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SIS MIXER DESIGN BY FREQUENCY SCALING

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SUMMARY

In designing superconductor-insulator-superconductor tunnel junction mixers, the saturation power level is an important and often overlooked parameter. If practical values are chosen for the input and output embedding impedance levels, desired conversion loss, pumping parameter, and $\omega R_n C_j$-product, only one other quantity -- the saturation power -- is needed to determine the junction size, the number of junctions, and the local oscillator power required for an SIS mixer.

Equations are given for the number of junctions, junction size, critical current density, array length, and LO power, all as functions of frequency and saturation level. It is shown that, in some circumstances, the physical length of an array with enough junctions to avoid saturation is not electrically small. As an example, the parameters of a prototype 115 GHz mixer are scaled to 38, 230, and 345 GHz.

CONTENTS

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1. Introduction

Frequency scaling is the process in which the dimensions of a microwave circuit are changed by a certain factor D, thereby shifting in frequency the electrical characteristics (e.g., impedance, filter pass-band, etc.) of the original circuit by a factor 1/D. In classical (e.g., Schottky diode) mixer design, frequency scaling of waveguide mounts and diode parameters has been widely used. A mixer optimized for one frequency serves as a prototype for the design of a mixer to operate in another frequency band. In this report we apply the principles of frequency scaling to the design of superconductor-insulator-superconductor tunnel junction mixers.

Saturation is said to occur in a mixer when the input signal power is sufficient to cause the conversion loss to increase above its small-signal value by some fixed amount, typically 1 dB -- hence the so-called "1 dB gain-compression point." This saturation power is independent of frequency scaling in classical mixers provided the diode's junction capacitance and parasitic reactances are properly scaled. In non-classical mixers, such as the SIS mixer, the very sharp nonlinearity of the I-V curve requires the more general quantum mixer theory [1] to be used. Mixer properties become strongly dependent on the pumping parameter $\alpha = eV_{LO}/hf$. Clearly, frequency scaling requires scaling the local oscillator voltage V_{LO}, and the saturation power will therefore depend on frequency. In this report we show that by using a series array of N SIS junctions, it is possible to frequency scale an SIS mixer design so that all its operating characteristics, including the saturation level, remain constant.

We assume that a good SIS mixer design exists, and we want to scale this design to other frequency bands. The prototype determines the normal resistance and susceptance of the SIS element at all frequencies. We also make the following assumpions: (i) The junctions used at all frequencies have the same gap voltage, V_g (typically ~ 3 mV) and specific capacitance C_s. (ii) The pumping parameter, α, is approximately independent of frequency [2] for optimum performance. (iii) There is an optimum value of conversion loss, L, determined by overall system considerations, which can be achieved by the SIS mixers and is the same at all frequencies. Factors affecting this optimum value include: the noise temperature of the IF amplifier, the desired instantaneous bandwidth, maintaining stability of the mixer, and the desired saturation power. (iv) The desired input saturation temperature is set by system requirements and is independent of frequency. For example, it may be required to design a series of SIS receivers which can look at the sun (~ 6000 K) without overloading.

2. SIS Mixer Theory

It has been shown [3] that the small-signal properties of an SIS mixer using a series array of N identical SIS junctions are the same as those of a mixer using an equivalent single junction; the normal resistance and junction capacitance of the single junction are equal to the total normal resistance and capacitance of the entire array. In particular, the noise of the mixer is independent of the number of junctions, as now appears to be supported by experimental evidence [4]. The LO power and saturation power of the array mixer are N^2 times those of the single-junction mixer. The saturation level of an SIS mixer thus can be conveniently increased by the use of a series array of junctions.

In frequency-scaling the mixer, the number of junctions, N, will be treated as a variable, as will the critical current density J_c and area A of each of the junctions. In practice, all of these variables are easily controlled within rather broad limits. Note that the dependence of the specific capacitance, C_s, upon J_c and A, less than about ± 10% for any given type of SIS junction, will be ignored.

For the chosen prototype mixer design, the embedding impedance of the microwave circuit (as seen by the array of junctions) is invariant under frequency scaling. Therefore the total normal resistance and susceptance of the array should be maintained constant, independent of frequency and the number of junctions; that is,

$$R_{n,array} = N\,R_n \qquad (1)$$

and

$$B_{array} = \frac{\omega C_j}{N} \qquad (2)$$

are independent of ω and N. It follows that the product $\omega R_n C_j$ should also be constant, independent of frequency and the number of junctions. Given the specific capacitance C_s,

$$B_{array} = \frac{\omega A C_s}{N} \quad . \qquad (3)$$

Following the approach of Smith and Richards [5], saturation in SIS mixers has been analysed in [6] and [7]. To first order, the (large-signal) conversion loss of a mixer with small-signal conversion loss L, operating into an IF load R_L, is given by

$$L_s = L/\left(1 - P_{sig}/P_x\right) \qquad (4)$$

where $$P_x = 0.10 \left(\frac{\hbar\omega}{e}\right)^2 \frac{N^2 L}{R_L} \quad . \tag{5}$$

The conversion loss here is the transducer conversion loss, as opposed to the available conversion loss, and is single sideband. Our experience with Pb-alloy, Nb/Pb-alloy, and all-niobium arrays of two and four junctions agrees with the general form of equation (4) up to a gain compression of about 3 dB. However, we find that the constant 0.1 in (5) is too small by a factor of 2 to 4. In the present paper we shall retain the value 0.1, a conservative value, for want of a better understanding of the actual mechanism of saturation.

Defining P_{sat} as the input power at which 1% gain compression occurs, we have from (4) and (5)

$$P_{sat} = 0.001 \left(\frac{\hbar\omega}{e}\right)^2 \cdot \frac{N^2 L}{R_L} \tag{6}$$

For the saturation power to be unchanged during frequency scaling, it follows that

$$N = \frac{e}{\hbar\omega} \left(1000 \frac{P_{sat} R_L}{L}\right)^{1/2} \tag{7}$$

3. Mixer Parameters

3.1 Junction Size

It is now possible to deduce the size of the junctions required. Assume square junctions of side a. From (3), $A = a^2 = N B_{array}/(\omega C_s)$. Using (7) to eliminate N:

$$a = \frac{1}{\omega} \left(\frac{e}{\hbar}\right)^{1/2} \cdot \left(1000 \frac{P_{sat} R_L}{L}\right)^{1/4} \cdot \left(\frac{B_{array}}{C_s}\right)^{1/2} . \tag{8}$$

3.2 Critical Current Density

The critical current density can be written as $J_c = (I_J R_n)/R_n A$, where I_J is the Josephson critical current of a junction. The quantity $(I_J R_n)$ is a characteristic constant for a particular type of junction at a given operating temperature. For a weakly coupled junction at T = 0, the theoretical value [8] is $I_J R_n = \pi V_g/4$. For the present paper we shall adopt the value $I_J R_n = 1.7$ mV, which is an average for Pb-alloy (1.6 mV) and Nb/Al-Al_2O_3/Nb (1.8 mV) junctions operating at 4.2K. From the definition of specific capacitance, $C_s = C_j/A$, we obtain the desired critical current density

$$J_c \approx 1.7 \times 10^{-3} \frac{\omega C_s}{(\omega R_n C_j)} \quad . \tag{9}$$

Equation (9) shows that for a given $\omega R_n C_j$ product, J_c is proportional to ω, and independent of N and P_{sat}.

3.3 Local Oscillator Power

The local oscillator power required by an SIS mixer is given by Tucker and Feldman [9] as

$$P_{LO} = \frac{(N \alpha \hbar \omega)^2}{2 e^2 R_{n,array}} \tag{10}$$

where α is a pumping parameter (usually close to 1.2). We can eliminate N from this equation using (7) to obtain an expression for P_{LO} as a function of P_{sat}:

$$P_{LO} = 500 \frac{\alpha^2}{L R_{n,array}} R_L P_{sat} \tag{11}$$

3.4 Array Length

The length of an SIS array can be important in designing mixers, especially if an inductive tuning element is connected directly across the array [10], or if the length is great enough for electrical phase shifts to occur within the array. For a linear array of N square junctions of side a, the total length is

$$l_{array} = N a K_1(a), \tag{12}$$

where $a \cdot K_1(a)$ is the size of a unit cell of the array. For very small junctions ($a < 2$ μm), practical considerations such as mask alignment accuracy tend to set a lower limit to the size of the unit cell, so $a \cdot K_1 \rightarrow$ const. For very large junctions ($a > 20$ μm) the unit cells can be nearly equal to the junction size a, so $K_1 \rightarrow$ const. (Most junctions used in SIS mixers are in the range 1 to 5 μm square.) For the two extreme cases we can write (12) as

$$l_{array} = d_1 N \quad \text{(small junctions)} \tag{13a}$$

and

$$l_{array} = d_2 N a \quad \text{(large junctions)} \tag{13b}$$

where d_1 has the dimension of length, and d_2 is dimensionless. Using (7) and (8) we obtain

$$l_{array} = \frac{d_1}{\omega} \frac{e}{\hbar} \left(1000 \frac{P_{sat} R_L}{L}\right)^{1/2} \quad \text{(small junctions)}, \tag{14a}$$

$$l_{array} = \frac{d_2}{\omega^2} \left(\frac{e}{\hbar}\right)^{3/2} \cdot \left(1000 \frac{P_{sat} R_L}{L}\right)^{3/4} \cdot \left(\frac{B_{array}}{C_s}\right)^{1/2} \quad \text{(large junctions)}. \tag{14b}$$

4. Thermal Source

Equations (7), (8), and (14) give the number of junctions, the size of a junction, and the length of the array required to provide the desired saturation power at any frequency. In many cases of interest, saturation will be caused by a broadband source with an equivalent black-body noise temperature T, for example, the sun with T = 6000K. It is then convenient to use the saturation temperature, T_{sat}, defined by

$$P_{sat} = k\, T_{sat}\, B\,, \tag{15}$$

where B is the input noise bandwidth of the receiver. If there is no special filter to restrict B, it will scale as 1/D, the reciprocal of our geometrical scaling factor. As the operating frequency ω also scales as 1/D, the fractional bandwidth b is independent of frequency. Therefore, in (15),

$$P_{sat} = k\, T_{sat}\, b\omega/2\pi \tag{16}$$

Substituting for P_{sat} in (7), (8), (11), and (14) gives the junction parameters required to achieve a given saturation temperature:

$$N = \frac{1}{\omega^{1/2}} \frac{e}{\hbar} \left(500 \frac{b\,k\,T_{sat} R_L}{\pi\,L} \right)^{1/2} \tag{17}$$

$$a = \frac{1}{\omega^{3/4}} \left(\frac{e}{\hbar} \right)^{1/2} \cdot \left(500 \frac{b\,k\,T_{sat} R_L}{\pi\,L} \right)^{1/4} \cdot \left(\frac{B_{array}}{C_s} \right)^{1/2} \tag{18}$$

$$P_{LO} = 250 \frac{\omega\,\alpha^2}{\pi\,L\,R_{n,array}}\, b\,k\,T_{sat} R_L \tag{19}$$

$$l_{array} = \frac{d_1}{\omega^{1/2}} \frac{e}{\hbar} \left(\frac{500\,b\,k\,T_{sat} R_L}{\pi\,L} \right)^{1/2} \quad \text{(small junctions)} \tag{20a}$$

$$l_{array} = \frac{d_2}{\omega^{5/4}} \left(\frac{e}{\hbar} \right)^{3/2} \cdot \left(\frac{500\,b\,k\,T_{sat} R_L}{\pi\,L} \right)^{3/4} \cdot \left(\frac{B_{array}}{C_s} \right)^{1/2} \quad \text{(large junctions)} \tag{20b}$$

5. Examples

To understand the implications of the equations developed above, it is useful to consider the design of SIS mixers for several different frequency bands. It is assumed that a prototype mixer exists, which will be scaled for operation at these frequencies. The prototype will be assumed to operate with a junction (array) of normal resistance $R_{n,\ array}$ = 50 ohms, and B_{array} = 0.1 Ω^{-1}, and to have conversion loss L = 1 (0 dB) and RF bandwidth of 10% (b = 0.1) for pumping parameter α = 1.2 and an IF load of 50 ohms. The SIS junctions are assumed to have a specific capacitance C_s = 60 fF/μm^2, corresponding to the Nb/Al-Al_2O_3/Nb devices developed by Huggins and Gurvitch [11]. The mixers will be required to have 1% gain compression for three different situations: (A) when looking at the sun (a broadband source with T ≈ 6000K), (B) when looking at a broadband room-temperature source (T = 300K) and, (C) when subjected to a narrow-band input signal of 1 nW. For these three cases the calculated parameters are presented in Tables I - III.

The parameters chosen for the prototype mixer are roughly optimum values for an SIS mixer at ~ 115 GHz. The array resistance and susceptance correspond to an $\omega R_n C_j$ product of 5; experimental evidence shows this to give superior performance at 115 GHz [9]. The conversion loss is chosen to be unity as a reasonable compromise between a larger L, which depresses the receiver sensitivity, and a much smaller L (i.e., gain), which limits the bandwidth and the saturation level and increases the risk of instability while giving only a marginal increase in receiver sensitivity.

The thermal source temperature at which 1% gain compression occurs is shown graphically in Figs. 1 - 4 as a function of the number of junctions and the RF bandwidth of the mixer, for mixers at 38, 115, 230, and 345 GHz. The broken curves are the corresponding LO powers. In calculating T_{sat} from eqn.(17) we have assumed L = 0.0 dB and R_L = 50 Ω. In calculating P_{LO} from eqn.(10) we have assumed α = 1.2 and $R_{n,array}$ = 50 Ω.

6. Discussion

The array lengths given in the tables are expressed in units of the free-space wavelength. The array length is significant in two contexts: the parasitic inductance of the array, and electrical phase shifts along the array. The parasitic series inductance is an important factor in designing a mixer with an inductive tuning element connected directly across the array. This is discussed in detail in [10]. The significance of electrical phase shifts along an array as a result of its length is not yet well understood. However, it is obvious that if an appreciable phase shift existed between individual junction voltages, the treatment of the array as an equivalent single junction would no longer be valid. The mixer configuration is a factor here: For mounts in which the array is parallel to the electric field, such as a waveguide mount with the array suspended between the broad walls, the fundamental mode couples to all junctions of the array with the same phase (higher-order, evanescent modes generated by the presence of the array may cause phase differences). For mounts in which the array is not aligned with the fundamental mode E-field, such as a microstrip transmission line with the array connected in series, phase differences are more likely to occur. Evidence of phase shifts in long microstrip arrays has been reported by Rudner *et al.* [12].

Are the prototype mixer parameters both optimum and practicable over the range of frequencies considered? Although there is no evidence that $\omega R_n C_j$ = 5 is optimum at frequencies above 115 GHz, the relatively large capacitance can cause no problem as long as the external microwave tuning circuit can be successfully scaled in frequency (to resonate the capacitance) as we assume here. The choice of $R_{n,array}$ = 50 ohms has been found by computer simulation and in practice to result in good mixer performance with practical values of source (RF) and load (IF) impedance. It remains to be seen whether unity conversion loss can be attained at frequencies above 115 GHz, but, in theory at least, conversion gain is possible at frequencies much higher than those considered in the Tables. The optimum value of the pumping parameter α is approximately independent of frequency [2]. We have not considered the effects of Josephson noise [12], which may interfere with high frequency SIS mixer operation. This was not a limiting problem in recent experiments [13] covering the frequency range of the Tables. The results in this paper are changed only slightly if junctions other than Nb/Al-Al_2O_3/Nb or Pb-alloy are used. The quantity $I_J R_n$ in the discussion leading to Eqn.(9) may be slightly modified, and of course, the appropriate value of the specific capacitance C_s must be used. In sum, it appears that the prototype mixer parameters we have chosen are realistic and close to optimum over a wide range of frequencies.

7. References

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TABLE I

CASE A: T_{sat} = 6000K (the sun)

$R_{n,array}$ = 50 Ω, B_{array} = 0.1 $Ω^{-1}$, C_s = 60 fF/$μm^2$, α = 1.2, b = 10%, L = 0.0 dB, R_L = 50 Ω

		38 GHz	115 GHz	230 GHz	345 GHz
From (17)	N =	25	14	10	8
From (18)	a =	13.2 μm	5.8 μm	3.4 μm	2.5 μm
From (9)	J_c =	490 A/cm^2	1470 A/cm^2	2950 A/cm^2	4420 A/cm^2
From (20a) small jn. approx. with d_1 = 10 μm	l_{array} =	0.032 $λ_o$	0.055 $λ_o$	0.078 $λ_o$	0.096 $λ_o$
From (20b) large jn. approx. with d_2 = 2	l_{array} =	0.084 $λ_o$	0.064 $λ_o$	0.054 $λ_o$	0.049 $λ_o$
From (19)	P_{LO} =	0.22 μW	0.68 μW	1.4 μW	2.0 μW

TABLE II

CASE B: T_{sat} = 300K (room temperature)

$R_{n,array}$ = 50 Ω, B_{array} = 0.1 $Ω^{-1}$, C_s = 60 fF/$μm^2$, α = 1.2, b = 10%, L = 0.0 dB, R_L = 50 Ω

		38 GHz	115 GHz	230 GHz	345 GHz
From (17)	N =	6	3	2	2
From (18)	a =	6.3 μm	2.7 μm	1.6 μm	1.2 μm
From (9)	J_c =	490 A/cm^2	1470 A/cm^2	2950 A/cm^2	4420 A/cm^2
From (20a) small jn. approx. with d_1 = 10 μm	l_{array} =	0.007 $λ_o$	0.012 $λ_o$	0.017 $λ_o$	0.021 $λ_o$
From (20b) large jn. approx. with d_2 = 2	l_{array} =	0.009 $λ_o$	0.007 $λ_o$	0.006 $λ_o$	0.005 $λ_o$
From (19)	P_{LO} =	0.011 μW	0.034 μW	0.068 μW	0.102 μW

TABLE III

CASE C: $P_{sat} = 10^{-9}$ W (narrow band)

$R_{n,array} = 50\ \Omega$, $B_{array} = 0.1\ \Omega^{-1}$, $C_s = 60$ fF/μm^2, $\alpha = 1.2$, $b = 10\%$, $L = 0.0$ dB, $R_L = 50\ \Omega$

		38 GHz	115 GHz	230 GHz	345 GHz
From (7)	N =	45	15	7	5
From (8)	a =	17.7 μm	5.8 μm	2.9 μm	1.9 μm
From (9)	J_c =	490 A/cm^2	1470 A/cm^2	2950 A/cm^2	4420 A/cm^2
From (14a) small jn. approx. with $d_1 = 10$ μm	l_{array} =	0.057 λ_o	0.057 λ_o	0.057 λ_o	0.057 λ_o
From (14b) large jn. approx. with $d_2 = 2$	l_{array} =	0.201 λ_o	0.066 λ_o	0.033 λ_o	0.022 λ_o
From (11)	P_{LO} =	0.72 μW	0.72 μW	0.72 μW	0.72 μW

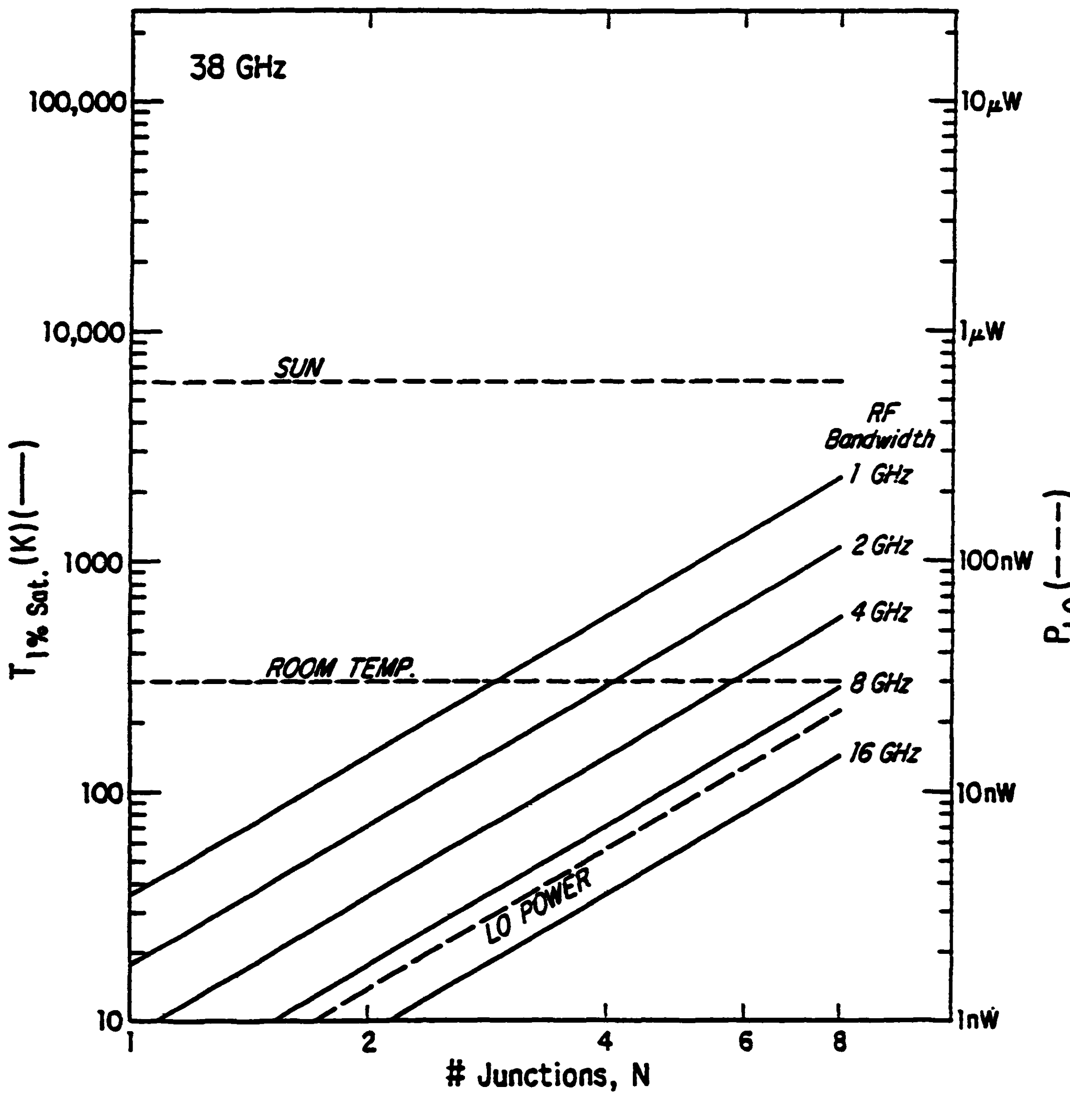

Fig. 1. Thermal source temperature to give 1% gain compression in a 38 GHz SIS mixer as a function of the number of junctions N, for various RF bandwidths. Also shown is the LO power as a function of N. Other parameters assumed in calculating T_{sat} from eqn.(17) are L = 0.0 dB and R_L = 50 Ω. Other parameters assumed in calculating P_{LO} from eqn.(10) are α = 1.2 and $R_{n,array}$ = 50 Ω.

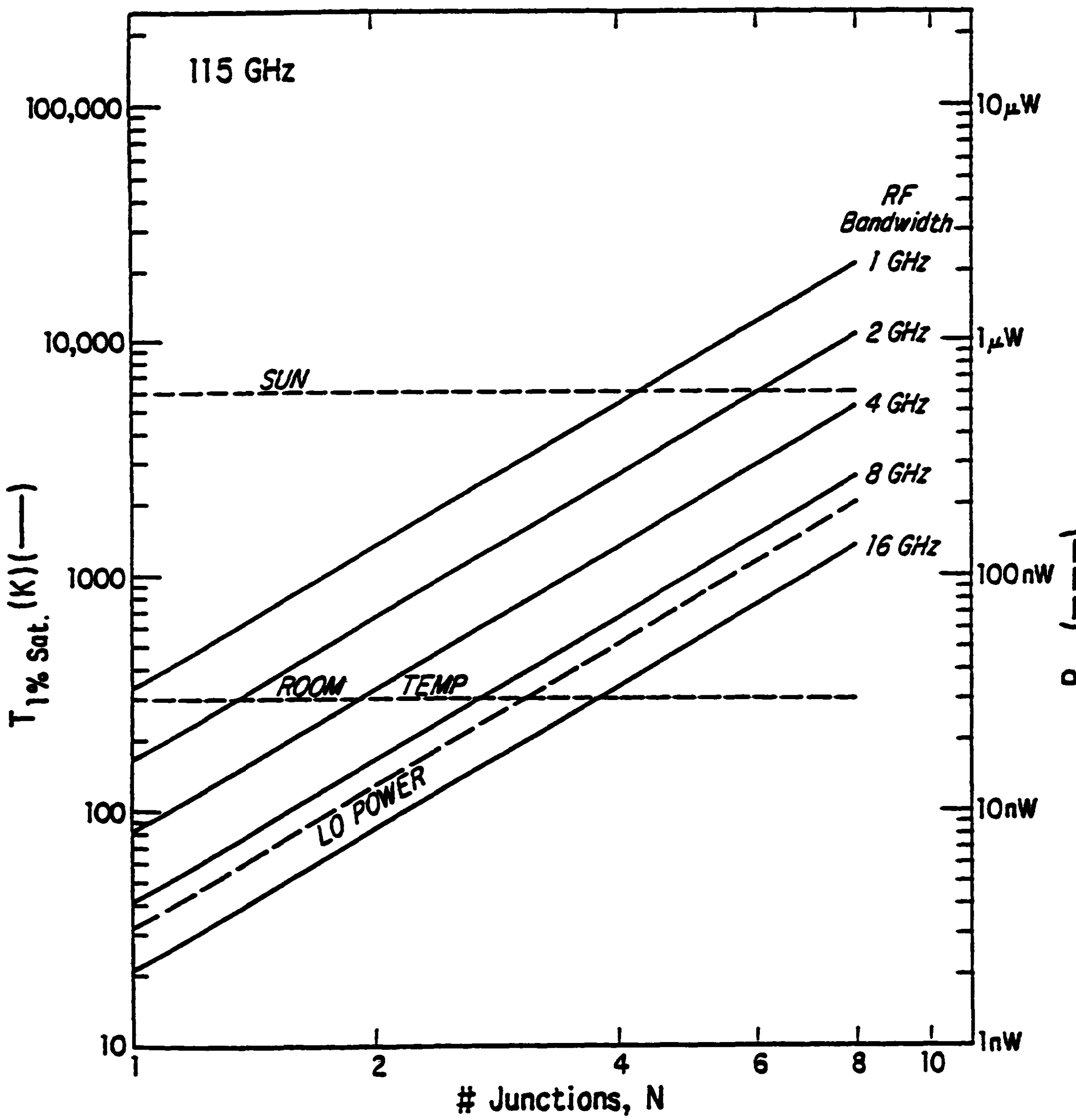

Fig. 2. Thermal source temperature to give 1% gain compression in a 115 GHz SIS mixer as a function of the number of junctions N, for various RF bandwidths. Also shown is the LO power as a function of N. Other parameters assumed in calculating T_{sat} from eqn.(17) are L = 0.0 dB and R_L = 50 Ω. Other parameters assumed in calculating P_{LO} from eqn.(10) are α = 1.2 and $R_{n,array}$ = 50 Ω.

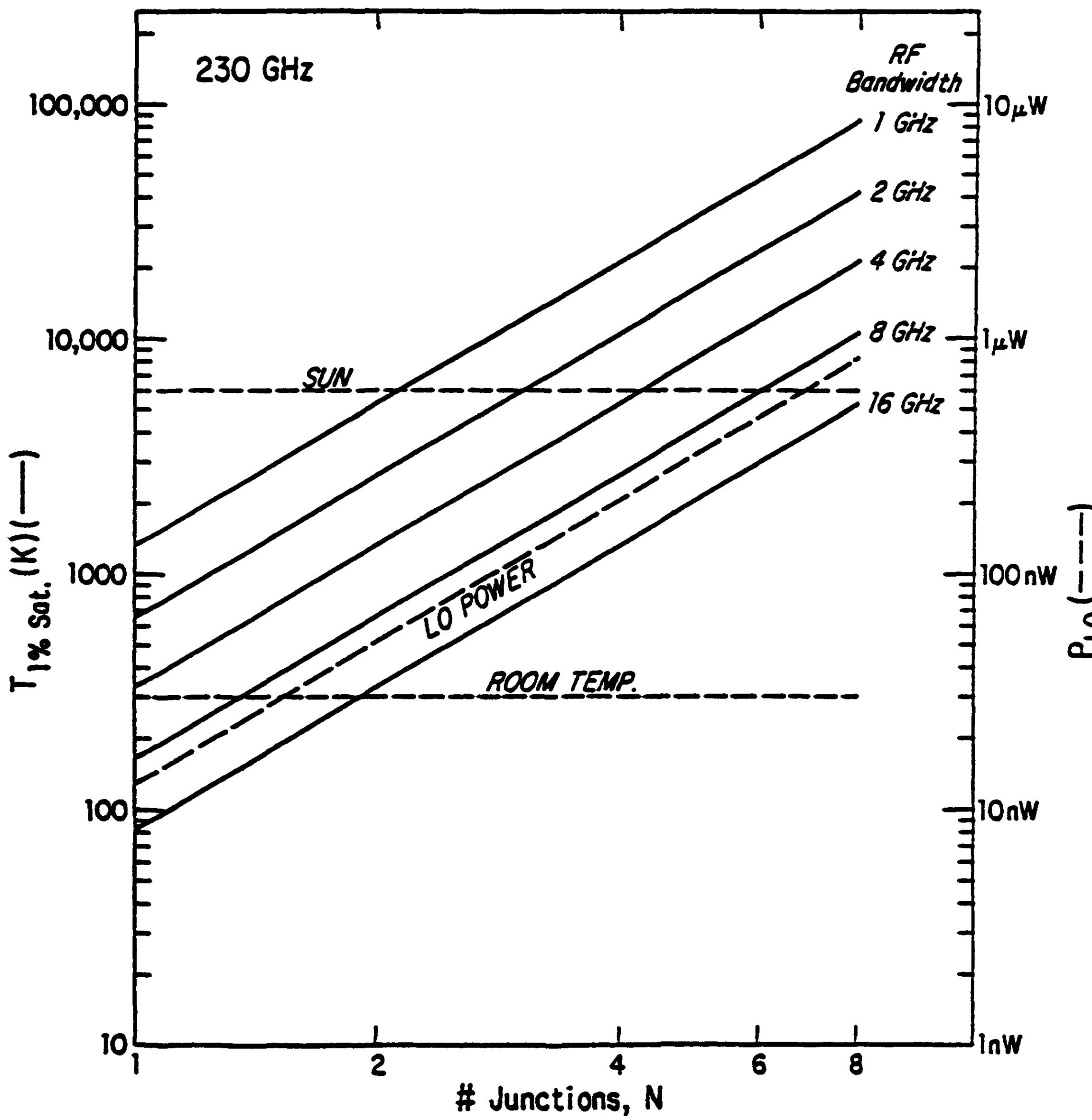

Fig. 3. Thermal source temperature to give 1% gain compression in a 230 GHz SIS mixer as a function of the number of junctions N, for various RF bandwidths. Also shown is the LO power as a function of N. Other parameters assumed in calculating T_{sat} from eqn.(17) are L = 0.0 dB and R_L = 50 Ω. Other parameters assumed in calculating P_{LO} from eqn.(10) are α = 1.2 and $R_{n,array}$ = 50 Ω.

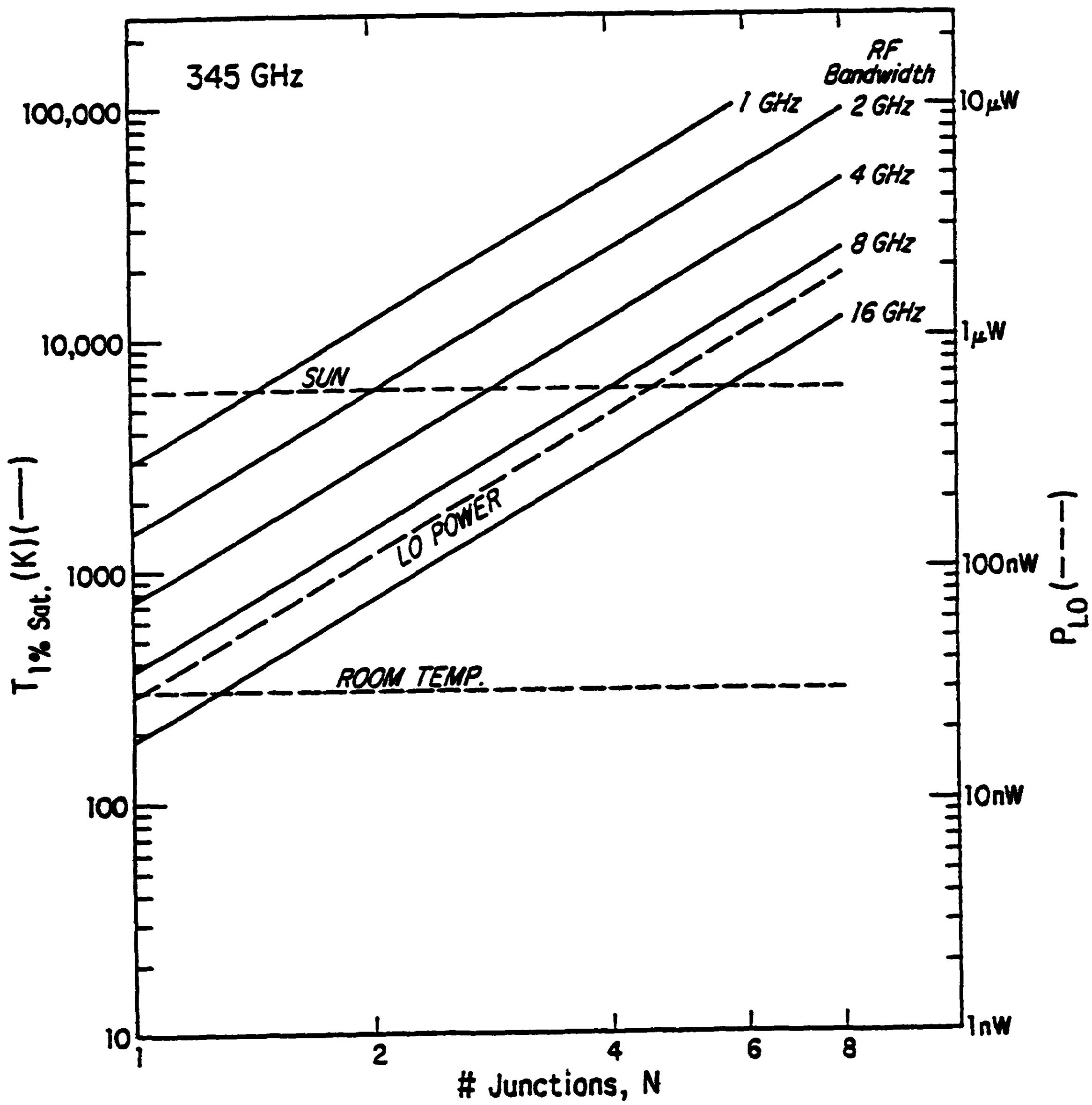

Fig. 4. Thermal source temperature to give 1% gain compression in a 345 GHz SIS mixer as a function of the number of junctions N, for various RF bandwidths. Also shown is the LO power as a function of N. Other parameters assumed in calculating T_{sat} from eqn.(17) are L = 0.0 dB and R_L = 50 Ω.* Other parameters assumed in calculating P_{LO} from eqn.(10) are α = 1.2 and $R_{n,array}$ = 50 Ω.