compare the fringe amplitude (T_U) with the total power interferometer output (addition, square-law detection and integration). A total power interferometer record is shown in Figure V(a). In Figure V(b) the increase in average (DC) level in the region of the fringes is due to a finite source, so that the source is partially resolved at the interferometer spacing considered. This is equivalent to a temporary increase in system noise and must be taken into account when determining fringe amplitudes.

III. Pratical System

The maximum interferometer fringe rate is given by:

$$\frac{dH}{dt} = \frac{dH}{dt}\frac{b}{\lambda}\cos d \text{ (fringes per second).}$$

For a 1200 meter baseline at 1420 MHz, the NRAO interferometer fringe rate would be 0.385 per second. Hence the longest integration (after primary cross-correlation) should be < 0.4 seconds if optimum fringe signal-to-noise ratios are required. For longer baselines the maximum integration time should be proportionately shorter. We will see that integration times should actually be somewhat shorter than $\frac{1}{\omega_{fringe}}$, since smearing of the correlation coefficient occurs due to the variation of fringe amplitude -- especially in the region of the fringe zero. In the NRAO autocorrelation receiver, the maximum autocorrelation delay corresponds to 100 samples, so that primary correlation takes

$$\frac{100}{2B} \text{ seconds.}$$

For highest spectral resolution (B = 62.5 kHz) the primary correlation takes 800 microseconds, and for the lowest resolution (B = 2.5 MHz) the primary correlation time is 20 microseconds. In order to permit approximately the same integration time for correlation coefficients of all delays (maximum delay $\equiv$ 100 samples for all bandwidths), continuous correlation should occur for at least 10 times the maximum delay time. For the fringe rate considered, an eight millisecond continuous correlation would cause a two percent "smearing" of the correlation coefficient in the region of fringe amplitude zero, and less ($\sim \cos \phi$, where ϕ is the fringe phase relative to a point of maximum amplitude) away from this region. We have to switch between the spectral region containing hydrogen line emission (or absorption) and the region outside it, by switching the local oscillator frequency. If this switching rate is 100 Hz, there will be 5 milliseconds of integration with each LO frequency: the period of this switch driver should be the continuous correlation interval, and should be driven by the correlation receiver logic. Even if the integrators are cleared every 10 switching cycles (every 50 msec) the integration time is too short for the existing autocorrelation receiver, unless some considerable alterations were to be made. However, such integration times could easily be attained in a new autocorrelation receiver. The major limitation of such short integrating times is the magnetic tape recording speed: an on-line computer is the only method by which the data could be removed from the hundred channels if $\frac{1}{10}$ fringe resolution is required at this fringe frequency. It is noteworthy that 5 milliseconds is less than the correlation time at the narrowest bandwidth (highest resolution) by a factor of two. Corrections for this could be applied in the computer. For the moment, however, we concern ourselves with the extragalactic possibilities, and hence we consider the wider bandwidths.

Since correlation coefficients for the various delays are very closely spaced in time, any variation of power spectrum with fringe phase represents a relative shift in the positions of the various velocity components.

IV. Equipment Considerations

The system is shown in Figure VI. The cross-correlation system is for less sensitive to gain fluctuations than the autocorrelation system.

An interesting point arises in using clipped digital cross-correlation, as is demonstrated in equation (1) and Figure IV. This is the relative unimportance of the clipping correction for sources whose (unresolved) antenna temperatures are much lower than system noise temperatures -- as is the normal case in radio astronomy.

Since we are considering a single passband (image rejection) system, IF delay tracking now results in a movement of the fringes on the sky as IF delays are switched in and out. This means that only relative positions of the various velocity components may be determined, since absolute position calibrations are much more difficult to achieve than in the double-passband (continuum) interferometer.

For convenience we consider the widest bandwidth, such as would be used for extragalactic observations. From the point of view of spectral resolution, the autocorrelation receiver is equivalent to a very good 50-channel multifilter receiver, although from the point of view of system noise, we must assume 100 channels; with a 2.5 MHz total bandpass we have individual equivalent single filters of 25 kHz passband. Hence 200 °K system temperatures will give a rms noise of

$$\frac{\pi}{2} \cdot \frac{\cdot\, 200}{\sqrt{2.5 \times 10^4 \times 0.5\, N}} = \frac{10}{\sqrt{N}}\ ^\circ K$$

using a correlation system which switches from hydrogen to continuum.

We measure N continuous correlation coefficients, stacking observations according to the calculated fringe phase. Hence, this is the noise per channel (assuming all noise to be gaussian). Assuming a fringe amplitude corresponding to $\frac{1}{2}$ °K, a signal-to-noise ratio of 10 is attainable if N = 40,000 (we have to remember that we do not have sine and cosine interferometer responses). This represents a 33 minute integration time, during which time the projected baseline length and orientation on the source will have changed appreciably. In five minutes, however, integration will give a smearing of the fourier component smaller than the effect of finite apertures in the interferometer at a baseline of 1200 m. It should be borne in mind that the fourier components are slightly different for each equivalent passband, this effect being < 0.2 percent. However, it is necessary to take this effect into account since it will cause a phase shift, which could be mistaken for a position shift of one velocity component relative to another.

If the local oscillator were not switched, a signal-to-noise improvement of $\sqrt{2}$ would shorten required integrating times by a factor of two. The problem of gain variations would not concern us here, since receiver noise voltages are uncorrelated [2]. This method is used at present by Weinreb [1] for single antenna work.

Conclusions

A system has been outlined which uses the principle of the present autocorrelation receiver, in conjunction with an on-line computer, to determine relative strengths and positions of small size velocity components in radio sources displaying significant hydrogen line emission or absorption. Spatial resolution better than 5 arc seconds should be attainable. A seven-day integration of one region (dictated by individual antenna beamwidths) can give a signal-to-noise ratio of 10 for a $\frac{1}{2}$°K signal. Except for the difficulty of obtaining absolute phases of various velocity components, and the phase corrections required for spectral interferometry, the data available from such a system is similar to that obtained from the NRAO continuum radiometer [2]. For an interferometer baseline of 100 meters, integrators could be cleared every half minute. This would permit use of the present autocorrelation receiver without an on-line computer. Spatial resolution would not now be sensibly better than for observations using the 300-foot antenna, but contributions due to spatial continuum hydrogen emission (or absroption) would be resolved out by the interferometer.

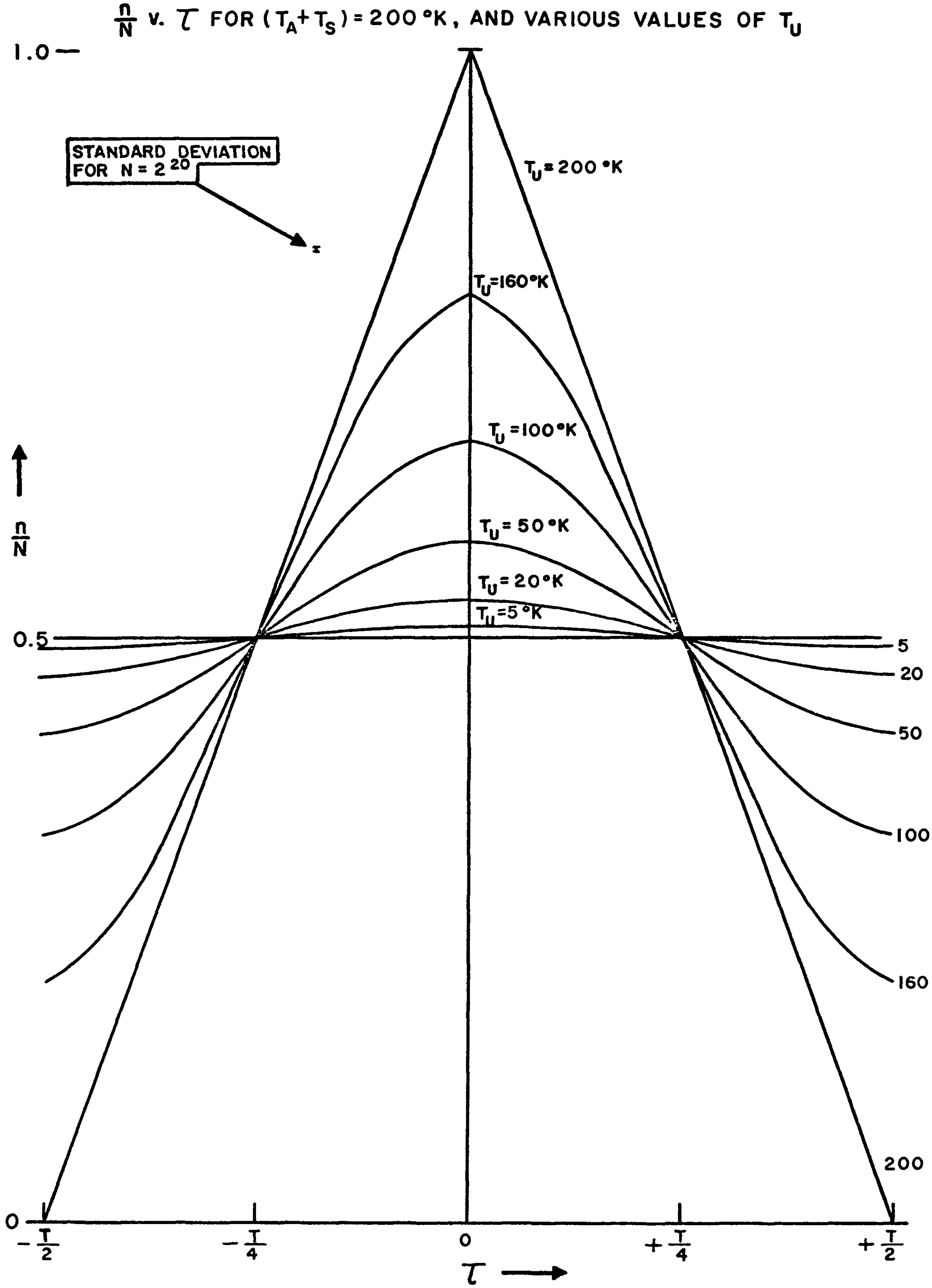
The use of a "lobe rotator" would slow down the fringes, although the use of two local oscillators might complicate this problem. For the unswitched system, however, the solution is relatively straightforward, and lobe-rotating facilities exist in the delay switching computer at NRAO [3], requiring only minor modifications to existing equipment.

References

[1] Weinreb, S., 1963, MIT, Electronics Research Laboratory Technical Report No. 412.

[2] Keen, N. J., 1964, NRAO Electronics Division Internal Report No. 40.

[3] Keen, N. J., 1964, NRAO Electronics Division Internal Report No. 41.



$\frac{n}{N}$ v. τ FOR $(T_A + T_S) = 200\,°K$, AND VARIOUS VALUES OF T_U
1.0
STANDARD DEVIATION FOR $N = 2^{20}$
$T_U = 200\,°K$
$T_U = 160\,°K$
$T_U = 100\,°K$
$T_U = 50\,°K$
$T_U = 20\,°K$
$T_U = 5\,°K$
$\frac{n}{N}$
0.5
5
20
50
100
160
200
0
$-\frac{T}{2}$
$-\frac{T}{4}$
0
$+\frac{T}{4}$
$+\frac{T}{2}$
τ

Fig. IV

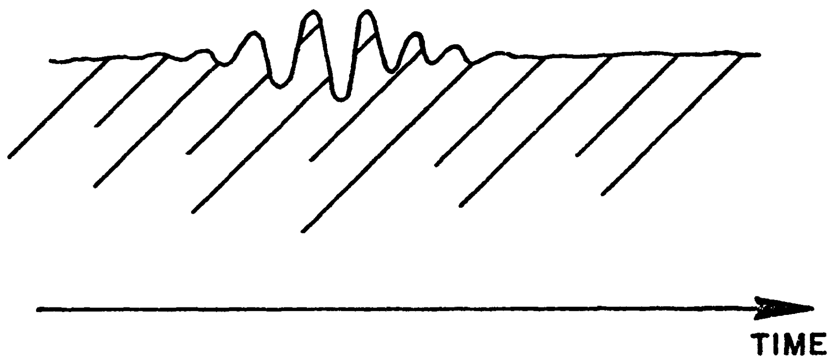


FIGURE V (a)

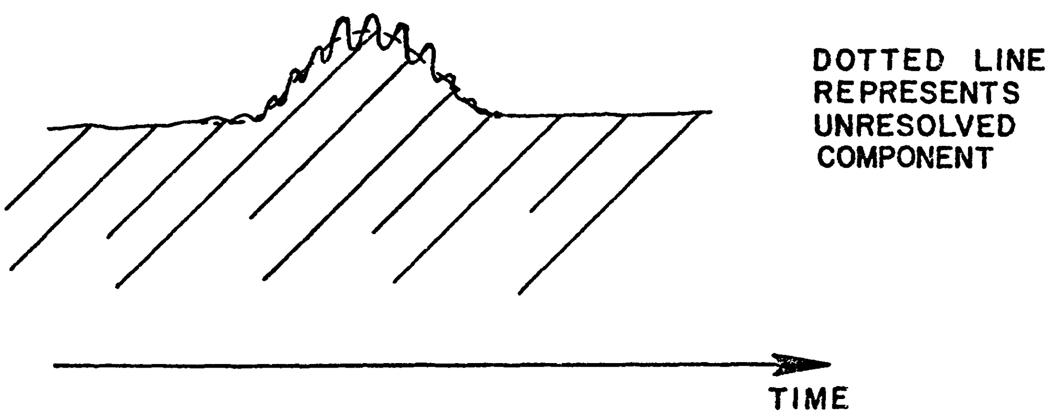


FIGURE V (b)

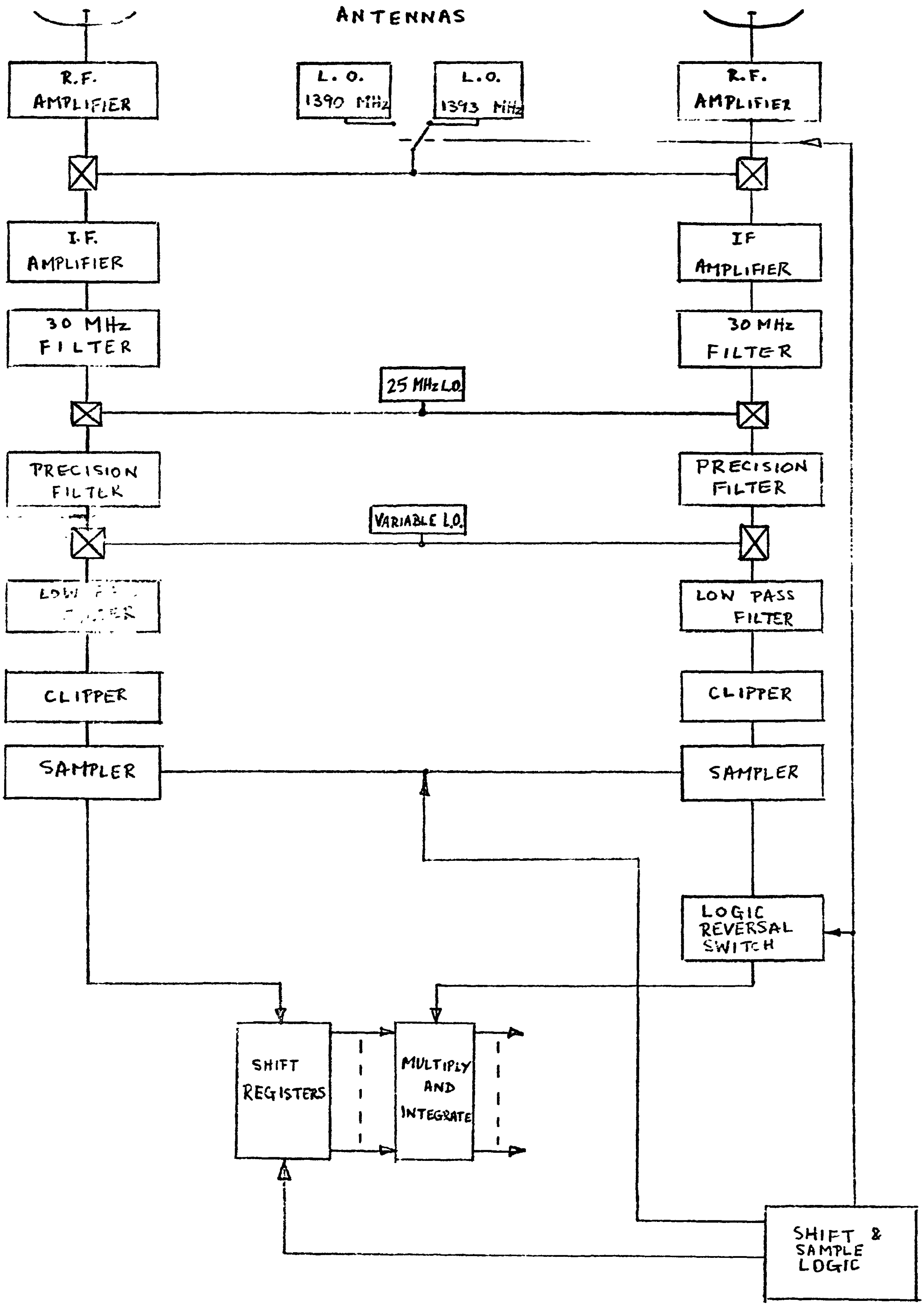

ANTENNAS
R.F. AMPLIFIER
L.O. 1390 MHz
L.O. 1393 MHz
R.F. AMPLIFIER
I.F. AMPLIFIER
IF AMPLIFIER
30 MHz FILTER
30 MHz FILTER
25 MHz L.O.
PRECISION FILTER
PRECISION FILTER
VARIABLE L.O.
LOW PASS FILTER
CLIPPER
CLIPPER
SAMPLER
SAMPLER
LOGIC REVERSAL SWITCH
SHIFT REGISTERS
MULTIPLY AND INTEGRATE
SHIFT & SAMPLE LOGIC

FIGURE VI