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140-FT. POINTING PROGRAM, AND THERMAL SHIELDING OF SHAFT AND YOKE

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Summary

Astronomical pointing data, taken by K. Kellermann on Sept. 20 - 23, are analyzed for thermal pointing errors. Thermal shielding had been installed on Sept. 3 - 7 ($14,449.- total costs), with 3 inch foam sprayed on yoke arms and polar shaft, and 1/2 inch on tower walls. Several parts of the building platform were covered with electric heat pads which were not yet electrically connected, thus leaving the tower-platform combination still inactive. The weather-dependent refraction correction was installed and active.

The standard pointing parameters (Kellermann 1975) were used on-line. The analysis showed that two parameters needed a significant change, reducing the rms HA error from 12.4 to 6.2 arcsec. Our present on-line pointing program needs a general revision. It contains one unphysical and two redundant terms to be omitted, and it misses one physical term to be added. The considerable numerical uncertainty of the parameters is discussed.

The large sun-induced pointing errors seem to have disappeared after shielding. With the present (insufficient) data, they are decreased by at least a factor 6. An apparent slow drift of the declination error may still be caused by thermal deformations of platform and tower.

The total observed pointing errors, for day and night, are now 6.2 arcsec rms (and 26 arcsec peak-to-peak) in hour angle, and 6.4 arcsec rms (30 ptp) in declination. As compared with similar observing periods before shielding, the improvement is a factor 2 in hour angle, and a factor 3 in declination.

For a future pointing program, we suggest a reduction from 15 to 11 parameters, and a least-squares procedure which yields also their mean errors and correlations.

1. Thermal Shielding

Previous investigations of the 140-ft pointing errors (Reference 1 and 2) gave the following results. First, astronomical pointing data from K. Kellermann (runs of several days with ambient air changes of 15 ^{o}C peak-to-peak), after reduction with their own best-fit 15 pointing parameters, showed the following residual pointing errors (day and night) which should be mostly due to thermal deformations:

	rms	ptp		
hour angle, $\Delta h = \cos D\ \Delta H$	13	68	arcsec	(1)
declination, ΔD	19	80	arcsec	

Second, thermal deformations were directly monitored with electronic levels and thermistors at various locations, during 3/4 year, covering ambient air changes of 40 ^{o}C ptp. The measured ptp deformations were

	shaft + yoke (sunshine)	concrete building (ambient air temperature)		
hour angle	37	9	arcsec	(2)
declination	105	42	arcsec	

Third, it was suggested to cover polar shaft and yoke arms with a sprayed-on layer of 3 inch of foam (topped with a hard surface) for obtaining a long thermal constant, smoothing out all daily effects of sunshine. Covering the whole concrete building would have been best but too expensive. A thermal model of building and tower suggested a compromise: if the tower is sprayed with 1/2 inch foam, and parts of the platform (deck) are covered with electric heat pads simulating a constant room temperature, then the thermal deformations might cancel each other such that the polar shaft moves only up and down in parallel, without changing its direction.

These shieldings were installed on Sept. 3 - 7, 1976. The foam was sprayed on by John L. Renshaw Inc., of Beltsville, Maryland, for $11,329.00 and the total cost (including heat pads, transformers, concrete, ...) amounts to $14,449.00. During the observing period, the heat pads were not yet connected; the tower-platform combination thus was not active, while the shielding of shaft and yoke was complete.

A weather-dependent refraction correction (Ref. 3) was installed and active during the observations, using data from the interferometer about air temperature, air pressure, and dew point temperature.

2. The Observations

During 58 hours of VLBI-observations, at $\lambda = 2.8$ cm on Sept. 20 - 23, 1976, K. Kellermann provided a set of 50 pointing errors in HA, and 54 in Dec, see Table 1. The errors were obtained by the telescope operator while scanning the sources. The on-line pointing program used the 1975 standard parameters, obtained with a least-squares program of Claude Williams from Kellermann's observations of January and March 1975, see Table 2.

About 1/3 of the observing time, the sky was completely overcast with some rain; 1/3 was mostly clear, and 1/3 exceptionally clear. Unfortunately, 8 hours about noon were blocked on September 22 by maintenance, and the observations terminated on September 23 before noon; thus the two periods of strongest sunshine were omitted. But still, the ambient air covered a large range of 23 ^{o}C ptp during the observations, see Fig. 1a.

Table 1. VLBI Observations of K. Kellermann during Sept. 20-23, 1976, with number of observed pointing errors.

Run No.	Source	Flux (Jy)	Number of Observations H	D	Duration (hours)
1	4C 39.25	8	2	2	2.0
2	3C 371	3	3	5	6.2
3	3C 111	5	4	4	5.0
4	4C 39.25	8	7	9	9.6
5	3C 345	6	10	10	8.1
6	3C 84	50	1	1	0
7	3C 345	6	6	6	3.0
8	DR 21	18	4	4	2.1
9	2005+40	4	1	1	0
10	0133+47	2	1	1	0
11	0235+16	2	1	1	0
12	3C 120	6	9	9	9.5
13	3C 273	50	1	1	0
		Total:	50	54	

Fig. 1b shows hour angle and declination of all sources observed, and Fig. 1c gives the hour angle error, $\Delta h = \cos D\ \Delta H$ (note that $\Delta H = -\Delta\alpha$), as obtained with the standard parameters. It is immediately seen that Δh moves parallel with H unless D is very small, which means that the pointing term A_7 (sinD sinH) must be changed. A plot of the remaining residuals indicated that also A_9 (sinD) wants some change.

Table 2. Two of the 140-ft Standard Pointing Parameters (A_7, A_9) must be changed for present observations, see Figs. 1c and 1d. Some terms are redundant. All values in minutes of arc.

	Pauliny-Toth August 1968	Gordon, et al. June 1973	"Standard" Kellermann Jan.+March 1975	Needed at present Sept. 20-23 1976	Remove redundant terms from standard
A_3	1.19	1.08	1.04		
A_4	.37	2.32	1.54		
A_5	1.48	22.04	23.14		0.00
A_7	-.40	-1.98	-1.18	-1.52	
A_8	-.26	-.53	-.52		
A_9	.93	1.32	1.12	1.31	
A_{10}	.50	1.53	.60		
A_{11}	-.19	-2.00	-1.52		
A_{12}	-1.58	-16.64	-17.19		.94
A_{13}	1.53	.24	-.19		
A_{14}	-.29	.43	-.93		-2.11
A_{15}	.23	12.72	13.52		-.85

Old, standard and present values are given in Table 2. We see that the change of A_7 and A_9 as needed at present is within the general range of uncertainty. The resulting residuals are given in Table 3, where the change of A_7 and A_9 yielded an improvement by a factor 2.

Table 3. Change of two pointing parameters, and resulting hour angle error, $\Delta h = \cos D \, \Delta H$.

used	Δh(arcsec)	
	rms	peak-to-peak
standard parameters	12.4	58
change A_7 only	7.8	30
change A_7 and A_9	6.2	26

3. The Pointing Program

Our present pointing program (Ref. 4) needs several revisions. First, regarding internal consistency, the program contains two redundant parameters which should be cancelled. For brevity, we omit the complicated refraction term, and we call s = sin, c = cos. With H = hour angle, D = declination, and L = geographic latitude of Green Bank, and $\Delta h = cD \, \Delta H = -cD \, \Delta\alpha$, the equations (1) and (2) of Reference 4 then read:

$$\Delta h = -A_1 cD + A_4 sH - A_6 + A_7 sDsH - A_8 sD\, cH - A_9 sD + A_{10} sD\, cD\, sH + A_{11} cD\, sH,$$

$$\Delta D = A_2 + A_5(cL\, sD\, cH - sL\, cD) + A_7 cH + A_8 sH + A_{12} sD\, cH + A_{13} sD + A_{14} cH + A_{15} cD$$

We see that A_7 in equation (4) is redundant, since it can be absorbed by A_{14}, both having the same angular dependence, cH. Also, A_5 is redundant in (4), since L is a constant as long as the 140-ft stays at Green Bank; the first part of the parenthesis thus has the constant $A_5 cL$ which can be absorbed by A_{12}, and the second part, $A_5 sL$, can be absorbed by A_{15}. The result is shown in the last column of Table 2.

Cancelling A_5 has a very pleasing result. It bothered me for years that three of the 15 terms should be 10 times larger than all the remaining ones, and should have been completely different in 1968, see Table 2, where the readjustment of the surface done between 1968 and 1973 could only have acted like a box offset plus a beam shift, which can only change A_2 and A_6, but none of the parameters listed in Table 2 (see the physical explanations of Table 4). This discrepancy is now removed. It should also be noted that the accuracy of a least-squares fit decreases with the number of parameters to be solved for; the cancellation of redundant parameters thus should make future determinations of the parameters somewhat more accurate.

Second, we must ask which parameters are physically relevant, since the six parameters A_{10} ... A_{15} were introduced empirically in 1969 (Appendix 1 to Ref. 4). This was investigated by Victor Herrero in 1972 (Ref. 5). He started out with listing all possible physical causes of pointing changes (box offsets, dial errors, misalignments, gravitational deformations, encoder eccentricities). By coordinate transformations he derived 19 parameters with different angular dependence for Δh and ΔD, which we call V_1 ... V_{19} in Table 4. After this careful derivation he concludes a bit hasty that "the empirical terms ... are all recovered in this treatment", whereas Table 4 shows two exceptions: the empirical term A_{10} of Δh (sD cD sH) has no physical counterpart and should thus be omitted, and the physical term V_{16} of Δh (cD cH) is missing in our present program and should be added.

We should emphasize again (Ref. 1) the uncertainty of the observational determinations for the numerical values of the pointing parameters. From one determination to the other, only the offsets of the receiver box will show a larger difference, changing A_2 and A_6, see Table 4. If dial errors are different, this changes A_1 and A_2. Because of different weather conditions, the refraction term A_3 may have changed slightly. But all remaining nine terms (resulting from misalignments, gravity, eccentricity) must be exactly the same if redundancies are removed. In Table 5 we have listed these nine constant terms, after removing the redundancies.

Table 4. Comparison of Herrero's physically derived intermediate (I) and final (V) pointing terms, with the empirical terms (A) as used in present on-line pointing program. Omitted are: refraction (V_4, A_3), the redundant terms of ΔD (A_5, A_7), and the cyclic errors (V_8, V_9, V_{18}, V_{19}). NP = not perpendicular, N = north, E = east.

	for Dec error, ΔD									for H error, Δh = cD ΔH									
angular dependence ⟶	const	sD	cD	sH	cH	sDsH	sDcH	cDsH	cDcH	const	sD	cD	sH	cH	sDsH	sDcH	cDsH	cDcH	sDcDsH
Dec dial error (box off N)	I_1																		
HA dial error												I_2							
Beam NP Dec axis (box off E)										I_3									
Dec axis NP polar axis											I_4								
Polar axis off, I_5=E, I_6=N				I_5	I_6										I_6	I_5			
Gravity dish, I_8=N, I_9=E			I_8				I_8						I_9						
Gravity yoke + polar axis					I_{12}										I_{10}		I_{11}		
Encoder eccentric, Dec +HA		I_{13}	I_{14}														I_{15}	I_{16}	
Final terms Herrero	V_1	V_6	V_7	V_2	V_3		V_5			V_{11}	V_{12}	V_{10}	V_{15}		V_{14}	V_2	V_{17}	V_{16}	
Present pointing program, A_{10} ... A_{15} are empirical	A_2	A_{13}	A_{15}	A_8	A_{14}		A_{12}			A_6	A_9	A_1	A_4		A_7	A_8	A_{11}		A_{10}

Table 5. The nine pointing terms which should have the same numerical value in all observational observations, after redundancy is removed.

term	angular dependence		physical cause	numerical value (arc min)			
				1968	1973	1975	present
	ΔD	ΔH		λ=11 cm	λ=2.8 cm	λ=2.8 cm	λ=2.8 cm
A_4		sH	gravity dish E	+ .37	+2.32	+1.54	
A_7		sDsH	polar axis N, gravity mount	- .40	-1.98	-1.18	-1.52
A_8	sH	sDcH	polar axis E	- .26	- .53	- .52	
A_9		sD	dec axis NP polar axis	+ .93	+1.32	+1.12	+1.31
A_{11}		cDsH	gravity mount, encoder exc.	- .19	-2.00	-1.52	
A_{12}	sDcH		gravity dish N	- .42	+ .63	+ .94	
A_{13}	sD		encoder eccentricity	+1.53	+ .24	- .19	
A_{14}	cH		polar axis N, gravity mount	- .69	-1.55	-2.11	
A_{15}	cD		gravity dish N, encoder exc.	+ .28	- .97	- .85	

Table 5 shows a rather unsatisfactory situation. Some of the differences may be explained. First, the values of 1968 are more uncertain than the others because of the longer wavelength used giving a larger beam (λ = 11 and 6 cm, versus 2.8 cm). Second, if thermal deformations are present, they will be absorbed into the pointing parameters. The 1975 determinations were confined to night observations only, and the 1973 observations were done during an overcast and rainy period; whereas the 1968 determinations covered day and night and no weather data are available; if some sunshine was included, then again the 1968 values have a reason to be more different. But even disregarding the 1968 values, the differences between 1973 and 1975 in Table 5 are still too large to be tolerated.

Which parameters could have changed, by physical causes, from 1973 to 1975? First, we consider the difference in thermal deformations. The 1973 determinations (overcast and rainy) and those of 1975 (nights only) should not have different EW deformations but may have different NS deformations, see the time scales of Figs. 8a and 8b of Reference 1. The NS deformation of shaft and yoke can only give an offset of D without angular dependence, which is the same as an NS box offset, A_2, which occurs anyway and thus is omitted in Table 5. The NS deformation of the concrete building (plus elongation of platform) will tilt the polar axis, I_6 in Table 4, affecting A_7 and A_{14} in Table 5. Both should have changed by the same amount, however, which clearly is not the case:

	1973	1975	change		
A_7	- 1.98	-1.18	+ .80	not equal.	(5)
A_{14}	+ .43	- .93	- 1.36		

Second, the feed legs were braced by cables in the fall of 1974. According to a computer analysis of W. Y. Wong, this increases the lateral stiffness of the feed point by 40%. This changes I_8 and I_9 of Table 4, effecting the parameters A_4, A_{12} and A_{15} of Table 5 by amounts which can be calculated. But no agreement is found between observed and calculated changes:

	1973	1975	change observed	change calculated
A_4	+ 2.32	+ 1.54	- .78	- .40
A_{12}	+ .63	+ .94	+ .31	- .40
A_{15}	- .97	- .85	+ .12	+ .31

no agreement (6)

Third, Cassegrain cabin and additional counterweight were added in the fall of 1974. This increased the weight on the elevation axis by about 22,000 lb, which is about 3% of the total. Thus, I_{10}, I_{11} and I_{12} of Table 4 may have changed by 3%, which then also holds for A_7, A_{11} and A_{14}. But A_7 and A_{14} may have changed by larger amounts from thermal deformations, see (5), which leaves only A_{11}. The observed change, however, is eight times larger:

	1973	1975	change observed	change calculated
A_{11}	-2.00	-1.52	24	3

eight times larger. (7)

We see from (5), (6) and (7), that the observed changes never agree with the physically permitted ones. Furthermore, there is no physical cause known which could have changed A_8, A_9 and A_{13}, whereas the observed changes of two of them are considerable:

	1973	1975	change	
A_8	- .53	- .52	+ .01	
A_9	+1.32	+1.12	- .20	} not zero. (8)
A_{13}	+ .24	- .19	- .43	

How large is the present overall uncertainty? We calculate the rms change for all nine parameters of Table 5, between their values of 1975 and 1973. Regarding the resulting pointing errors, we see from the pointing program that six terms are multiplied by single angular functions (sD, ... cH) which, in the average over most observations, may give a factor about 1/2; four terms have double angular functions (sDsH ... cDsH), with an average factor about 1/4; all terms together thus have a factor about 0.40. Thus, we multiply the rms changes by 0.40 to obtain the average pointing error, which then is in minutes of arc, and we multiply by 60 to obtain seconds of arc. For comparison, all this was done also for the changes from 1968 to 1973:

	rms change of parameters	resulting rms pointing errors
1968 - 1973	1.29	31 arc sec
1973 - 1975	.49	12 arcsec

(9)

In summary, the present uncertainty is very large. There is no agreement between observed and calculated changes; some parameters have changed without physical cause; and the resulting uncertainty adds 12 arcsec to the rms pointing errors.

We hope that a better future consistency will now be reached after the thermal shielding; also, the weather-dependent refraction correction might help. If possible, future determinations of the pointing parameters should give more emphasis to positions close to horizon, pole, and zenith, where the angular dependences show their largest differences. Also, the determinations should cover much more data (large number of partly dependent parameters), and should be extended over long periods (smoothing-out of thermal effects). Extension over long periods seems possible, if VLBI experiments will use the same receiver, in the same rotation angle of the sterling mount, and if the box is mounted always at exactly the same location. With a new set of dowel pins we may assume a box offset of only $b = \pm 0.2$ mm, and with a beam deviation factor BDF = 0.848 (for F/D = 0.425 and 16 db taper, see Ref. 6), the resulting change of the beam offset $\Delta\phi$ then is tolerably small:

$$\Delta\phi = \frac{\Delta b}{F} \, \mathrm{BDF} = \pm \, 2.3 \text{ arcsec.} \tag{10}$$

Once the pointing parameters are well determined, will different weights of the receiver boxes give cause to pointing errors? From the analysis of W. Y. Wong it follows that a weight difference of, say, 200 lb will give, in horizon position, a downward feed offset of 0.25 mm, which is a pointing error of 2.9 arcsec; at declination zero it is only 0.6 arcsec. Thus, weight differences between boxes of $\pm$ 200 lb may be neglected. But the pointing errors for Cassegrain observations would need a separate investigation, including angular deformations at the Sterling mount.

4. Pointing Errors Before and After Thermal Shielding

The thermal pointing errors before shielding are given in (1) and (2). In order to tell how much improvement was achieved by the shielding, we must estimate the expected amount of thermal pointing error which would have been present during the observations of Sept. 20 - 23 if no shielding had been applied. These expected values then are to be compared with the actually observed ones.

The largest thermal errors arise from solar radiation on shaft and yoke. They are proportional to the amount of sunshine, and a useful measure for this amount is given by the rise ΔT of ambient air temperature during the day. Table 6 gives five determinations for these coefficients of proportionality. They are fairly consistent, and we adopt, for the maximum thermal error of a day,

$$\begin{aligned} \Delta h_{max} &= 1.42\ \Delta T\ \text{arcsec}/^{o}\text{C, at noon,} \\ \Delta D_{max} &= 3.80\ \Delta T\ \text{arcsec}/^{o}\text{C, at 4 PM.} \end{aligned} \tag{11}$$

Table 6. Observed maximum thermal pointing errors, ΔH and ΔD, as functions of maximum rise of air temperature, ΔT, during sunny days before thermal shielding ($\Delta h = \cos D\ \Delta H$).

Method	Day	ΔT ^{o}C	Δh arcsec	$\frac{\Delta h}{\Delta T}$	ΔD arcsec	$\frac{\Delta D}{\Delta T}$
Electronic levels	Nov. 11, 1975	19	-27	1.42	-76	4.0
on Dec axis,	Nov. 15, 1975	15	-21	1.40	-54	3.6
(S. von Hoerner)	Nov. 16, 1975	12	-18	1.50	-64	4.5
Astron. Observ.	March 21, 1975	8	-14	1.75	-28	3.5
(K. Kellermann)	March 23, 1975	15	-22	1.47	-51	3.4
	Adopted coefficients:			1.42		3.8

The measured values of ΔT on Sept. 21 - 23 (see Fig. 1a) are given in Table 7, and by multiplication with the coefficients of (11) we obtain the expected maximum values for Δh and ΔD as entered in Table 7. The previously observed time-dependence (Fig. 8 of Ref. 1) then was normalized to these maxima, and also the difference in times of sunrise and sunset was taken care of by expanding the time scale. The resulting expectations are plotted in Fig. 1e for hour angle, and in Fig. 2b for declination.

Table 7. Expected maximum thermal pointing errors without thermal shielding, using adopted coefficients from Table 6.

Day	ΔT °C	Δh arcsec (Noon)	ΔD arcsec (4 PM)
Sept. 21, 1976	6	8.5	22.8
22	15	21.3	57.0
23	22	31.2	83.6

Comparison of actually observed values, with the values to be expected without shielding, gives the ratios shown in Table 8. We see that the sun-induced pointing errors after shielding are not larger than their error limits. The hour angle error is at least down by a factor 2, and the declination error by at least a factor 10. Since both should have changed by the same factor, we may conclude that shielding polar shaft and yoke arms has improved the large sun-induced pointing errors by at least a factor 6 and probably more.

Table 8. Improvement of sun-induced pointing errors, from thermal shielding. (Observed with shielding, expected without it.)

Day	observed/expected	
	for Δh	for ΔD
Sept. 21, 1976	+0.5 ± 0.6	-0.1 ± 0.1
22		+0.2 ± 0.2
23	+0.4 ± 0.3	0.0 ± 0.1

In general, the hour angle errors of Fig. 1d show larger jumps when going from one source to the next, which is difficult to explain. The declination errors of Fig. 2a start with a slow monotonous drift, which seems to be correlated with a smoothed drift of the air temperature, see Fig. 1a. If so, this may be caused by thermal deformations of the concrete building. A further investigation is planned by monitoring during maintenance days.

The over-all observed pointing errors (day and night, thermal and otherwise) are now

	rms	ptp		
hour angle	6.2	26	arcsec.	(12)
declination	6.4	30		

As compared with very similar observations before shielding, see (1), this means an improvement of a factor 2 in hour angle, and a factor 3 in declination.

5. Suggestions for Future Pointing Program

The revisions discussed in Section 3 (regarding redundancy and physical terms) lead to a total of p = 14 parameters, instead of the present 15. In addition, we now suggest a further reduction of the number of parameters to p = 11, and a least-squares determination not only of their numerical values A_k, but of their mean errors as well, $A_k \pm \sigma_k$, in order to tell whether a change from one determination to the next is significant or not.

First, it seems that the encoder eccentricity can be omitted ($I_{13} \ldots I_{16}$ in Table 4), just as the cyclic error (Ref. 5) had been omitted already and actually was never entered into the program. Fred Crews has the tests of the encoders made by the manufacturer. The measured error graphs show short ripples (1° length) from the cyclic errors, superimposed on a long wave (360°) from the eccentricity. The amplitudes are only

$$\begin{aligned} &\text{cyclic error} &&\approx 0.7 \text{ arcsec}, \\ &\text{eccentricity error} &&\approx 1.5 \text{ arcsec}. \end{aligned} \tag{13}$$

Since the intrinsic errors of our pointing system are about 2 - 3 arcsec, and the observational determination of a source position may add another 3 arcsec, we should neglect the encoder eccentricity. From Table 4 we see that this removes three parameters: A_{13}, V_{16}, and A_{15} (which now is $A_{15} = -A_{12} \tan L = -0.7926 A_{12}$). The number of parameters then is p = 11.

The past empirical values confirm this suggestion. V_{16} was not entered empirically at all, meaning it was small; A_{13} was small and about zero within its uncertainty (Table 5); and the difference $A_{15} + A_{12} \tan L = -0.10$ (1975) is also close enough to zero.

Second, for the least-squares determination of the parameters, one must use

$$\Delta h = \cos D \, \Delta H \tag{14}$$

instead of ΔH, because the use of ΔH gives unrealistically high weights to all observations close to the pole.

Third, we want to know the mean error σ of each parameter, and it will also be helpful to know the mutual correlation of these errors. For this purpose, we shall rediscuss the least-squares procedure. Let us call the eleven new parameters P_k (instead of the old A_k), with a new numbering sequence. Including refraction, equations (3) and (4) read

$$\Delta D = P_1 + P_2 sH + P_3 cH + P_4(sDch - 0.7926\ cD) + P_5\ Q(sL - sDcZ)/cD, \tag{15}$$

$$\Delta h = P_6 + P_7 sD + P_8 cD + P_9 sH + P_{10} sDsH + P_{11} cDsH + P_2 sDcH - P_5\ Q\ cLsH, \tag{16}$$

where c = cos, s = sin, D = declination, H = hour angle, L = 38.4°, Z = zenith distance, and

$$Q = \frac{K}{cZ + 0.00175 \tan(Z - 2.5^{o})} \tag{17}$$

and where K is the weather-dependent correction of the refraction term of Reference 3. Equations (15) and (16) shall be written in general form

$$\Delta D = \sum_{k=1}^{p} P_k f_k \quad \text{and} \quad \Delta h = \sum_{k=1}^{p} P_k g_k \quad \text{with } p = 11. \tag{18}$$

The f and g are the angular terms, for example $f_2 = sH$, $g_2 = sDcH$, $f_6 = f_7 = \ldots = f_{11} = 0$, and $f_1 = g_6 = 1$ (the box offsets are P_1 and P_6).

We call n the number of observations available for the determination of the p parameters, and we call f_{ki} and g_{ki} the angular terms of observation number i, for example $f_{2i} = sH_i$, and so on. The residual R (sum of the squares of all pointing errors) then is

$$R = \sum_{i=1}^{n} \left\{ \left(\sum_{k=1}^{p} P_k f_{ki} - \Delta D_i\right)^2 + \left(\sum_{k=1}^{p} P_k g_{ki} - \Delta h_i\right)^2 \right\} . \tag{19}$$

The least-squares values of the P_k then are found by letting all $\partial R/\partial P_k = 0$, which gives a set of p linear equations, $\ell = 1 \ldots p$,

$$\sum_{k=1}^{p} M_{\ell k} P_k = B_\ell , \tag{20}$$

with

$$M_{\ell k} = \sum_{i=1}^{n} \left\{ f_{\ell i} f_{ki} + g_{\ell i} g_{ki} \right\} \tag{21}$$

and

$$B_\ell = \sum_{i=1}^{n} \left\{ f_{\ell i} \Delta D_i + g_{\ell i} \Delta h_i \right\} . \tag{22}$$

Our old program solved (20) directly by eliminations. The new program should invert Matrix M, since the inverse, M^{-1}, is needed also for finding the errors. The best-fit values of the parameters then are obtained as

$$P_k = \sum_{\ell=1}^{p} M^{-1}_{k\ell} B_\ell . \tag{23}$$

Actually, our observed values of ΔD_i and Δh_i contain measuring errors, ε_i and η_i, thus (18) reads, calling $\hat{P}_k$ the true values,

$$\Delta D_i = \sum_{k=1}^{p} \hat{P}_k f_{ki} + \varepsilon_i \quad \text{and} \quad \Delta h_i = \sum_{k=1}^{p} \hat{P}_k g_{ki} + \eta_i . \tag{24}$$

If these errors are introduced into (19), letting $\partial R / \partial \hat{P}_k = 0$ then leads, instead of (20), to

$$\sum_{k=1}^{p} M_{\ell k} \hat{P}_k = B_\ell - E_\ell \tag{25}$$

with

$$E_\ell = \sum_{i=1}^{n} \left\{ f_{\ell i} \varepsilon_i + g_{\ell i} \eta_i \right\} . \tag{26}$$

Just as the best-fit parameters are defined by (23), the true parameters are now defined by

$$\hat{P}_k = \sum_{\ell=1}^{p} M^{-1}_{k\ell} (B_\ell - E_\ell) \tag{27}$$

and the error or parameter P_k is

$$\Delta P_k = P_k - \hat{P}_k = \sum_{\ell=1}^{p} M^{-1}_{k\ell} E_\ell . \tag{28}$$

The mean of the product of any two such errors (their correlation) then is

$$\rho_{kr} = \overline{\Delta P_k \Delta P_r} = \sum_{\ell=1}^{p} \sum_{s=1}^{p} M^{-1}_{k\ell} \, M^{-1}_{rs} \, \overline{E_\ell E_s} \qquad (29)$$

where

$$E_\ell E_s = \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ f_{\ell i} \, \varepsilon_i + g_{\ell i} \, \eta_i \right\} \left\{ f_{sj} \, \varepsilon_j + g_{sj} \, \eta_j \right\} , \qquad (30)$$

We now assume that the errors are all uncorrelated

$$\overline{\varepsilon_i \, \varepsilon_j} = \overline{\eta_i \, \eta_j} = 0 \qquad \text{for} \quad i \neq j,$$

and (31)

$$\overline{\varepsilon_i \, \eta_j} = 0 \qquad \text{for any } i, j.$$

We further assume equal rms values, ε_o and η_o,

$$\overline{\varepsilon_i^{\,2}} = \varepsilon_o^{\,2} = \overline{\eta_i^{\,2}} = \eta_o^{\,2} . \qquad (32)$$

Then, from (30) and (21),

$$\overline{E_\ell E_s} = (\varepsilon_o^{\,2} + \eta_o^{\,2}) \, M_{\ell s} = 2 \, \varepsilon_o^{\,2} \, M_{\ell s} = 2\varepsilon_o^{\,2} M_{s\ell} \qquad (33)$$

and from (29) and (33)

$$\rho_{kr} = 2 \, \varepsilon_o^{\,2} \, M^{-1}_{kr} \, . \qquad (34)$$

It remains to be shown that the (sum squares) residual R, when using the best-fit parameters P_k from (23), is given by

$$R = (n-p)\ (\varepsilon_o{}^2 + \eta_o{}^2) = 2\ (n-p)\ \varepsilon_o{}^2, \tag{35}$$

but sine this is generally known we skip its lengthy derivation. The average of the product of any two errors then is

$$\rho_{kr} = \frac{R\ M^{-1}{}_{kr}}{n - p} \tag{36}$$

and the wanted mean error of each parameter is $\sigma_k = \sqrt{\rho_{kk}}$, or

$$\sigma_k = \left(\frac{R\ M^{-1}{}_{kk}}{n - p}\right)^{\frac{1}{2}} . \tag{37}$$

This is a nice result, since these errors σ are easily obtained from quantities already available in the computer.

Finally, the normalized error correlation is

$$c_{kr} = \frac{\rho_{kr}}{\sigma_k\ \sigma_r} , \tag{38}$$

normalized to

$$-1 \leq c_{kr} \leq +1.$$

This correlation matrix should also be printed out. It tells us which correlations are largest, and by looking up the corresponding angular terms, we may be able in future determinations to decorrelate the parameters by putting more emphasis on certain parts of the sky, and omitting other parts.

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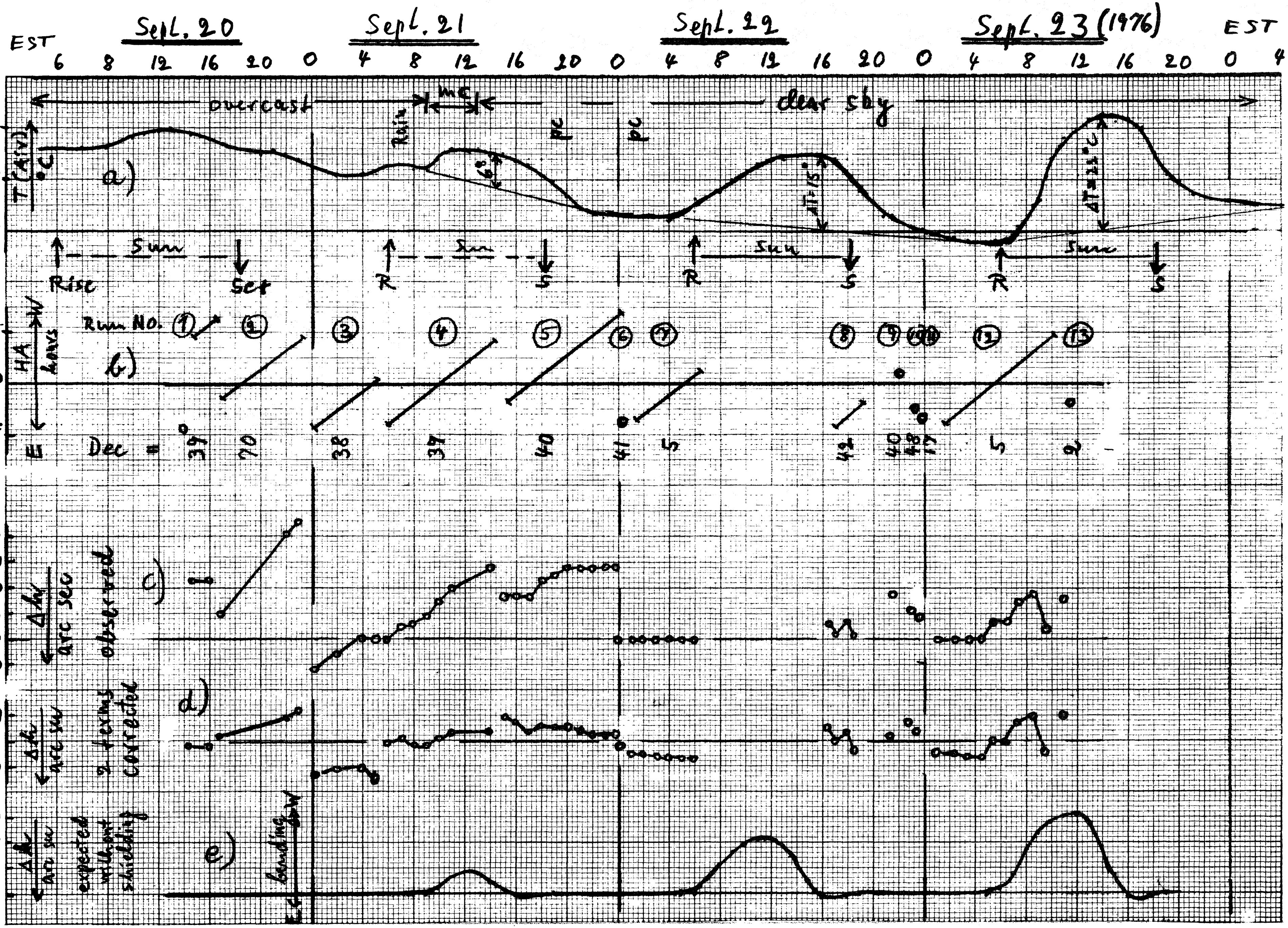

Fig. 1. Hour angle error ΔH. a) Weather: sky, air temp., sun; b) Run number, hour angle, declination; c) Observed ΔH, standard pointing terms; d) Two terms changed; e) Before thermal shielding, same weather. ($\Delta h = \cos D \, \Delta H$)

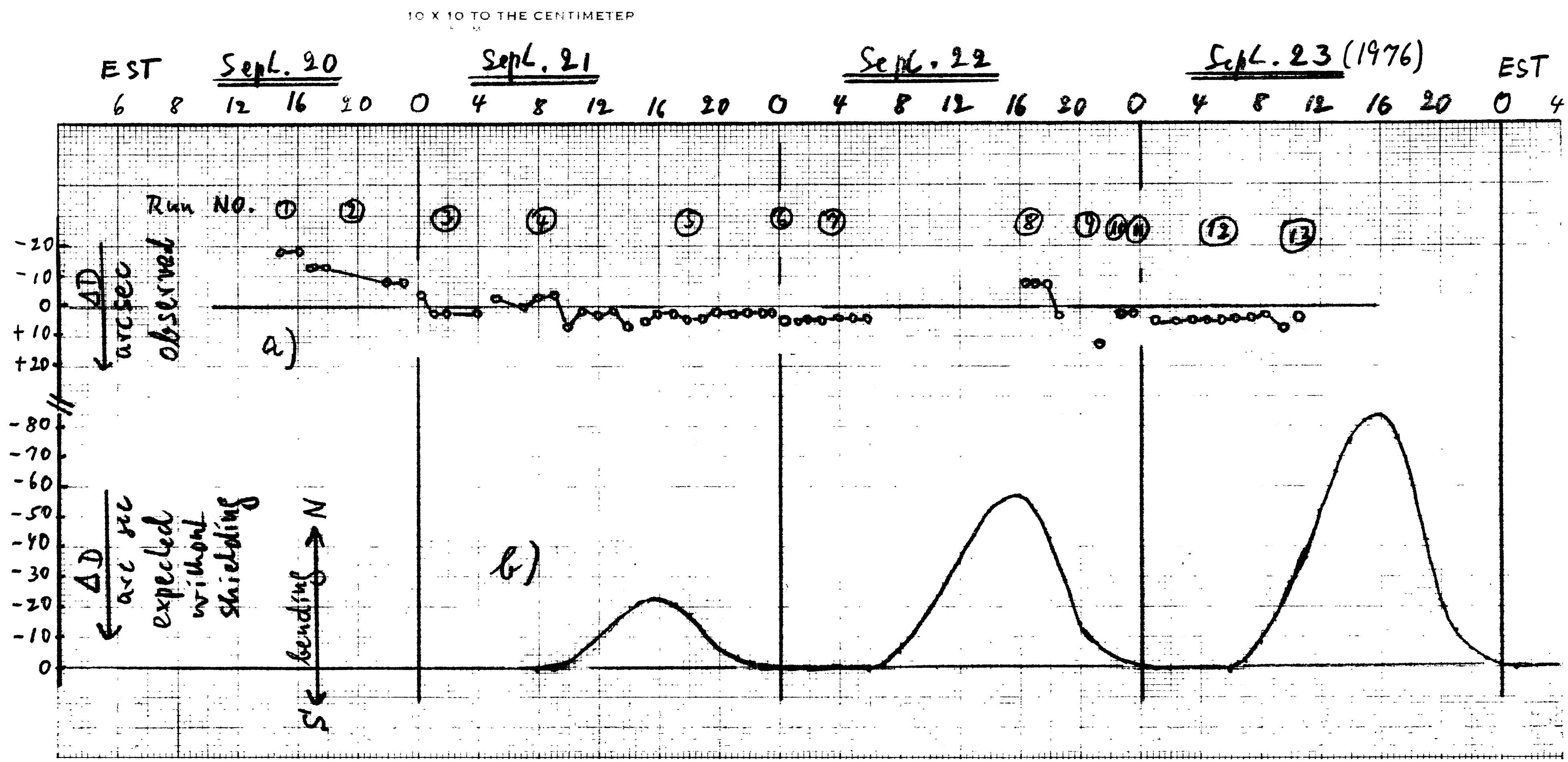

Fig. 2. Declination error, ΔD.

a) Observed ΔD, standard poining terms.

b) Before thermal shielding, for same weather.