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CHIRP TRANSFORM SPECTROMETERS FOR MILLIMETER SPECTROSCOPY

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1. Introduction

The compressive or microscan receiver [1] is a long-known method of implementing a sensitive, high speed spectrometer. It involves the use of a swept local oscillator and a dispersive delay line in the i.f. path, where the rate of frequency sweep df/dt and the dispersion rate $d\tau/df$ are reciprocals. Under a few simple conditions, this results in a dispersed i.f. signal whose average power vs. time is proportional to the input signal's average power vs. frequency. This holds over the time of one l.o. sweep, and can be repeated for successive sweeps if the input signal has a stationary power spectrum. Given sufficient bandwidth in the delay line, the full input bandwidth is being processed at all times, so there is no loss of sensitivity relative to a filter bank spectrometer. The frequency resolution is the reciprocal of the sweep time.

The output of the i.f. delay line is sometimes called the chirp transform of the input signal. If multiplied by another swept signal with the correct initial phase, it becomes either the real or imaginary part of the Fourier transform of the input.

In this report, I first develop the mathematical expressions describing the operation of this device. I then consider applying it to the construction of a radiotelescope back end by putting in reasonable specifications and developing a block diagram. Several alternate configurations are considered. The critical components of this system are then examined in relation to the present state of the art. Finally, the practical value of such a spectrometer is assessed in comparison to competing technologies.

2. The Greek

Let a chirp function be defined by

$$C_k(t) = \prod(t/T_k) \cos(\omega_k t + \mu_k t^2) \tag{1}$$

with parameters T_k (duration), ω_k (center frequency), and μ_k (slope). Then a *chirp filtcr* has an impulse response of the form $h(t) = C_k(t-\tau)$, where $\tau > T_k/2$ so that h(t) is causal. We will ignore causality in these calculations by taking $\tau = 0$ (in order to avoid carrying extra constants around), remembering at the end that an overall delay should be added.

Consider a system in which a real signal x(t) is first multiplied by a chirp function C_1 and then passed through a chirp filter with impulse response C_2 . This is known as the *multiply-convolve* or MC configuration. The output is

$$y(t) = \int_{-\infty}^{\infty} x(t') C_{1}(t') C_{2}(t-t') dt'$$

$$= \int_{-\infty}^{\infty} \prod(\frac{t'}{T_{1}}) \prod(\frac{t-t'}{T_{2}}) x(t') \cos(\omega_{1}t' + \mu_{1}t'^{2}) \cos(\omega_{2}(t-t') + \mu_{2}(t-t')^{2}) dt'$$

$$= \frac{1}{2} \int_{a}^{b} x(t') [\cos((\omega_{1} - \omega_{2})t' + \omega_{2}t + (\mu_{1} + \mu_{2})t'^{2} + \mu_{2}t^{2} - 2\mu_{2}tt') + \cos((\omega_{1} + \omega_{2})t' - \omega_{2}t + (\mu_{1} - \mu_{2})t'^{2} - \mu_{2}t^{2} + 2\mu_{2}tt')] dt'$$

$$= \frac{1}{2} \operatorname{Re} \left\{ e^{j(\omega_{2}t + \mu_{2}t^{2})} \int_{a}^{b} x(t') e^{-j\mu_{2}tt'} + e^{-j(\omega_{1} + \omega_{2})t' - j(\mu_{1} - \mu_{2})t'^{2}} \right] dt' \right\}.$$

$$(3)$$

Here a, b have been used in the limits of integration to account for the finite duration of the chirps; their values are worked out later. We shall attempt to manipulate the integral in (3) so that it

looks like a Fourier transform. Let's assume that x(t) is bandlimited with center frequency ω_0 and bandwidth B. Then by proper choice of the parameters, it should be possible to make one of the exponential terms in brackets contain only frequencies that are large compared to all frequencies in x(t), so that that term does not contribute to the integral. For convenience, let this be the second term. If we now let $\mu_2 = -\mu_1$, eliminating the quadratic term in the exponent, (3) reduces to

$$y(t) = \frac{1}{2} \operatorname{Re} \left\{ e^{j(\omega_2 t + \mu_2 t^2)} \int_a^b x(t') e^{-j\Omega t'} dt' \right\},$$
(4)

where $\Omega = \omega_2 - \omega_1 + \mu_2 t$. The integral is the Fourier transform of a certain segment of the input signal, namely that segment in [a, b]; calling this transform $X_{ab}(\Omega)$, we have

$$y(t) = \frac{1}{2} \left[\cos(\omega_2 t + \mu_2 t^2) \operatorname{Re} \{ X_{ab}(\Omega) \} - \sin(\omega_2 t + \mu_2 t^2) \operatorname{Im} \{ X_{ab}(\Omega) \} \right].$$
(5)

Next we need to assume that over the interesting range of Ω , namely $\Omega = \omega_0 \pm B/2$, $X_{ab}(\Omega)$ changes slowly compared to the cosine and sine factors. Then the mean square value of y(t), for suitable averaging times, will be

$$\overline{y(t)^2} \approx \frac{1}{4} |X_{ab}(\Omega)|^2.$$
(6)

Incidentally, one could also get the real and imaginary parts of the Fourier transform of the input segment by multiplying y(t) by cosine and sine chirps, respectively, and averaging. The times t corresponding to the interesting range of Ω are such that

$$\omega_0 - (\omega_2 - \omega_1) - B/2 \le \mu_1 t \le \omega_0 - (\omega_2 - \omega_1) + B/2.$$
(7)

Let's review the assumptions and special conditions leading to (6): (a) The two chirps should have opposite slopes, $\mu_1 = -\mu_2$. (b) The "sum term" should contain frequencies large compared with the largest frequency in the signal; this requires $\omega_1 + \omega_2 - |\mu_1 T_1| \gg \omega_0 + B/2$. (c) The frequencies in the second chirp (the filter's passband, roughly) should all be large compared with $\mu_1/\Delta\omega$, where $\Delta\omega$ is the *resolution* required in measuring the spectrum of the signal; this is because the detected output signal $y(t)^2$ needs to be averaged for at least a few cycles of $\omega_2 + \mu_2 t$.

Now consider the limits of the integral. They cover the range of t' over which $\Box(t'/T_1) \Box((t-t')/T_2)$ is non-zero. As illustrated in Figure 1, there are three distinct time periods during which the integral (and hence the output y(t)) is non-zero:

$$-(T_2 + T_1)/2 < t < -|T_2 - T_1|/2,$$

$$-|T_2 - T_1|/2 < t < +|T_2 - T_1|/2,$$

$$+|T_2 - T_1|/2 < t < +(T_2 + T_1)/2.$$
(8)

The integration limits are a bit different depending on whether $T_2 < T_1$ or the reverse, but in either case the length of the integration is constant during the middle time period and given by $b-a = \min(T_1, T_2)$, and it falls off linearly to zero during each of the outer time periods.

The case $T_2 > T_1$ (Figure 1b) is slightly neater because the limits for the middle period are then fixed at $a = -T_1/2$, $b = T_1/2$. From (8), we see that the duration of the output over which we get this full integration time is $T_3 = T_2 - T_1$. The particular choice $T_3 = T_1$ or $T_2 = 2T_1$ then takes on special significance: in this case the segment of x(t) occuring during the l.o. sweep (of duration T_1) is transformed to an output signal y(t) of exactly the same duration, when $y(\Omega/\mu_1)$ has amplitude proportional to that of $X_{ab}(\Omega - \omega_2 + \omega_1)$. This arrangement, where the filter impulse response is twice as long as the l.o. chirp, is known as multiply short-convolve long or M(s)C(1)[2]. The opposite case, where $T_1 = 2T_2$, gives a similar result, except that the segment of x(t) being transformed is selected by a sliding window; this produces the same result if the input is a stationary random process. Here the output duration is equal to the length of the filter impulse response, and the l.o. chirp must be twice as long; naturally this is called M(1)C(s).



Figure 1: Integration limits and input function windowing.

Notice that, from (7), the frequency range of the output sweep is centered on the input band only if $\omega_2 - \omega_1 = \omega_0$; roughly, this means that the l.o. chirp sweeps the input band across the filter bandwidth. Furthermore, to cover the input bandwidth using the full integration time we need $\mu_1 T_1 \ge B$. Thus we have three more conditions to add to our list if everything is to work properly: (d) $|T_2 - T_1| = \min(T_2, T_1)$; (e) $\omega_2 - \omega_1 = \omega_0$; and (f) $\mu_1 T_1 \ge B$.

Finally, from (8), it seems that there can be non-zero output before and after the output of interest, where the transform is taken over a shorter segment of x(t). But these times correspond to input frequencies outside the signal bandwidth if conditions (e) and (f) are met, so we should have y(t) = 0. This means that the process can be repeated (by starting another l.o. sweep) every $\min(T_1, T_2)$ in order to observe another segment of the input; the output sweeps will not interfere with each other, and the input segments will neither overlap nor have gaps.

It should be mentioned that another chrip transform arrangement is possible in which the input signal is first passed through a dispersive filter, then multiplied by a chirp, then filtered again; this is called CMC [2]. This arrangement is not particularly useful for spectroscopy, so I do not consider it here, although it does allow the Fourier transform of the input to be computed.

3. A Sample Design

Figure 2 shows a basic block diagram of a spectrometer with a bandwidth of 100 MHz.

While this may not be enough bandwidth to be of practical interest in millimeter astronomy, it can be built with available components and with minimal compromise in the design. It serves to illustrate some of the problems, and provides a point from which to extrapolate to larger bandwidths.

The design uses the M(s)C(l) configuration, with the l.o. chirp being generated by applying a short pulse to a chirp filter of duration 10 μ sec, slope 10 MHz/ μ sec, and center frequency 200 MHz. The i.f. chirp filter, which must have twice the duration and bandwidth, is centered at 400 MHz in order to keep its fractional bandwidth well inside the one-octave limit of surface acoustic wave devices (discussed later). The input signal band, the l.o. chirp, and harmonics of the l.o. should all be kept outside the i.f. band, and this dictates that both should be above the i.f. Either USB or LSB mixing could be used, and we have chosen LSB with the signal at 550 to 650 MHz. The l.o. chirp then needs to be 950 to 1050 MHz, so the output of the first chirp filter is upconverted



Figure 2: Sample design for a 100 MHz bandwidth spectrometer.

using a fixed l.o. at 1400 MHz. Notice that the latter mixing reverses the direction of the chirp, so that equal-slope chirps are required in each filter rather than opposites. Amplifiers are required at various points to overcome the insertion loss of the chirp filters, expected to be about 30 to 40 dB.

Since the input is measured in segments equal to the sweep length of $10 \mu sec$, the measured spectrum will be convolved with $sinc^2(f \times 10\mu sec)$, which has a 6 dB width of 0.1 MHz. The detected signal will have a video bandwidth equal to the input bandwidth of 100 MHz. This means that if the full frequency resolution of the spectrometer is to be preserved, then the detector output must be sampled at 200 Msamples/sec (every 5 nsec). Figure 2 indicates digitizing and accumulating occuring at this rate. The resolution could be deliberately degraded to allow sampling more slowly by reducing the bandwidth of the post-detector low pass filter. To avoid further degradation, the video bandwidth of the detector should be larger than that of the low pass filter.

This report will not consider the analog to digital conversion or the digital circuitry in any detail, but it appears that conversions at the rate of 100 to 200 Msamp/sec with 2 to 6 bits of precision are feasible. Fast shift registers could then be used to divide the data into several slower streams for parallel processing with lower speed logic. The question of how many bits are needed in the digitization is interesting. Notice that the predetection and postdetection bandwidths are always equal (if the full resolution is kept), so the detected signal will always be quite noisy, with a standard deviation about equal to its average value $(T/\Delta T \approx \sqrt{B\tau} = 1)$. Nevertheless, the sample-to-sample dynamic range must accomodate that of the final spectra; if we want to handle strong lines, then each sample must have enough bits for several times the strongest line. I have suggested 6 bits on Figure 2, which allows for peak line temperatures about 20 times the system temperature. However, the nature of the noise on strong lines suggests that a logarithmic digitization would be better; that is, let the quantizer thresholds be spaced exponentially rather than linearly. Then about six levels would give the same dynamic range, and these could be encoded in three bits.

The chirp filters required for the design of Figure 2 are available on the commercial market. Each would cost \$1,000 to \$2,000. The high speed DAC is the only other difficult part. Appropriate chips are probably available, but I have not checked this carefully; if they are not, the DAC could be built out of discrete components. All other parts of the spectrometer are standard.

4. Wider Bandwidths

Figure 3 is an attempt to design a similar spectrometer with a bandwidth of 500 MHz. Here I have not been constrained by requiring that the parts be available, but I have tried to keep to a minimum the amount by which the state of the art is strained.



Figure 3: A 500MHz bandwidth spectrometer.

I have kept the time-bandwidth product at 1000, implying a $2 \mu sec$ sweep time. The M(s)C(l) configuration used in Figure 2 would here require a $4 \mu sec$ long and 1000 MHz wide i.f. filter, which is quite far from what can be produced today; so I have selected instead the M(l)C(s) configuration. The i.f. filter is then $2 \mu sec$ by 500 MHz, which is still hard to do. The longer l.o. chirp can be made up in any of several ways without the use of longer chirp filters.

The same considerations as for the 100 MHz spectrometer come into the selection of frequencies. Naturally, the larger bandwidth leads us to higher center frequencies for all of the signals. I ended up with the input at 1750 to 2250 MHz, and the i.f. at 750 to 1250 MHz. It is possible to use identical chirp filters for the l.o. generation and i.f. filtering, and this is the approach taken here. Although the l.o. sweeps need to cover 1000 MHz and last 4 μ sec, they need to start every 2 μ sec so that they overlap. By pulsing a 2 μ sec by 500 MHz filter every 2 μ sec and mixing the output with fixed oscillators at frequencies differing by 500 MHz, we can get a 1000 MHz long sweep. As before, the fixed oscillators are placed above the chirp frequencies so as to change the sign of the chirp slope. The two half-sweeps could be combined before mixing with the input signal, but instead I have connected them to separate mixers in order to avoid intermodulation between them. The use of identical chirp filters for l.o. generation and i.f. filtering may make it easier to ensure that the chirp slopes are matched.

The digitization and accumulation have the same parameters as for the 100 MHz spectrometer, in particular a 5 nsec sampling period, but since the sweep lasts only 2μ sec only 400 samples are taken per sweep. This amounts to throwing away much of the available resolution. The averaging is accomplished in the low pass filter, which has only 100 MHz bandwidth compared with the 500 MHz video bandwidth of the detector output.

5. Errors and Limitations

The results of section 2, especially equation (5), depend on the accuracy of the two chirp functions. In practice, the slopes of the chirps might not be perfectly matched, and the functions may depart in more general ways from the ideal form. We can write

$$C_k(t) = \prod(t/T_k) \left[1 + \delta(t)\right] \cos[\omega_k t + \mu t^2 + \phi(t)]$$
(9)

where $\delta(t)$ is the amplitude error and $\phi(t)$ is the phase error, including any departure of the slope from the nominal value μ . To see the effects of this, we could substitute (9) into (2) in place of (1). There is no simple, general result. If the errors are small, they can be regarded as causing a distortion of the shape of the channel bandpass, nominally $\operatorname{sinc}(fT_{\min})$. Errors in the multiplying chirp C_1 will affect the response in the same way at all frequencies, but errors in the convolving chirp C_2 will cause the response to be frequency dependent. In a system where the detector output is being heavily averaged so as to discard much of the inherent resolution (as in Figure 3), small errors in the chirps will have little effect because the response shape is dictated by the low pass filter bandwidth.

Another type of error in SAW filters is called the triple transit error. This is the result of reflections at the transducers causing a spurious response at a time corresponding to three crossings of the substrate. Such responses seem to be around 20 dB down, and this may be adequate for our purposes. If not, it is possible to implement a scheme in which two i.f. filters are used, each being switched into place on alternate sweeps; it can then be arranged that the triple transit response of each filter occurs when it is not being used.

In practice, errors in chirp filters are usually specified in the frequency domain. The transfer function (Fourier transform) of a chirp is mathematically complicated (involving Fresnel integrals), but approaches a rectangular bandpass in amplitude and a quadratic in phase as the time-bandwidth product gets large. Examples are given in Figure 4 (from reference [6]).

6. Available Components

Consideration of spectrometers of this type is being prompted by the development of suitable chirp filters using surface acoustic wave (SAW) technology [3]. This development is in turn motivated by applications quite different from ours, especially detection of short pulses of unknown timing and frequency (usually from hostile radars). There, a high speed sweep is desired in order to avoid missing a pulse; for us, the input is stationary and a slow sweep is desirable.

The SAW filters use several different construction techniques, the details of which will not be considered here. Generally, they involve generation of an acoustic wave on the surface of a piezoelectric crystal by means of an electrostatic transducer, and then detecting the wave with a similar transducer at another point on the crystal's surface. The wave propogation is mostly non-dispersive, with the dispersion being introduced by the transducers or by frequency-selective structures along the propogation path. Elements of the transducers must have dimensional accuracy better than the acoustic wavelength, and this limits the upper frequency of the filters; the required accuracy becomes sub-micron for frequencies above about 1 GHz.

Some engineering characteristics of SAW chirp filters are given below. This information is obtained largely from telephone discussions with the manufacturers listed in the Appendix.

- Maximum feasible bandwidth: 500 MHz for most manufacturers; up to 1,000 MHz has been demonstrated, but at considerable reduction in accuracy. For off-the-shelf designs, bandwidths up to 200 MHz only.
- Chirp duration at 500 MHz: 0.5μ sec. At least three manufacturers have designs or are developing designs with these dimensions. Up to 2μ sec is considered possible but very difficult, and would require considerable NRE money.
- Accuracy: flatness $\pm 2 \,dB$ across passband; slope error 0.5%; phase deviation 10 deg. (The phase accuracy stated is much better than would be expected from the slope error alone, so I take the phase error to be in addition to the slope error.)
- Time-Bandwidth product: 250 at 500 MHz to 25,000 at 2.5 MHz bandwidth.
- Insertion loss: 30 to 40 dB.
- Cost: Few companies offer catalog items at fixed prices. Bugetary estimates for a 500 MHz by $0.5\,\mu$ sec device vary from \$1000 to \$4000 each in quantity 100, times 3 for small quantities. Anything fancier would require NRE of \$20k to \$40k. Closest catalog item (Anderson Labs) was 100 MHz by 20 μ sec at \$13k each for 1-4.





Figure 5, from an Anderson Labs brochure, gives some idea of the possible range of bandwidth and time. The upper ends of this chart appear to be overly optimistic.

7. An Analog Sample-and-Integrate Scheme

As we have seen, increasing the bandwidth for a fixed time-bandwidth product leads to very short sweep times and a requirement for very fast digitizing of the output. Yet we do not really want to look at the output of a single sweep, since it's mostly noise. What we would really like is to be able to add several sweeps (better yet, several thousand) on top of each other before digitizing. It is thus worth inquiring whether an analog processor can be devised to do this.

One such scheme is illustrated in Figure 6.

The idea is to let the output of our compressive receiver propogate along a tapped transmission line, where the propogation time interval between taps is 1/2B. Each tap is connected to a sampler consisting of a switch and a capacitor, with the switch being driven by a short pulse once per sweep. The pulse must occur at just the right time so as to sample the same point in each sweep; to achieve this, each pulse is taken from a tap on another, identical transmission line which is driven in the opposite direction from a single pulse generator. Each sample adds a small amount of charge to its capacitor. Finally, after a suitable integration time, the charges on all capacitors can be measured

Range of Performance for SAW and IMCON Dispersive Delay Lines



Figure 5: Possible parameter ranges for SAW dispersive filters (Anderson Laboratories).



Figure 6: An analog sample-and-integrate scheme based on tap transmission lines.

and digitized by a relatively slow analog to digital converter.

A transmission line with 1 ns taps would be quite feasible; they would be about 1 ft apart if the line has a velocity factor of 0.7. Of course, for 1000 taps this gets to be a bit large. Pulse widths of a few times 10 ps and samplers with aperture times of the same order are also feasible. The accuracy, stability, and dynamic range of such a system might be problems.

8. Discussion

The chirp transform spectrometer has a fundamental weakness: it provides no data rate reduc-

tion. The full-resolution sampling rate required at its output is the same as at its input, since the bandwidth is the same. This is a consequence of having only one detector, so that no averaging can be done if the resolution is to be preserved.

Consider some competing technologies. An analog filter bank provides a detector for each channel, so that the detected signals can be integrated for an arbitrary time before being read out. In a digital autocorrelator, an array of multipliers is operating in parallel, so that the output of each can be integrated for an arbitrary time. An acousto-optical spectrometer is logically equivalent to the filter bank, with the detectors being implemented as photodiodes.

In the present state of the art, a purely digital spectrometer (using an autocorrelator) for 500 MHz bandwidth is barely feasible. Assuming 1000 channels across this band, it would be large, expensive, and power hungry. An analog filter bank is also large and expensive. The cost can be minimized by building a hybrid filter bank/autocorrelator [4], as is already being done for the 12 meter telescope.

Acousto-optical spectrometers may represent a better choice than chirp spectrometers at this time. Bandwidths approaching 1,000 MHz seem to be feasible. AOSs are in routine use on several radiotelescopes, apparently with good results.

In my opinion, it is not reasonable to pursue the use of chirp transforms based on SAW filters for construction of wideband spectrometers. The available devices are only marginally able to achieve the required bandwidth, and the high speed digital processing required would still be formidable.

It should be mentioned that SAW devices have also been used to implement a kind of filter bank [5], an approach quite different from that considered here. But the published results are for a low-bandwidth, low-resolution instrument. It is possible that this technique could be extended to the point where it becomes useful to us, but unless we are willing to support the development of the necessary SAW devices this is unlikely to happen in the forseeable future.

APPENDIX: MANUFACTURERS OF SAW CHIRP FILTERS

- 1. Anderson Laboratories, Bloomfield, CT 06002; 203/242-0761 (Bob King).
- 2. Crystal Technology, Palo Alto, CA; 415/856-7911 (Dr Don Allen).
- 3. Phonon Corporation, Simsbury, CT 06070; 203/651-0211 (Richard Farley).
- 4. Sawtek Incorporated, Orlando, FL 32860; 305/886-8860. (Not yet contacted.)
- 5. RF Monolithics Inc., Dallax, TX 75234; 214/233-2903. (Probably only narrow bandwidths.)
- 6. Texas Instruments. (Devices made for internal use only.)
- 7. Hughes Aircraft. (Devices made for internal use only.)
- 8. TRW. (Devices made for internal use only.)
- 9. Eaton Corp., AIL Division, Melville, NY 11747; 516/595-4566 (Klaus Breuer). (Devices for internal use only.)

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Note: Not many papers on chirp filters are published in the regular literature. Most references are to the annual *Proc. IEEE Ultrasonics Symposium* from 1975 onwards.