EDTN 196

Sideband Calibration of Single-Output Sideband-Separating SIS Receivers

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ALMA Memo 357 [1] describes the measurement of the image rejection and single-sideband noise temperature of sideband-separating receivers with the usual two IF output ports (nominally the USB and LSB output ports). The measurement uses CW test signals in each sideband, but the levels of the test signals do not need to be known. If one of the IF output ports of the sideband-separating receiver is internally terminated, the sideband ratio measurement cannot be made as described in Memo 357. This note describes a modification of the measurement procedure which allows the sideband ratio and SSB noise temperature to be determined accurately for a single-output SIS receiver. The new procedure relies on the fact that SIS mixers have anti-symmetric I(V) curves, I(-V) = -I(V), so that reversing the bias polarity simply changes the phase of the IF output by π. This means that in a sideband-separating mixer, reversing the bias polarity on one of the two component mixers swaps the USB and LSB output ports.

Fig. 1 depicts a single-output sideband-separating SIS receiver with equal (a) and opposite (b) bias polarities on the two component mixers. (It is arbitrarily assumed that the internally terminated IF port is the LSB port when the bias polarities are equal.) If this is not the case, the same analysis applies but with the + and — superscripts interchanged. The upper and lower sideband conversion gains are (a) G_U^+ and G_L^+, (b) G_U^- and G_L^-. The desired image rejection ratios for equal and opposite bias polarities are

$$R^+ = \frac{G_U^+}{G_L^+} \qquad \text{and} \qquad R^- = \frac{G_L^-}{G_U^-} \; . \tag{1}$$

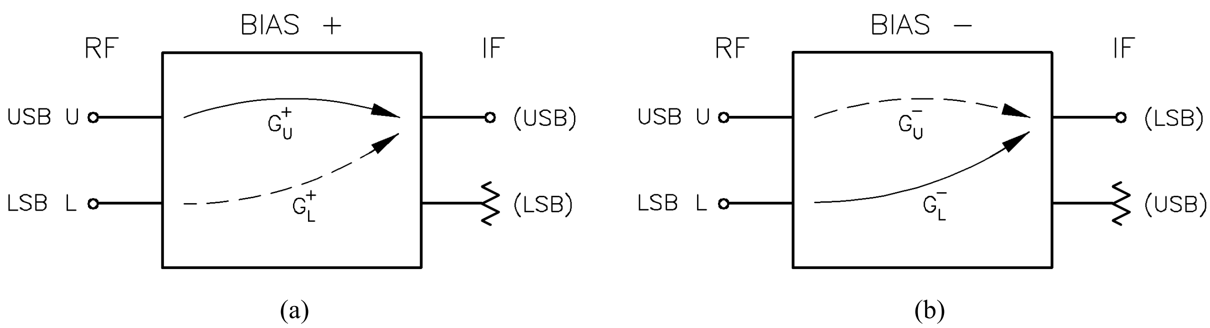

Fig. 1. Power gains of the single-output sideband-separating receiver with equal (+) and opposite (–) bias polarity on the component mixers. The RF upper- and lower-sideband ports are normally the same waveguide or transmission line, but are shown separately here for clarity.

Image Rejection

The following measurements are made:

(i) With a CW test signal (of unknown amplitude) in the upper sideband, the corresponding IF signals are measured with equal and opposite bias polarities (denoted + and –). The ratio of these powers is

$$M_U = \frac{G_U^+}{G_U^-} \; . \tag{2}$$

(ii) With a CW test signal (of unknown amplitude) in the lower sideband, the corresponding IF signals are measured with equal and opposite bias polarities. The ratio of these powers is

$$M_L = \frac{G_L^-}{G_L^+} \; . \tag{3}$$

(iii) The changes, ΔP^+ and ΔP^-, of output power when a cold load at the receiver input is replaced by a hot load are measured with equal and opposite bias polarities. If the difference in noise temperatures of the hot and cold loads is ΔT, then

$$\Delta P^+ = kB\Delta T\left(G_U^+ + G_L^+\right) \ , \tag{4a}$$

$$\Delta P^- = kB\Delta T\left(G_U^- + G_L^-\right) \ . \tag{4b}$$

Define
$$M_{DSB} = \frac{\Delta P^+}{\Delta P^-} = \frac{G_U^+ + G_L^+}{G_U^- + G_L^-} \quad . \tag{5}$$

The measured quantities M_U, M_L, and M_{DSB} can now be used to determine the sideband separation ratios R^+ and R^-. From (5),
$$M_{DSB}\left[G_U^- + G_L^-\right] = G_U^+ + G_L^+ \ . \tag{6}$$

With (2) and (3),
$$M_{DSB}\left[\frac{G_U^+}{M_U} + M_L G_L^+\right] = G_U^+ + G_L^+ \ , \tag{7}$$

so
$$G_U^+\left[\frac{M_{DSB}}{M_U} - 1\right] = G_L^+\left[1 - M_{DSB} M_L\right] \tag{8}$$

and the USB image rejection
$$R^+ = \frac{G_U^+}{G_L^+} = M_U\left[\frac{M_L M_{DSB} - 1}{M_U - M_{DSB}}\right] . \tag{9}$$

Similarly, from (6) with (2) and (3),
$$M_{DSB}\left[G_U^- + G_L^-\right] = M_U G_U^- + \frac{G_L^-}{M_L} \ , \tag{10}$$

so
$$G_U^-\left[M_{DSB} - M_U\right] = G_L^-\left[\frac{1}{M_L} - M_{DSB}\right] \tag{11}$$

and the LSB image rejection
$$R^- = \frac{G_L^-}{G_U^-} = M_L\left[\frac{M_{DSB} - M_U}{1 - M_L M_{DSB}}\right] . \tag{12}$$

<u>SSB Receiver Noise Temperature</u>

The DSB noise temperature of a receiver can be measured by the usual Y-factor method using hot and cold loads with known noise temperatures:

$$T_{R,DSB} = \frac{T_{hot} - Y\,T_{cold}}{Y - 1} \ . \tag{13}$$

This is the noise temperature of the RF source which, when connected at the input (*i.e.,* to both upper and lower sidebands) of a noiseless but otherwise identical receiver, would produce the same output power at the IF port as the actual receiver would produce with its input connected to a source with zero noise temperature.*

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* The concept of zero noise temperature, implying a noise power density of zero W/Hz, is non-physical but is a convenient abstraction.

The single-sideband noise temperature of a receiver is obtained by ascribing all the receiver noise to an equivalent input source in one sideband. It is the noise temperature of the RF source which, when connected to one sideband input of a noiseless but otherwise identical receiver, with the other sideband input connected to a source with zero noise temperature, would produce the same output power at the IF port as the actual receiver with both sideband inputs connected to sources with zero noise temperature. The SSB noise temperature of a DSB receiver with sideband ratio R ($0 \le R \le \infty$) is obtained by correcting the DSB noise temperature for the image contribution to the IF output:

$$T_{R,\ SSB} = T_{R,\ DSB}\left(1 + \frac{1}{R}\right) \ . \tag{14}$$

For a single-output sideband-separating receiver, the SSB noise temperatures are likewise

$$T_{R,\ USB} = T_{R,\ DSB}\left(1 + \frac{1}{R^+}\right) \quad \text{and} \quad T_{R,\ LSB} = T_{R,\ DSB}\left(1 + \frac{1}{R^-}\right) \ . \tag{15}$$

where R^+ and R^- are the image rejections with like and opposite mixer bias.

Conversion from Higher Harmonic Sidebands

In the above, it has been assumed that there is no significant conversion from the higher harmonic sidebands, $nf_{LO} \pm f_{IF}$ (n = 2, 3, ...), to f_{IF} . When conversion from the higher harmonic sidebands is significant, the measurement procedure can be modified to take this into account. The procedure uses an inclined dichroic plate in front of the receiver which reflects the upper and lower sidebands to one side while coupling the higher frequency components from the hot and cold loads to the receiver. It is essentially the same procedure as described in ALMA Memo 357 [1] for a sideband-separating mixer in which both the IF output ports are accessible. The subscripts 1 and 2 in [1], denoting the two output ports, are replaced by superscripts + and **–** denoting like and opposite bias polarities.

Conclusion and Discussion

The image rejection of a sideband-separating mixer with a single IF output port can be measured accurately using CW test signals in the upper and lower sidebands, even when the relative power levels of the test signals are not known. This allows accurate determination of the upper- and lower-sideband gains and the single-sideband noise temperatures, even if the receiver has poor image rejection. (Note that there is no simple and accurate way to determine the sideband ratio and SSB noise temperature of a DSB receiver.)

The frequencies of the upper- and lower-sideband CW test signals used in determining M_U and M_L must be chosen to give the same intermediate frequency f_0. During the measurements with the RF noise sources, a narrow-band IF filter centered at the same f_0 should be used. The bandwidth B of the filter should be smaller than the width of any features on the receiver gain, noise, or image rejection characteristics.

The levels of the CW test signals need not be known, but they must be low enough to avoid saturation of the mixer and the IF measuring system, and large enough to give a measurable response above the noise floor with both bias polarities. The measured IF signal includes the noise of the measuring system. When a power meter or other square-law detector is used, an indicated signal level a factor H above the noise floor of the measuring system can be corrected by a factor (1 - 1/H) to obtain the actual IF signal level. If a spectrum analyzer is used to measure the IF signals, a different correction factor must be used because most spectrum analyzers use an envelope detector rather than a square-law detector. This is discussed in more detail in [1].

When measuring the noise temperature of a receiver using hot and cold loads, the noise temperatures of the loads must be accurately known*. At millimeter wavelengths, the noise temperatures of hot and cold loads are not exactly equal to their physical temperatures [2, 3]. This is because: (i) the Rayleigh-Jeans radiation law deviates from the

* The noise temperature of a load is its radiated noise power density (W/Hz) divided by Boltzmann's constant, k.

Planck law at low temperatures and high frequencies, and (ii) the zero-point (vacuum fluctuation) noise has a significant magnitude at millimeter wavelengths ($hf/2k = 5.5$ K at 230 GHz). In the millimeter-wave range, when liquid nitrogen and room temperature loads are used, these two corrections to the Rayleigh-Jeans law almost cancel out and sufficient accuracy is obtained for most receiver measurements if it is assumed that the noise temperature of a load is equal to its physical temperature. This gives results close to those which would be obtained using the Callen and Welton radiation law in calculating the hot and cold load noise temperatures, which treats the zero-point noise as part of the noise temperature of the source rather than part of the receiver noise temperature. At 230 GHz, the difference between the Callen and Welton noise temperature and the physical temperature of black bodies at 77 K and 300 K is 0.13 K and 0.03 K, respectively.

References

[1] A. R. Kerr, S.-K. Pan and J. E. Effland, "Sideband Calibration of Millimeter-Wave Receivers," ALMA Memo 357, 27 March 2001. See http://www.alma.nrao.edu/memos/.

[2] A. R. Kerr, M. J. Feldman, and S.-K. Pan, "Receiver Noise Temperature, the Quantum Noise Limit, and the Role of the Zero-Point Fluctuations," *Proc. Eighth International Symposium on Space Terahertz Technology*, pp. 101-111, 25-27 March 1997. See ALMA Memo 161 at http://www.alma.nrao.edu/memos/.

[3] A. R. Kerr, "Suggestions for Revised Definitions of Noise Quantities, Including Quantum Effects," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 3, pp. 325-329, March 1999.