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ANTENNA FEED EFFICIENCY AND SPILLOVER CALCULATION PROGRAM

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A. Introduction

The program that is described in this report calculates the reduction in aperture efficiency of a paraboloidal antenna due to tapered illumination and spillover. The input of the program is the paraboloid F/D ratio and the feed pattern, specified in decibels relative to the maximum gain, at angular increments (typically 10°) from 0° to 180°. The output of the program is the taper efficiency, the spillover efficiency, the total efficiency (product of taper and spillover), and the spillover temperature at zenith, assuming a ground radiation temperature of 290 °K.

The program is intended as an easily used evaluation of feeds and does not predict the total antenna efficiency. This total efficiency will be reduced by feed support blockage, reflector phase errors, and usually to a much smaller extent by feed phase errors, cross polarization, and feed VSWR.

A one dimensional feed pattern is accepted by the program and the results are calculated assuming this pattern is circularly symmetric. Since the E- and H-plane patterns of a feed are usually different, the program should be used separately for each pattern and the results should then be averaged.

B. Theory

The taper efficiency, η_T, is defined as the ratio of the effective area of a paraboloid with tapered illumination to the physical projected area of the paraboloid. Various texts express η_T in terms of a radial distribution of electric field, E(r), over the aperture,

$$\eta_T = \frac{2\left|\int_0^1 E(r)rdr\right|^2}{\int_0^1 \left|E(r)\right|^2 rdr}$$

where r is normalized to the paraboloid radius and a uniform phase distribution is assumed. The field distribution is then expressed in terms of the feed power pattern,

$G(\Theta)$, using the relations,

$$E(r) = \sqrt{G(\Theta)}\ \cos^2 \Theta/2$$

$$rdr = 8\left(\frac{F}{D}\right)^2 \cdot \frac{\sin\ \Theta/2}{\cos^3 \Theta/2} \cdot d\Theta$$

where Θ is the angle with respect to the feed pattern maximum.

The taper efficiency can then be expressed as

$$\eta_T = 32\left(\frac{F}{D}\right)^2 \frac{\int_0^{\Theta_0} G(\Theta) \tan \Theta/2\ d\Theta^2}{\int_0^{\Theta_0} G(\Theta) \sin \Theta\ d\Theta}$$

where $\Theta_0 = 2 \tan^{-1} D/4F$ is the angle subtended by the edge of the paraboloid.

The spillover efficiency, η_s, is defined as the ratio of energy incident upon the reflector to the total energy emitted by the feed. It is calculated by integration of the feed radiation in rings of solid angle, $2\pi \sin \Theta\ d\Theta$, to give,

$$\eta_s = \frac{\int_0^{\Theta_0} G(\Theta) \sin \Theta\ d\Theta}{\int_0^{\pi} G(\Theta) \sin \Theta\ d\Theta}$$

The program computes and prints η_T, η_s, and the total efficiency, $\eta_T\eta_s$, all expressed as percentages.

The antenna temperature due to spillover with the antenna pointed at zenith is proportional to the fraction of feed radiation striking the ground, i.e., the radiation between Θ_0 and 90°. The zenith antenna temperature, T_z, is then given by

$$T_z = 290 \cdot \frac{\int_{\Theta_0}^{\pi/2} G(\Theta) \sin \Theta\ d\Theta}{\int_0^{\pi} G(\Theta) \sin \Theta\ d\Theta}$$

where a ground brightness temperature of 290 °K is assumed. This quantity is also computed and printed by the program.

The antenna temperature due to spillover with the antenna pointed at the horizon is given by 290 $(1 - \eta_s)/2$, where a 290 °K ground brightness temperature is assumed. This quantity is not printed by the program but is easily computed from η_s.

C. Program Description

A listing of the program is given in Figure 1. The program is written in Fortran IV for an IBM 360 Model 50 computer.

The integrations in the program are performed by the subroutine "QTFE" which is in the IBM subroutine library. This subroutine calculates integrals using a trapezoid area element between data points. An earlier version of the program used the subroutine "QSF" which calculates integrals by using a parabolic area element. This subroutine produced erroneous results because the integral value was slightly dependent on whether there was an odd or even number of points defining the integrand; the spillover calculation was critical to this error.

The program interpolates between data points at the reflector edge; i.e., integrals will be calculated to a reflector edge angle of 61° even though the pattern is specified only at 60° and 70°.

D. Preparation of Data Cards

There are two data cards for each pattern. Examples of data cards are shown in Figure 2. The format is as follows:

First Card

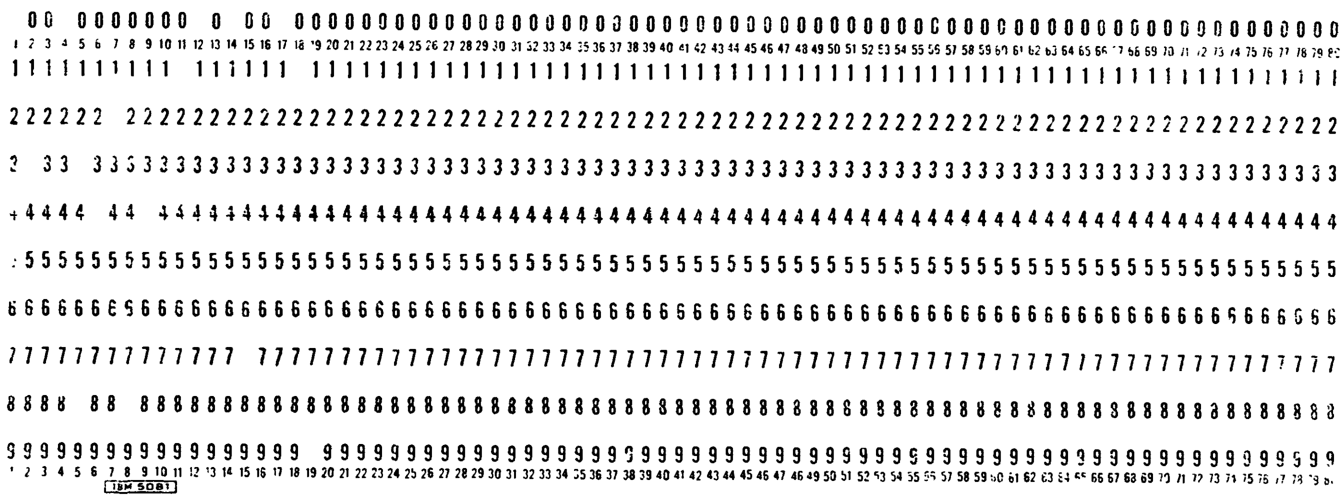
Columns

1-2	A data identification number; this number will be printed with the result. Example: 03.
4-9	Paraboloid F/D ratio expressed with 4 decimal places. Example: 0.4284.
11-12	Incremental angle between pattern points. Example: 10.

```
      DIMENSION G(50),G2(50),G3(50),F2(50),F3(50)
      INTEGER*4A,B,C
      WRITE(6,3)
    3 FORMAT('1')
    6 READ(5,1,END=10)L,FD,H,A,N,(G(I),I=1,N)
    1 FORMAT(I2,1X,F6.4,1X,F2.0,1X,I2,1X,I3/(20F4.1))
      B=A+1
      C=A-1
      K=90./H
      K1=K+1
      K2=K+2
      HR=H*3.1415926535/180.
      TH=2*ATAN(1./(4.*FD))
      DO 20 I=1,N
   20 G2(I)=1./(10.**(G(I)/10.))*SIN(HR*FLOAT(I-1))
      CALL QTFE (HR,G2,F2,N)
      VAL1=F2(N)
      VAL2=F2(A)
      VAL12=F2(B)
      VAL30=F2(K1)
      VAL31=F2(K2)
      DO 30 I=1,B
   30 G3(I)=1./(10.**(G(I)/20.))*TAN(HR*FLOAT(I-1)/2.)
      CALL QTFE (HR,G3,F3,B)
      VAL3=F3(A)
      VAL13=F3(B)
      VAL22=VAL2+((VAL12-VAL2)*(TH-(HR*FLOAT(C)))/HR)
      VAL23=VAL3+((VAL13-VAL3)*(TH-(HR*FLOAT(C)))/HR)
      VAL33=VAL30+(VAL30-VAL31)*(90./H-FLOAT(K))
      VAL4=100.*32.*(FD*VAL23)**2/VAL22
      VAL5=100.*VAL22/VAL1
      VAL6=VAL4*VAL5/100.
      VAL7=290.*(VAL33-VAL22)/VAL1
      WRITE(6,2)L,VAL4,VAL5,VAL6,VAL7
    2 FORMAT(////,2X,I2,6X,'TAPER EFF',8X,F4.1//,10X,'SPILLOVER EFF',4X,
     1 F4.1//
     1 10X,'TOTAL EFF',8X,F4.1//,10X,'SPILLOVER TEMP',2X,F5.1)
      GO TO 6
   10 STOP
      END
```

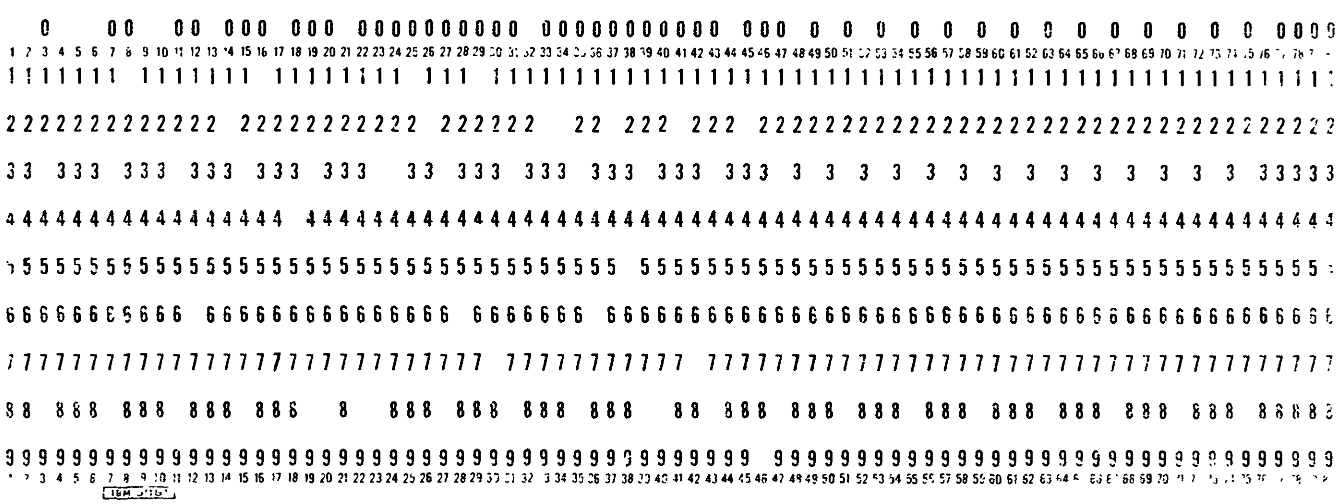
Figure 1 — Program Listing Written in Fortran IV for an IBM 360 Model 50 Computer.

```
03 0.4284 10 07 019
```



First Card

```
00.000.100.602.104.808.312.617.022.625.827.029.030.030.030.030.030.030.030.0
```



Second Card

Figure 2 — Examples of data cards. As many pairs of cards as desired may be run in one program.

First Card (continued):

Columns

14-16	Number of pattern points falling on the reflector. Example: For F/D = 0.42 the reflector edge is 61° and hence for 10° increments this number is 7.
17-19	Number of pattern points to 180°. Example: For 10° increments this number is 19.

Second Card

This card has the feed gain in dB (positive values) starting at the beam center and going to 180°. The value is given by two digits, decimal point, and another digit. There is no space between values. Example:

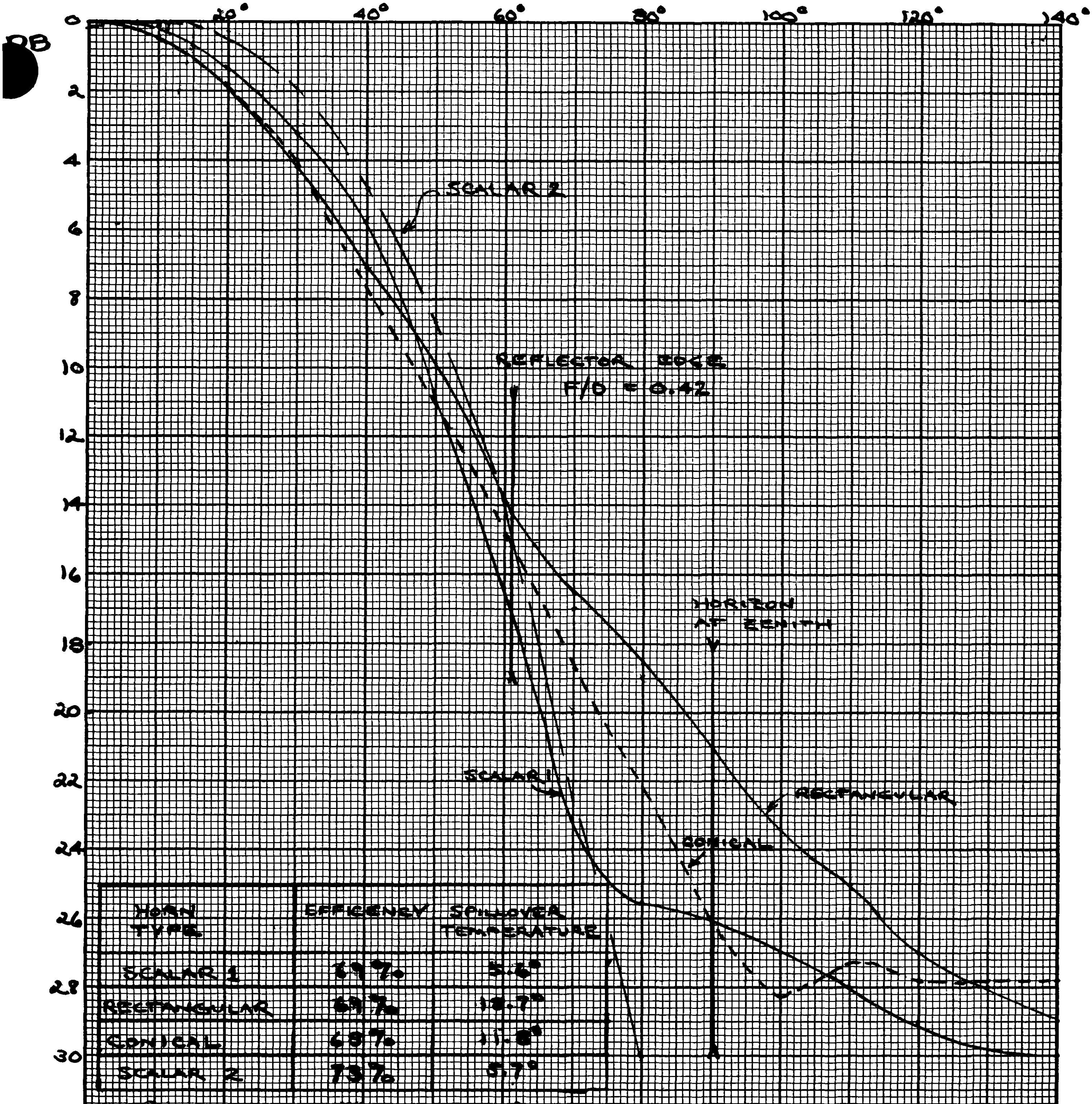
Columns

1-4	00.0
5-8	00.1
9-12	02.1
⋮	⋮
73-76	30.0

E. Typical Results

The program output for various types of feeds is given in Table 1. Patterns, total efficiency, and spillover temperature (all average of E- and H-plane values) for four of these feeds are shown in Figure 3.

Results on the square horn are in good agreement with results calculated by a program developed by Scientific-Atlanta, Inc. The NRAO program gives an E-H average of 86.2% and 86.8% for taper and spillover efficiency, whereas the alternate program gives 84.8% and 89.1%. The small differences may be due to the 10° pattern increments and E-H plane averaging in the NRAO program.



HORN TYPE	EFFICIENCY	SPILLOVER TEMPERATURE
SCALAR 1	[illegible]	[illegible]
RECTANGULAR	69%	[illegible]
CONICAL	68%	11.8°
SCALAR 2	73%	5.7°

Figure 3 — The patterns, total efficiency, and spillover temperature for 4 feeds is shown. The plotted pattern is the average of E- and H-plane patterns and the efficiency and spillover temperature are the average results of two computations assuming the E- and H-plane patterns are circularly symmetric. "Scalar 1" is a 10.69 GHz, 6.55 λ scalar feed while "Scalar 2" is a 1.40 GHz, 2.85 λ sealar feed. The rectangular horn was 1.16 λ by 0.85 λ, and the conical horn had 1.0 λ inside diameter.

TABLE 1

Results of Efficiency and Spillover Temperature Calculations for Various Feed Horns

Data Number	Feed Description	Plane	Taper Efficiency %	Spillover Efficiency %	Total Efficiency %	Spillover Temperature °K
1	Variable Polarization	E	76.3	95.1	72.6	11.6
2	1.67 GHz Scalar	H	77.6	95.7	74.3	10.9
22	10.69 GHz Scalar	E	68.7	97.0	66.6	5.5
21	6.55 λ Diameter	H	73.7	97.3	71.7	5.7
32	1.67 GHz Scalar	E	74.9	96.6	72.3	7.2
31	7.55 λ	H	78.2	96.5	75.5	7.9
29	1.4 GHz Scalar	E	72.4	97.9	70.8	5.0
30	2.85 λ Diameter	H	77.2	97.4	75.2	6.4
24	10.69 GHz Rectangular	E	76.1	90.0	68.5	22.0
23	Horn, 1.16 λ x 0.85 λ	H	75.5	93.2	70.3	15.4
27	Square Horn	E	85.7	88.1	75.5	28.0
28	0.9 λ Square	H	86.8	85.6	74.3	34.9
26	Conical Horn	E	73.4	94.2	69.1	11.7
25	1.0 λ Inside Diameter	H	70.5	95.1	67.0	11.9

F. Some Useful Approximations

In order to develop some feeling for the relation between the feed pattern and spillover, some approximate formulas and examples will be given.

The fraction of power, ϵ, transmitted by an antenna of gain, $G(\Omega)$, into a solid angle, Ω_0, is given by

$$\epsilon = \frac{1}{4\pi} \int^{\Omega_0} G(\Omega)\, d\Omega$$

If $G(\Omega)$ is assumed constant over Ω_0, we have,

$$\epsilon = \frac{\Omega_0}{4\pi} G(\Omega_0)$$

The feed gain can be expressed as the ratio of the on-axis feed gain, G_0, to the pattern function, P. For an F/D = 0.42 paraboloid, 10 log G_0 is approximately 10 dB. Thus the fraction of power in Ω_0 can be expressed as

$$\epsilon = \frac{\Omega_0}{4\pi} \cdot \frac{10}{P}$$

where $P = 10^{P_{dB}/10}$ is the average pattern in Ω_0.

Example 1

Suppose Ω_0 is the complete half-sphere extending 90° to 180° from the feed axis. Thus $\Omega_0 = 2\pi$ and we can make the following table:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Horizon
20 dB	5.0 %	7.5°
25 dB	1.7 %	2.5°
30 dB	0.5 %	0.7°

Example 1 (continued):

The conclusion is that patterns greater than 30 dB down have negligible effect on efficiency or antenna temperature. The feed pattern can be measured on a pattern range with 30 dB dynamic range.

Example 2

Suppose Ω_0 is the segment extending from 60° to 90° from the feed axis. This is the solid angle which the ground subtends for an F/D = 0.42 paraboloid pointed at zenith. The value of Ω_0 is π and we have:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Zenith
15 dB	7.5 %	21.8°
20 dB	2.5 %	7.3°
25 dB	0.7 %	2.2°
30 dB	0.2 %	0.7°

Thus the spillover is very critical to average patterns of 15 dB (poor) to 20 dB (good) in this solid angle.

Example 3

Considering the range of 60° to 70° from the feed axis, $\Omega_0 = .316\,\pi$, and we obtain:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Zenith
13 dB	4.0 %	11.6°
14 dB	3.2 %	9.3°
15 dB	2.5 %	7.3°
16 dB	2.0 %	5.8°
17 dB	1.6 %	4.6°
18 dB	1.3 %	3.7°
20 dB	0.8 %	2.3°
23 dB	0.4 %	1.2°

Example 3 (continued):

(Note that P_{dB} refers to pattern measurements; do not include 3 dB for space attenuation.) Since Ω_0 will be approximately the same for 70° to 80° and 80° to 90° the above table can also be used for these increments. The zenith spillover temperature can then be evaluated by use of the table. For example, the rectangular horn pattern plotted in Figure 1 gives the following results:

Angle Range	Average Pattern Value	Spillover Temperature at Zenith
60° - 70°	15	7.3°
70° - 80°	20	2.3°
80° - 90°	23	1.2°
Total	-------------------------	10.8°

This is in good agreement with the 11.8° value computed by the program.