A PROGRESS REPORT ON JOSEPHSON JUNCTION MIXERS
FOR MILLIMETER WAVE RADIO ASTRONOMY

D. R. Decker

FEBRUARY 1978

NUMBER OF COPIES: 150
A PROGRESS REPORT ON JOSEPHSON JUNCTION MIXERS
FOR MILLIMETER WAVE RADIO ASTRONOMY

D. R. Decker

ABSTRACT

Interest in use of Josephson junction devices for millimeter and sub-
millimeter wave detection and mixing is high because these devices potentially
offer high-sensitivity, low-noise detection capability to about 1000 GHz
(0.3 mm wavelength)\(^{1-4}\). However, many practical problems have impeded their
early application for this purpose. An ongoing program of research and develop-
ment of Josephson junction mixers for application to millimeter wave radio
astronomy at NRAO has focused on use of Nb microbridge junctions in 80-120 GHz
mixers. This report gives an account of the status of the current program and
evaluates the relative merit of alternative approaches to continuing the develop-
ment of these devices for mixer application.
1. Introduction

The potential utilization of Josephson junction devices as low noise heterodyne mixers in the millimeter and submillimeter bands has been under investigation for several years\(^1\)\(^-\)\(^10\). Recently, encouraging mixer performance has been reported for niobium point contact junctions at millimeter wave frequencies\(^11\). Also, mixer operation of variable thickness microbridges (VTB's) has been observed at X-band\(^12\). Although the point contact junctions have to date exhibited better performance than the microbridges, difficulties with mechanical stability and reproducibility have precluded usage outside the laboratory environment and have led to a search for more stable, reproducible junctions for use in practical mixers. The mechanical stability and reproducibility of point contacts appears to be improved by recent attempts to fabricate stable junctions\(^11,13\). However, there is still strong interest in using variable thickness microbridge junctions in millimeter wave mixers for improved stability and cyclability.

Understanding of the performance limitations of variable thickness microbridges has evolved in recent years so that size effects and thermal effects are fairly well understood. It is generally agreed at present that both size and self-heating limit the performance of present (submicron) bridges. The following sections describe further the electrical characteristics and mixing theory for Josephson junctions, the design and performance analysis of an experimental mixer for 80-120 GHz including cryogenics and electrical shielding, and experimental results and conclusions achieved. Several appendices are included to provide further detail of device structure and technology, salient physics of operation, and self-heating effects. The paper concludes by establishing new directions in which it is felt effort must be directed to obtain useful Josephson junction mixers for application to radio astronomy.
2. Electrical Characteristics and Mixer Theory

The electrical characteristics of junctions which are adequately modeled by the RSJ model (see appendix B) obey the following equations:

\[ I = I_c \sin \phi + \frac{V}{R} \]  
(1)

\[ \frac{d\phi}{dt} = \frac{2e}{h} V \]  
(2)

where \( I_c \) is the zero voltage critical current

\( R \) is the junction shunt resistance

and \( \phi \) is the phase difference of the order parameter across the junction.

Equations 1 and 2 have been widely studied in the literature. An analytic solution is available for the case of constant current bias. Numerical solutions are available for several specific cases of interest. However, no general solution is available which is directly applicable to the case of mm wave mixing.

In normalized variables, the solution to equations 1 and 2 for constant current bias is [14].

\[
\begin{align*}
\sin \phi &= i_o \quad (i_o < 1) \\
v &= 0 \\
\sin \phi &= \frac{[1 + i_o \cos (vt - \delta)]/[i_o + \cos (\omega t - \delta)]}{[1 + i_o \cos (\omega t - \delta)]} \\
v &= \frac{\phi}{v_o} \left( i_o^2 - 1 \right)^{1/2} \left[ i_o + \cos (\omega t - \delta) \right] \\
v_o &= <v> = \left( i_o^2 - 1 \right)^{1/2}
\end{align*}
\]  
(3a, 3b)

where \( i_o = i/I_c \), \( v = V/(I_c R) \)

and \( t \rightarrow t/(2eI_c R/h) \)

Taken in conjunction, equations 3a and 3b describe an i-v characteristic which is composed of a superconducting branch, \( v=0 \), up to \( i_o=1 \), followed by a resistive-superconductive branch of hyperbolic shape asymptotic to \( i_o=v_o \).
(an unnormalized slope of 1/R) as shown in Figure 1. The Josephson self oscillation is observed to occur at the frequency \( \omega = 2eV/\hbar \) and harmonics thereof as seen by the expression for \( v = \dot{\phi} \). Current oscillations do not appear for this case since we are driving the junction from a constant current source.

Solutions of equations 1 and 2 for the case \( i(t) \neq 0 \) are much more difficult to obtain. Perturbation theory analyses have been performed by Thompson and others. Also, an analytic solution (technique) has been obtained by Renne and Polder that is valid for \( i(t) < 1 \). Several workers have developed analog computer equivalents for the RSJ model (it is essentially the same as a phase locked loop) and others have developed numerical solutions for various special cases using digital computers. All of the above analyses shed light on the modes of operation of the RSJ model in their own manner. However, in order to obtain design information for specific mixer configurations and bias and drive conditions it is still necessary to resort to numerical modeling for the design under consideration.

When a superconducting weak link is biased with both d.c. and r.f. current sources, the i-v characteristic will (under appropriate circumstances) exhibit constant voltage steps as shown in Figure 1 when the Josephson frequency \( f = 2eV/\hbar \) equals the r.f. frequency or a harmonic thereof. The superconducting branch of the i-v characteristic becomes the zeroth constant voltage step. The heights of the constant voltage steps are a function of the amplitude of the applied r.f. current source. The variation of step height with r.f. amplitude permits a mode of mixer operation whereby the junction is biased between the zeroth and first constant voltage steps and the current is modulated at \( f_{if} = |f_s - f_{LO}| \) by the amplitude beat between the signal and local oscillator. This mode of operation has been most successful for heterodyne mixing and is the only mode that will be discussed in this paper.

For the above mixing mode, the conversion efficiency has been shown to be dependent on the change of the zeroth step height with r.f. current amplitude, the ratio of i.f. to signal impedance, and the power coupling efficiencies (impedance matching) at signal and i.f. frequencies as expressed by the following equation:
\[ \eta = C_s C_{if} \cdot \frac{R_{if}}{R} \cdot a^2 \]  

(4)

where \( C_s \) and \( C_{if} \) are the power coupling efficiencies at signal and i.f. frequencies, 

\( R_{if} \) is the slope of the v-i curve at the bias point and \( R \) is the normal junction resistance.

The value of \( a^2 \) is defined by

\[ a^2 = \left( \frac{\Omega_{eff}}{\Omega} - 1 \right) \left( \frac{\partial I_o}{\partial I_{rf}} \right)^2 \]

(5)

where \( \Omega_{eff} \) is the effective normalized frequency of the Josephson junction loaded by the additional shunt resistance \( R_s \) (broad-band coupling assumed).

This simple model has yielded good agreement with successful millimeter mixing results to date. A curve of \( \left| \frac{\partial I_o}{\partial I_{rf}} \right|^2 \) versus \( \Omega_{eff} \) is shown in Figure 2.

As is apparent from the simple mixing model depicted above, the essential requirement to get mixing action in a superconducting weak link device is the ability to modulate the i.f. current by the beat frequency of the signal and local oscillator currents. Thus it is possible to get appreciable mixing action in a junction which does not exhibit true Josephson effect yet is highly non-linear in i-v characteristic (at mm wave frequencies) due to any of the physical processes that may occur in these junctions. Therefore, any analysis of mixing performance in these devices rests heavily on understanding the basic mechanisms involved and their intrinsic relaxation times.

It may be noted from equation 4 that conversion gain in mixing is possible for the case \( R_{if} > R \) (if \( C_s, C_{if}, \& a \neq 1 \)). The dynamic resistance \( R_{if} \) is in general larger than the junction shunt resistance \( R \). However, \( R_{if} \) is strongly affected by the external circuitry connected to the junction and by the presence of noise. Unfortunately, numerical techniques are required to obtain solutions to these cases and only certain specific situations have been studied.
The quantity $\alpha^2$ as obtained from an analysis of the RSJ model can be shown to be dependent on the normalized operating frequency $\Omega$ of the junction and the effective frequency $\Omega_{\text{eff}}$ caused by loading of the junction by the external circuitry (assumed to be a broad band resistive load). The optimum value $\alpha^2_{\text{opt}}$ is obtained for a given $\Omega$ by the appropriate choice of $R_s$ (which determines $\Omega_{\text{eff}}$). The dependence of $\alpha^2_{\text{opt}}$ on $\Omega$ is shown in Figure 3. It is to be noted that $\alpha^2_{\text{opt}} < 1$ for $\Omega > 0.19$, and that the optimum source resistance $R_s < R$ for $\Omega < 1$. These numbers imply that $\Omega$ should be kept as small as possible to promote low conversion loss. Also, the source resistance for the millimeter wave signal should be matched to an impedance somewhat smaller than the junction resistance.

Using the above theory it is possible to analyze the expected performance of a millimeter wave Josephson junction mixer. Calculation of the coupling coefficients $C_s$ and $C_{if}$ will be deferred to the next section. Values of $\alpha^2$ and $R_{if}/R$ will be estimated here that are valid for presently available niobium microbridge junctions. The normalized frequency of operation $\Omega$ can be gotten from knowledge of the junction $I$-$R$ product and the operating frequency. For present junctions, $R \approx 0.3\Omega$ and $I_c \approx 600\mu\text{A}$ maximum before the onset of hysteresis. This $I$-$R$ product is then 180$\mu\text{V}$ which translates to a junction figure of merit $\Omega_{O} \approx 90$ GHz. Thus, for a mixer operating at 90 GHz, the normalized frequency of operation $\Omega_{\text{eff}} = 1.429$. The value of $\Omega_{\text{eff}}$ is obtained from knowledge of the source resistance $R_s \approx 0.7\Omega$ at the best matched frequency giving a value of

$$\Omega_{\text{eff}} = (1 + R/R_s) \Omega = 1.429$$

For $\Omega_{\text{eff}} = 1.429$, a value of $\left(\frac{\partial I_s}{\partial I_{rf}}\right)^2 \approx 0.16$ is obtained form Figure 2. Calculating $\alpha^2$ from equation 5 we obtain $\alpha^2 \approx 0.1686 = -11.6$ dB.

The ratio $R_{if}/R$ is estimated to be about five from the best data obtained to date ($R = 0.3\Omega$, $R_{if} \approx 1.5\Omega$) and from published results of other workers. As stated previously, this ratio can be substantially degraded by
external noise and is strongly influenced by parasitic (inductive or capacitive) circuit elements. The impedance gain for a ratio $R_{if}/R = 5$ is $+7$ dB which, when added to $a^2$, gives a conversion loss of $4.6$ dB for the 90 GHz Josephson mixer excluding impedance mismatch losses $C_s$ and $C_{if}$.

The noise temperature of the junction is expected to be in the range of 50 to 100$^\circ$K. Thus the mixer temperature, exclusive of i.f. amplifier noise contribution, would be about 300 to 600$^\circ$K (allowing 3.4 dB mismatch and other losses, see next section). Present cryogenic GaAs Schottky diode mixers are capable of SSB $T_m \approx 400^\circ$K with full 70 to 120 GHz tunability. Therefore, the theoretical performance of present Nb microbridges does not represent any significant improvement over existing mixers and in fact would give less frequency coverage.

The main factor limiting performance of present Nb microbridge mixers is a low value of $R$ which leads to a low figure of merit $\tilde{\Omega}$ and low impedance. In fact, microscopic examination of Nb microbridges typical of those tested shows $l \sim 0.5$ to 0.7 $\mu$m and $w \sim 3$ $\mu$m. Reduction of the width to the 0.5 to 0.7 $\mu$m range would give $R = 1$ to 1.5$\Omega$. For a maximum critical current of 500 $\mu$A (assumed self-heating limit) the $I_R$ product would rise to 750 $\mu$V $\equiv$ 375 GHz giving a normalized frequency of $\tilde{\Omega} = 0.24$ ($f = 90$ GHz). In this instance, $a^2$ improves to $-0.5$ dB and the overall conversion gain predicted is about $+5.5$ dB. Obviously the junction resistance is a very important factor in the performance of these junctions as mixers, as is the effect of self-heating on limiting the maximum critical current without hysteresis. Point contact junctions typically have higher resistance and less self-heating (higher $I_R$ product) than variable thickness microbridges of the same material.
3. Mixer Design

3.1 Electrical Analysis

The mixer block design is patterned after the NRAO 3 mm Schottky diode mixers. The Josephson junction is fabricated on a 6 mil. thick quartz substrate. Also fabricated on the same substrate are choke structures consisting of cascades of quarter wave sections of alternating low and high impedance microstrip lines. The function of these chokes is to keep the L.O. and signal frequencies out of the i.f. port and simultaneously to provide a low input impedance for these frequencies. The quartz microstrip circuit containing the junction and chokes is mounted across a reduced height WR-10 waveguide with the junction parallel to the electric field direction in the waveguide. The reduced height guide is matched to full height guide at the mixer input flange by an electroformed 3-section quarter wave transformer. A sliding backshort with metallic contact fingers in located behind the junction to provide tuning capability.

The waveguide height is reduced by 10:1 to better match the very low impedance of the Josephson microbridges. Standard dimensions of WR-10 guide are width \( a = 0.100" \), height \( b = 0.050" \). Thus the reduced height section is only \( 0.005" \) high which is about minimum from a machinability standpoint. The characteristic impedance of this section at 100 GHz is about \( 45\Omega \).

Input impedance of the r.f. chokes is series resonant above 100 GHz. Thus the impedance is mildly capacitive across the range of interest with a small series resistance component. This slight capacitive reactance of the chokes helps to resonate the series inductance of the microstrip line which crosses the waveguide. Both above and below this series resonance the junction impedance is transformed to match the waveguide impedance. The excess reactance can be matched by tuning the waveguide backshort. However, for low junction impedance, the range over which the signal is well matched is small as may be seen in the computed values of \( C_s \) versus frequency for \( R = 0.3 \) and \( R = 1.0 \) shown in Figure 4.

The i.f. impedance of the junction equals the dynamic resistance which is about a factor of 5 (this depends on noise and circuitry) greater than the junction resistance \( R \). The i.f. junction impedance can be matched to the 50\( \Omega \) i.f. amplifier using a quarter wavelength coaxial slug. This slug can
physically be positioned no closer than about $\lambda_{if}/2$ from the junction. For this configuration, the i.f. matching coefficient $C_{if}$ has been computed for $R_{if} = 2\Omega$ and $10\Omega$ as shown in Figure 5. It may be seen that the useful i.f. bandwidth is a strong function of $R_{if}$ in this range.

Based on the above calculations, a junction resistance of at least $1\Omega$ is desired to give useful results with the present mixer configuration. Even for this resistance, the usable range of signal frequencies over which $C_s$ is minimum is quite small. Therefore, if microbridge type junctions with maximum single junction impedance of about $1\Omega$ continue to be used for this work, it will be necessary to incorporate additional tuning elements into the mixer design and to explore the use of junction arrays.

3.2 Cryogenics

Josephson junction mixing experiments are presently being carried out in a 5l Superconducting Technology dewar. This dewar is capable of storing 1l of liquid He for about 24 hr. A cross-sectional view of the dewar and cryogenic apparatus is shown in Figure 6. All the electrical apparatus associated with the mixer is enclosed in a vacuum tight container which has the bottom section only immersed in the liquid He. The mechanical and electrical connections, which are fed through vacuum seals, required to operate the mixer are as follows:

1) backshort drive - by micrometer with stainless steel bellows and rod
2) WR-10 waveguide - stainless steel with mylar vacuum window
3) i.f. line - stainless .085 coaxial line with O-ring seals
4) several twisted pairs of varnish coated transformer wire (.007 Cu) for electrical connection to temperature sensors, heater, and d.c. bias.
   a) heating resistor - 1 pair
   b) temperature sensing resistor - 1 pair
   c) Lake Shore Cryotronics Si temperature sensor - 2 pair
   d) junction d.c. bias - 2 pair.
The above wires and transmission lines are fed down through a 1.25" diameter thin-wall steel tube to a Cu cold station covered by an aluminum can. The outside of the s.s. tube is ringed with baffles to minimize convection losses. The aluminum can is lead plated to provide magnetic shielding (by the superconducting Pb at 4.2°K). The waveguide and coaxial lines are copper strapped to the cold station to minimize conduction to the mixer block from room ambient.

Two types of temperature sensors are used as indicated above. The Lake Shore Cryotroncis unit includes a Si sensor in the dewar and a digital temperature indicating unit incorporating all the associated electronics. The other temperature sensor is a 220Ω carbon composition resistor which increases in resistance to over 3KΩ at 4°K. In the range 4.2 to 7.2°K this resistance changes of the order of 0.6Ω/°mK. The temperature sensing resistor has been calibrated using the Lake Shore unit and is used to obtain precise temperatures below 10°K. The calibration curve of this resistor is shown in Figure 7. The readings are quite precise, but the accuracy is obviously no better than that of the Lake Shore unit.

Temperature control above 4.2°K is achieved by injecting heat in the form of d.c. electrical power dissipated in a 1.6KΩ (at room temperature) wire wound resistor which is clamped to the copper bracket upon which the mixer block is mounted. A thermal drop is established across the stainless weak link to the aluminum can which is maintained at 4.2°K by the liquid He. A constant voltage d.c. power supply is used to bias the heating resistor. Temperature control of the order of 1°mk is obtained by adjusting the heat input and monitoring the temperature in an open loop mode. Some difficulty with thermal oscillations of amplitude 50°mK p-p was encountered but was eliminated by strapping the waveguide and coax lines to the cold station and stuffing the interior of the 1.25" stainless tube with polystyrene to prevent convection inside the pipe.

The backshort is driven by a micrometer outside the dewar. Linear motion is fed into the vacuum through a stainless steel bellows and transmitted to the mixer via a 1/8" diameter stainless tube. A pivot lever at the mixer converts the pushrod motion to backshort motion at 1:1 ratio. Some backlash is encountered with this design due to flex of the long connecting rod. However, settings are fairly reproducible, and redesign would necessitate much more complex mechanical linkage.
3.3 Noise Suppression

Josephson junction devices are extremely sensitive to noise from external sources. These devices are also quite sensitive detectors of small magnetic fields. Thus, every effort must be made in the operation of Josephson mixers to eliminate or minimize the effect of external noise sources and stray magnetic fields.

All wires to the dewar are coaxial or shielded twinax. All wires inside the dewar are twisted in pairs. Connections between wires to the dewar and wires that pass through into the vacuum are presently made at unshielded terminal strips on top of the dewar. These unshielded sections, although short, are a potential source of noise pickup and a redesign is underway to provide shielded connections throughout.

Magnetic shielding begins with the lead plating on the mixer vacuum can. At 4.2K lead is superconducting and will exclude any magnetic field by the Meissner effect. Additional magnetic shielding in the form of high permeability "conetic" alloy sheets was found to be effective in reducing observed junction noise. Several sheets of magnetic shielding material were wrapped around the outside of the dewar and several strips of shielding were placed beneath the dewar. More complete and permanent mu-metal shielding will be incorporated in future mixer cryogenic design.

Adequate grounding through the avoidance of ground loops is very necessary in the operation of Josephson junctions. The best solution seems to be to tie all separate pieces of equipment to a common ground point. The ground lead contained in the three prong electrical plug which is tied back to the building electrical ground does not appear to be adequate to minimize this noise source. The chart recorder ground is presently being used as common tie point for grounding all equipment. Direct connection of the waveguides between the klystron L.O. and dewar still produces a small low frequency ripple on the Josephson junction i-v characteristic which appears to be coming from the klystron power supply beam voltage circuit. Thus, provision of adequate grounding is a difficult problem which hasn't yet been entirely solved.
One additional source of noise was located and shielded to minimize its effect on the junction behavior. It was discovered that the 70-115 GHz BWO sweeper was radiating substantial power through the large screen wire that bridged the cooling fan holes in the enclosure. A finer shielding screen was installed which eliminated the leakage and still permits the fan to operate properly.

The digital volt/ohmmeter used to read the resistance of the temperature sensing resistor was found to be putting out a noisy sampling signal which seriously affected the junction characteristic. This instrument is now switched off and disconnected prior to making i-v measurements or attempting to mix.
4. Experimental Results

Testing of niobium microbridge weak links in millimeter mixer circuitry has been carried out in our laboratory. The microbridge junctions are fabricated and supplied to us through a cooperative effort between the U.Va. Physics and E.E. Departments. To date, no mm mixing action has been observed although some small constant voltage steps have been seen (see Figure 8a and d) corresponding to the mm wave L.O. pump frequency. It has been possible to suppress the zero voltage critical current with the applied L.O. power, (see Figure 8a), but the exact physical mechanism for this response is not known.

The mixer design and apparatus have evolved through the experimental work to the stage described in the preceding section. Much of the emphasis has been on obtaining precise temperature control and stability and reducing the influence of external noise. The basic structure of the microbridge junctions has not changed appreciably over this period of time. The most recent results are probably of greatest interest as they demonstrate the present capability of the technology as well as provide an indicator to how much work remains to be done to develop a Josephson junction mixer suitable for mm wave radio astronomy.

The primary observable quantity for a Josephson junction receiver besides i.f. output (which was not observed) is the d.c. current voltage characteristic of the junction. The nature of this characteristic reflects the junction type and size, the presence or absence of noise, and the response, if any, to millimeter wave radiation. The first and most important characteristic of the junction which is tested is the existence of a zero-voltage, superconducting branch of the i-v characteristic. For our mixer, this branch typically exhibits a resistance of about 30 to 40 mΩ which is the contact resistance of the gold foil at the ends of the choke sections. For $T \ll T_c$ the critical current $I_c$ is greater than 1 mA which is the maximum current out of our bias supply.

To reduce the critical current below 1 mA, the mixer temperature $T$ is increased by application of d.c. power to the heating resistor. An input voltage of about 10V ($P \approx 70$ mW) was adequate to raise the temperature to the operating range of 7.0 to 8.5°K. In this range, the critical current $I_c$ is
a sensitive function of T (see Fig. 9). It is desirable to operate at as large a value of \( I_c \) as is consistent with the absence of hysteresis on the \( i-v \) curve. Hysteresis occurs at high values of \( I_c \) (see Fig. 8b) as the result of self-heating induced by the increased power dissipation in the junction. Typically the onset of hysteresis for 0.3\( \Omega \) Nb microbridges has been in the range of 200 to 500 \( \mu \)A for recent junctions. Thus the temperature would be adjusted to obtain \( I_c \) in this range just below the onset of hysteresis. The \( I_c R \) product for such junctions is about 180 \( \mu \)V which is equivalent to a figure of merit \( \Omega_o = 90 \) GHz or a normalized operating frequency for \( f = 90 \) GHz of about \( \Omega = 1.0 \).

Once the operating temperature has been established as above, the junction sensitivity to millimeter wave radiation is tested. The klystron is tuned to the desired operating frequency and then the backshort is tuned to maximize the suppression of \( I_c \) by the applied signal. This is the condition for optimum coupling of the millimeter wave signal to the junction. Frequencies in the range of 65 to 85 GHz have been used for the local oscillator. The lower frequencies will reduce the normalized frequency \( \Omega \) and improve the junction response but at greatly reduced coupling efficiency to the present circuitry. More recent experiments are at 82 GHz with \( \Omega \approx 1 \) to take advantage of improved coupling efficiency.

In the most recent experiments it has been possible to completely suppress the critical current with an L.O. power of about 20 to 40 \( \mu \)W as seen in Figure 10. The maximum variation of \( I_o \) with \( i_{LO} \) is about 0.75 assuming a 7.5 dB coupling loss \( C_s \). This gives

\[
\left( \frac{\partial I_o}{\partial I_{rf}} \right)^2 \approx 0.5
\]

which is consistent with \( \Omega = 1 \) as previously estimated. The input waveguide loss has been estimated at 8 dB which gives an overall predicted mixer conversion loss of

\[
L \approx C_{if} \cdot C_s \cdot \frac{R_d}{R} \cdot \alpha^2 = C_{if} \cdot C_s \cdot \frac{R_d}{R} \cdot \frac{R}{s} \left( \frac{\partial I_o}{\partial I_{if}} \right)^2
\]
\[ L \approx 0 \text{ dB}(C_{\text{if}}) - (7.5 + 8)\text{ dB}(C_s) + 7 \text{ dB}(R_d/R_s) - 3 \text{ dB} \]

\[ L \approx -11.5 \text{ dB} \]

However, the dynamic resistance \( R_d \) was observed to be severely degraded to about 0.5\( \Omega \) (from a maximum of about 1.5\( \Omega \)) at this operating point which increases the overall loss estimate by about 5 dB (neglecting any decrease of \( C_{\text{if}} \)) giving a final estimated value of \( L = 16.5 \text{ dB} \).

Although no i.f. output was detected in this experiment, the small bump on the i.v. curve at 170 \( \mu \text{V} \) observable in Figure 8a is interpretable as a current step at 82 GHz (the Josephson constant is 0.484 GHz/\( \mu \text{V} \)). This step is evidence for synchronism of junction processes with the applied radiation as discussed in Appendix B.

Other notable features of the i-v characteristics for this junction are as follows:

1) a rounding (convexity) of the i-v curve in the direction of higher resistance for higher d.c. voltage indicating the effect of self-heating

2) abrupt fluctuations or jumps in the curve (see Figure 8c) sometimes appearing periodically at a very low frequency (< 1 Hz) which indicate either the influence of external magnetic fields or the existence of several stable junction states with slightly different i.v. characteristic

3) excess super-current greater than \( I_c/2 \) which is consistent with a large bridge \( l, w >> \xi \) and the flux flow model. (See Figure 8a, b, and c).

4) variation of \( R_d \) from curve to curve indicating some residual noise affecting the junction characteristics.

In a more recent experiment, the data shown in Figure 8d were obtained. The local oscillator frequency was 83 GHz. Subharmonic steps on the i-v curve are clearly visible. Also, small constant voltage steps corresponding to the fundamental Josephson frequency may be seen. These curves were obtained on the retrace of a hysteretic characteristic so that the region beneath the thermal switching off the superconducting branch could be observed. Thus the junction temperature is necessarily elevated by the power dissipation. These
curves are the result of incorporating all the noise reduction techniques discussed into the system and are quite stable and repeatable. The subharmonic steps are indicative of a nonsinusoidal current-phase relationship which implies activity of physical processes other than the pure Josephson effect$^{20}$ (see Appendix B).
5. Discussion

The analysis and experimental results give a good indication of the state of the art as regards millimeter wave mixing with niobium microbridges. Several important factors are apparent from the work done to date:

1) These devices are extremely sensitive to noise and magnetic field fluctuations, thus shielding and filtering requirements are very stringent in comparison to, for example, GaAs Schottky diode mixers.

2) A very high degree of temperature stability and precise control is required for present devices which must operate near Tc to avoid hysteresis.

3) The impedance of single junction microbridge devices is very low with \( R \approx 1 \) to 2\( \Omega \) being apparently achievable, but not presently available due to technological problems.

4) The physics of the microbridge devices is quite complex, particularly for the larger junctions, and it is not clear what performance limitations may result from the various phenomena.

Many of the shielding and screening problems have been solved in the present work. However, it must be remembered that further problems of this nature may occur when Josephson mixers are integrated into larger systems with attendant increased density of electromagnetic devices. Solution of shielding problems must be accomplished in any instance if Josephson junction receivers are to be utilized since point contact junctions are just as sensitive to this sort of interference as are microbridges.

The very stringent requirement for temperature stability and precise control with present junctions appears to be a factor which could severely limit practical application of these devices for radio astronomy. The temperature dependence of critical current of 2 \( \mu \)A/°mK could easily produce drift or spurious receiver response to small temperature fluctuations that are correlated with other aspects of receiver operation, e.g. beam switching. Such responses would be unacceptable for astronomical observation purposes. The temperature limitation arises due to self heating, and it would be desirable to operate colder to improve mixer
performance and reduce noise. This may be possible with Nb point contact junctions which have intrinsically better cooling capability or with variable thickness microbridges with greatly reduced self heating characteristics (this technology is probably quite difficult).

The low impedance level of single microbridge junctions is a serious drawback to design of wide bandwidth receivers. At these frequencies, additional tuning mechanisms are usually very lossy. An instantaneous bandwidth of 1 to 2 GHz is attainable with 500 MHz i.f. bandwidth using 1Ω junctions. The tunable backshort will not, however, enable the signal frequency to be changed more than a few GHz without significant performance degradation. Thus, the present design, based on conventional Schottky diode mixer design, behaves essentially as a fix-tuned mixer. To provide a broadband tunable mixer at these low junction impedance levels would require an entirely new design based on new tuning techniques. On the other hand, Nb point contact junctions have exhibited R in the range of 30 to 50 ohms which permits reasonably broadband (10-30 GHz) mixer design without unacceptable performance degradation. Another alternative is the use of microbridge arrays to obtain higher impedance. Fabrication of these arrays is planned but is again predicated on improvements in microbridge fabrication technology. There is, as well, the question of how to bias the individual junctions in an array; in series or in parallel? All junctions should theoretically be operated at the same bias voltage, but series biasing forces the junction currents equal and permits the individual junction voltage to vary. Parallel biasing is possible using high impedance bias lines which will not effectively propagate the mm wave or i.f. frequency signals. Successful application of junction arrays will require substantial experimental effort to determine optimum configuration and biasing arrangement.

Understanding the salient physics of the junctions used for mixing is needed to be able to guide the course of experimental work to eventual success. The area of Josephson junction mixing and phenomenological effects has been very active in recent years. The literature in this field is enormous and is still growing. Yet the application of these devices as practical low-noise, high-frequency detectors is long delayed. The physics is quite complex and many subtle effects occur. The state of the theory is rather behind the experimental work, and, thus, very little guidance comes from this area. The effects that occur in
APPENDIX A

JUNCTION STRUCTURE AND FABRICATION TECHNOLOGY

A brief review is given here of the actual structure and technology of realizable variable thickness microbridges (VTB's) and point contact Josephson junctions to provide a basis for understanding the physics of these devices and the differences between them. Figure 11 shows a sketch of the two structures to be described.

Several techniques have been developed to produce VTB's. Among them are the following:

1) the intersecting scratch technique
2) the quartz fiber technique
3) lithographic techniques including:
   a) photolithography
   b) electron-beam lithography
   c) x-ray lithography

The lithographic techniques are ordinarily combined with either wet or dry (gaseous) etching or sputter etching to remove the unmasked regions. The quartz fiber technique is usually used in conjunction with a lithographic technique so that the fiber defines the gap width (bridge length) and the lithography defines the bridge width. The objective of all the above approaches is to produce a small, narrow constriction between two (relatively) massive banks of superconducting material. The smallest (uniform thickness not VTB) Nb microbridge devices reported were fabricated by electron lithography with length \( l \approx 2000 \AA \) and width \( w \approx 2000 \AA \). Using photolithographic techniques it is difficult to achieve dimensions smaller than about 5000 to 7000\( \AA \).

The film thickness in the bridge region is typically 300 to 1000\( \AA \). The thickness of the superconducting banks at the ends of the bridge is made as large as possible (about 5000 to 10,000\( \AA \)) to minimize self-heating effects. However, the requirement of large bank thickness is incompatible with small gap width because of aspect ratio limitations arising from fabrication technology. An even more severe limitation is the diffraction effects caused during photolithography by the mask to substrate separation arising from the thick banks.
The process presently in use (at U.Va.) to produce niobium microbridges combines the quartz fiber and photolithographic techniques. The entire substrate (fused quartz, 6 mil. thick) is sputtered with a 300 to 600Å thick layer of niobium. The quartz fiber which is about 0.5 to 0.7 μm in diameter is placed across the substrate and the thick niobium film for the banks is deposited (5000 to 7000Å at present). The quartz fiber shields the region in which the bridges are to be formed so that the film thickness beneath the fiber remains 300 to 600Å. Subsequently, photolithography is performed to define the bridge width and the choke dimensions. Wet chemical etching is used to remove niobium outside the desired area. Bridge dimensions produced by this approach have been l ~0.5 to 0.7 μm and w ~3μm with film thicknesses as indicated above. The excessive bridge width arises partly as a result of the difficulty of obtaining high resolution exposures for mask to substrate separation of 5000 to 7000Å (diffraction problem) and partly as a result of the limitations of the wet chemical etching process which also attacks the substrate. A substantial amount of development would be required for this process to produce VTB's with both width and length about 0.5 to 0.7 μm, but this is the minimum improvement needed to get the present devices to perform at mm frequencies. Even smaller dimensions would be desirable.

The physics and electrical characteristics of VTB's will be more extensively discussed in Appendix B, but it should be here pointed out that even for w = l the bridge normal resistance $R$ will be about equal to the sheet resistance of the thin (300 to 600Å) Nb film. The film thickness is limited by suppression of the superconducting critical temperature $T_c$ from the bulk value. Thus a maximum sheet resistivity and bridge resistance are established by this minimum film thickness. For single niobium microbridges the maximum resistance appears to be in the range of 1 to 1.5 ohms at the operating temperature.

Point contact Josephson junctions may also be fabricated by a variety of techniques. Typically, an anvil is contacted by a point of the same superconducting material. Frequently, means of adjusting the point mechanically at the operating temperature is provided to achieve the desired critical current and resistance. This type of arrangement usually precludes cryogenic cycling of the junction without alteration of its characteristics.
Recently, point contact Josephson junctions have been assembled in a configuration similar to that of a GaAs Schottky diode for cryogenic mixers. The technique consists of mounting a small tab of niobium at the end of one quartz microstrip choke section and a pointed niobium whisker at the end of the other choke section. This whisker contact to the anvil is accomplished in the same manner that GaAs Schottky diodes are whiskered and the operation is monitored with a curve tracer to obtain the desired junction resistance. This approach appears to be quite suitable for assembly of mixers for operation at 1 to 3 mm wavelength and is reported to produce mechanically stable devices that withstand many cryogenic cycles without change.

The resistance of point contact Josephson junctions at operating temperature is in the range from 10 to 50 ohms which is much more suitable for impedance matching considerations than the 1Ω level of the VTB. It appears that contacts from which the best performance is obtained are fabricated from whiskers which have been exposed to "ambient" air for several days. Thus, the actual structure of successful point contacts is somewhat unclear. The structure may be superconductor-insulator-superconductor (SIS) or there may be bridging by fine metallic filaments in the junction region. In any event, the cross sectional area defined by contact of the whisker tip (r < 1 μm) to the anvil is small and the impedance is in a useful range for mixing. In fact, the description of the junction by the RSJ model as will be subsequently discussed is not strongly dependent upon the exact details of the point contact interface structure.
Current flow in VTB and point contact Josephson junctions may be described with the two fluid superconducting model. In this model, the electrons are in either a paired or an unpaired state. The paired electrons are described by an order parameter $\psi = \psi_0 e^{i\phi}$ where the phase factor $\phi$ is in general a function of position and time. The unpaired electrons are usually referred to as quasiparticles in this model. There is a temperature dependent energy gap of $2\Delta$ between the paired and unpaired electrons. For $T \ll T_c$, $\Delta \approx 2kT_c$ pairs may be excited across the energy gap (broken) by thermal phonons or by electromagnetic photons. Thus there is a continuous interchange of electrons between the paired and unpaired states within the junction depending on temperature, bias voltage, and coupling to electromagnetic fields. In addition, since the superconducting transition is dependent on the magnetic field strength, the junction is highly sensitive to magnetic fields.

Current flow across an ideal junction may be expressed by

$$i = I_c \sin \phi + [G_0(V) + G_1(V) \cos \phi]V \quad (B-1)$$

where the $I_c \sin \phi$ term represents supercurrent (pair) flow across the junction due to the phase difference $\phi$ of the order parameters on either side of the junction. The time dependence of the phase difference can be shown quite generally to be of the form

$$\frac{d\phi}{dt} = \frac{2e}{\hbar} V \quad (B-2)$$

where $V$ is the potential across the junction. The $G_0(V)V$ term represents quasiparticle current flow, and the $G_1(V)V \cos \phi$ term represents interference between pairs and quasiparticles. A very useful phenomenological model is obtained by neglecting the $\cos \phi$ dependent term and interpreting the $G_0(V)V$ term as being the result of a (constant) shunt conductance across the junction. This is called the resistively shunted junction (RSJ) model. In this instance, $R = 1/G_0$ becomes the normal resistance of the constriction in the VTB or of the point contact at the operating temperature.
Equation B-1 is valid for a junction which is small in comparison with the coherence length for the superconducting pairs. This is a requirement for the two superconductors on either side of the constriction (point) to be effectively coupled. The coherence length is a function of the material and the temperature of the following form:

$$\xi(T) = 0.74 (1 - t)^{-1/2} \xi_0$$  \hspace{1cm} (B-3)

$$t = T/T_c$$

where $\xi_0$ is material dependent and is about 390 $\AA$ for niobium. Thus it is usually necessary to operate niobium microbridges very near $T_c$ to realize a coherence length of the order of the bridge length so that equation B-1 is valid. In fact these bridges are not ordinarily operated near enough to $T_c$ for equation B-1 to be strictly valid and other physical mechanisms must be considered.

A figure of merit for the Josephson junction used as a mixer is its characteristic frequency defined as:

$$f_c = \frac{2e}{h} I_cR$$  \hspace{1cm} (B-4)

where $I_c$ is the critical (maximum) supercurrent and $R$ is the junction shunt resistance defined earlier. To first order, $R$ is independent of temperature. The temperature dependence of the critical current is complex but in general follows the temperature dependence of the pair density which is found experimentally to be $x = 1 - t^4$. Thus for $T = T_c$ the $I_cR$ product and junction characteristic frequency become small, but $T < 0.5 T_c$ the characteristic frequency is near its maximum value. The desirability of operating junctions well below $T_c$ to maximize the characteristic frequency and improve millimeter mixer performance is apparent.

In order to operate a niobium microbridge with $l = 0.7 \mu m$ so that $\xi = l$ the temperature would need to be within $(T_c - T)/T_c < 1.84 \times 10^{-3}$ according to equation B-3. In this instance, the critical current would be very low and the junction characteristic frequency would be in the microwave range ($f_c < 10$ GHz). Since niobium microbridges of length $l = 0.7 \mu m$ have been successfully operated at 10 GHz with $(T_c - T)/T_c \geq 0.1$, it is likely that some other mechanism(s) is
responsible for the observed behavior. In fact, two other mechanisms which produce Josephson like effects have been widely studied and discussed in regard to microbridge performance. The two processes are "phase slip" and "vortex flow" and will be described in simple terms below.

The phase slip process may arise in narrow superconducting regions longer than a coherence length. Phase slip occurs when the local critical current is exceeded at some point so that a finite potential appears across a small length of the microbridge. This potential accelerates the Cooper pairs according to equation B-2 until the critical pair velocity is reached at which all the pairs are simultaneously broken (locally). Once the pairs are broken, the phase slips by $2\pi$, the pairs reform, and the cycle repeats yielding an oscillation at the Josephson frequency $\omega = 2eV/h$. Thus, the phase slip process is highly indistinguishable from the true Josephson interaction except, possibly much more complex physics are involved. It is not well understood at present just what the limiting relaxation time is for the phase slip, so it is not possible to say whether junctions which exhibit response due to phase slip will be capable of millimeter wave mixing.

The remaining process which is capable of producing Josephson like behavior in superconducting weak links is flux flow which occurs when one or more flux quanta $\phi_0 = h/2e$ propagate transversely across the junction width. These flux quanta are only able to penetrate the weak link when the superconducting penetration depth $\lambda$ becomes substantially less than the transverse dimension of the bridge. Flux flow is basically the result of a breakdown in uniformity of the current in the direction transverse to the bridge axis. Vortex or flux flow leads to nonsinusoidal current phase relationship which may result in the presence of subharmonic steps on the i-v characteristic. The conditions and bridge dimensions for which flux flow occurs have been studied by Likharev, et al, but no estimate of mm wave mixing capability is given.

Another aspect of microbridge behavior that surfaces for $t \ll 1$ is self-heating effects. Self-heating refers to the temperature rise of the junction which occurs as a result of power dissipation within the junction. As the temperature is reduced below $T_c$ the critical current $I_c$ increases rapidly so that power dissipation increases. The accompanying rise of junction temperature can influence all aspects of microbridge behavior. Self-heating is discussed in more detail in Appendix C.
APPENDIX C

SELF-HEATING EFFECTS

Self-heating effects were first studied in connection with hysteretic i-v behavior of superconducting microbridges\(^{27,28,29}\). Since there is no power dissipation on the \(V = 0\) branch of the i-v curve, there is no temperature rise. Once the critical current is reached, the junction enters the resistive-superconductive state and power is dissipated. The junction temperature rise due to the dissipated power will alter the i-v characteristic in a manner that tends to reduce the critical current and increase the voltage. If the critical current is large enough prior to entering the resistive state, the junction may switch to a finite voltage (assuming constant current bias source) and follow a different i-v curve with reduced critical current. This situation is referred to as hysteresis.

The junction temperature profile depends on the solution in 3-dimensions of a highly non-linear heat-flow differential equation involving sources and material conductivities which are temperature dependent. Some solutions to this problem have been obtained by Skocpol et al under restrictive assumptions\(^{30}\). One important factor to determine is whether a normal hotspot is formed. The existence of such a hotspot can drastically alter the high frequency properties of these junctions.

If a normal hotspot is formed (see Figure 12), its size is defined by the region for which \(T > T_c\), i.e. at the hotspot edges the temperature equals the critical temperature \(T_c\). Phase coherence (coupling) can only be maintained across the hotspot if its length is of the order of a coherence length \(\xi\) or less. Typically, the hotspot will grow with increased power dissipation and shrink with reduced power dissipation. Since the power dissipation is approximately proportional to \(V^2/R\), the phase coherence will disappear when the bias voltage exceeds a particular value \(V_h\) which is structure and material dependent. This places an upper limit on frequency of the Josephson effect and consequently on mixing action which is thermally determined\(^{31,32}\).

Thermal transfer capability at low temperatures is quite limited. Cooper pairs, since they do not scatter from the lattice, do not transfer heat. Phonon mean free paths are quite large so boundary scattering predominates. The acoustic mismatch across the boundary interfaces restricts phonon transmission and causes an interfacial thermal resistance\(^{33}\). Heat transfer by normal electrons is
directly related to the electrical resistivity of the material through the Wiedemann-Franz law. Considering all contributions, the heat flow by normal electrons is the dominant factor in both microbridges and point contact junctions, and, therefore, the thermal resistance of the junction is directly proportional to the electrical resistance $R$. For microbridges there is some possibility of improving phonon heat transfer by reducing the interfacial thermal resistance. However, any reduction to be realized depends on using a high phonon conductivity crystalline substrate (quartz, sapphire, or silicon) and tightly coupling (acoustically matching) the interfaces for both longitudinal and shear waves, and is thus highly speculative at this time.

It is indeed unfortunate that the electrical and thermal properties of these junctions are so strongly interconnected. This implies that high resistance junctions will inevitably have a greater temperature rise than low resistance junctions and will thus have higher noise temperature and reduced performance. Unless some means of decoupling the thermal and electrical properties is found (such as improved phonon transfer for microbridges), there will always be a compromise between impedance matching and thermal characteristics for these devices.

One further difference between microbridge and point contact junctions in this area should be noted. The thermal resistance of a junction does not specify the temperature profile of the junction but only the maximum temperature rise as a function of input power. Thus the hotspot will have considerably different shape depending on the distribution of electrical resistance. For this reason, point contact junctions can run much hotter than microbridges without losing phase coherence since the size of the hotspot is much smaller due to the three dimensional heat flow. On the other hand, the hot spot must spread laterally and increase in size in a microbridge in order to handle increased heat flow thus losing phase coherence at lower voltages for the same material (compared to a point contact). The variable thickness microbridge discussed earlier is a structural modification which improves heat flow and actually makes the microbridge more "point-contact" like in configuration. However, it is unlikely that microbridges can approach point contacts in thermal performance (as a function of electrical resistance) unless a means is found to promote substantial heat removal through the substrate.
REFERENCES

1. P. L. Richards, "The Josephson Junction As a Detector of Microwave and Far Infrared Radiation", to be published in Semiconductors and Semimetals, Vol. 11 (or 12) - Infrared Detectors (II), Willardson and Beer, Ed.


LIST OF FIGURES

1. Theoretical i-v curve with and without r.f. $\left(\frac{3I_c}{I_{rf}}\right)^2$

2. $\frac{\partial I_c}{\partial I_{rf}}$ versus $\Omega_{\text{eff}}$

3. $a^2_{\text{opt}}$ and $R_{\text{opt}}R_{\text{sopt}}$ versus $\Omega$

4. Signal Coupling Coefficient versus Frequency for $R = 0.3$ and $1.0\Omega$.

5. I.F. Coupling Coefficient versus Frequency for $R_{\text{if}} = 2$ and $10\Omega$.


7. Temperature Sensing Resistance versus Temperature $(4 - 110^\circ K)$.

8. a, b, c, d Experimental i-v curves (raw data).

9. Critical Current $I_c$ versus Temperature

10. Critical Current $I_c$ versus $(L.O. \text{ Power})^{1/2}$

11. a, b Point Contact and Microbridge Geometry.

Figure 1 - Theoretical I-V Curve with and without RF

Without RF

With RF, \( \Delta \theta = 0.16 \)

I/Ic

V/(IcR)
Figure 2 - $\left( \frac{\partial I_0}{\partial I_{rf}} \right)^2$ vs. $\Omega_{eff}$
Fig. 3. Optimum coupling parameter and the corresponding RF source resistance computed from the RSJ model for broadband coupling.
Figure 4
Signal Mismatch Loss $C_s$
versus Frequency
FIGURE 5
IF Mismatch Loss \( C_{if} \)
versus Frequency
FIGURE 6 - CROSS SECTIONAL VIEW OF DEMAR-MIXER
Figure 7 - Temperature Sensing Resistance Versus Temperature

\[ T = \frac{4.57255}{(\log R + 2.64422)(\log R - 10.76648)} \]
FIGURE 9 - Critical Current $I_c$ versus Temperature

$\sim 2 \mu \text{A/}^{\circ}\text{mK}$
Figure 10 - Critical Current $I_c$ versus $(L.O. Power)^{1/2}$
Figure 11. a) VTB Geometry

b) Point Contact Geometry
FIGURE 12.
SELF-HEATING HOTSPOT ON MICROBRIDGE