NATIONAL RADIO ASTRONOMY OBSERVATORY
GREEN BANK, WEST VIRGINIA

ENGINEERING DIVISION INTERNAL REPORT No. 111

ERROR CONTRIBUTIONS FROM TRIANGULAR AND RECTANGULAR SURFACE PLATES OF REFLECTING TELESCOPES

SEBASTIAN VON HOERNER

APRIL 1980

NUMBER OF COPIES: 100

Summary

Simple approximations are presented to be used for the design, resurfacing or adjusting of radio telescopes. First, equations are given for the maximum value Δz_m (at plate center) of thermal and gravitational deformations, for surface plates of different type and shape. Second, it is shown that for both deformations the contribution to the telescope's error budget is rms$(\Delta z-\overline{\Delta z})$, which is only 1/4 to 1/3 of Δz_m in most cases, depending on type and shape of the plates. The approximations are checked with measurements of experimental plates, and some safety margins are suggested. Third, surface plates should be manufactured with some "gravitational offset" in horizontal position, having their desired shape at about 41° elevation, which reduces the error contribution by about a factor of three without additional manufacturing costs. Fourth, the rms(Δz) resulting from corner adjustment errors is calculated for triangular and rectangular plates; and the advantage of four versus three degrees of freedom in best-fit adjustments is discussed.

For equal thermal error contribution, triangles are better than rectangles if the simplicity of the backup structure is most cost-important. But if economy is more important in manufacturing, supporting and adjusting the plates, then rectangles are better than triangles for the shapes usually used. Finally, if adjustment errors and manufacturing tolerances of the plates are major contributions to the error budget, then the rectangular plates are of advantage for all shapes and sizes.

I. INTRODUCTION

Knowledge about various properties of surface plates (panels) is desired in three cases. First, during the design phase of a new telescope, the design of the backup structure is affected by the plate size which is limited by specified thermal and gravitational deformations, and also the plate type must be chosen (trapezoidal, triangular). Second, resurfacing an existing telescope leaves the choice of the new plate thickness which governs the deformations. Third, for readjusting an existing (or adjusting a new) surface, a least-squares procedure is needed for finding the best-fit adjustment height for the plate corners, because simply adjusting the corners to the design paraboloid would be a poor choice for plates with internal bulges and bumps.

For the final design, the plate deformations must be calculated with a detailed structural analysis (finite element method) and must finally be checked by actual measurements. But for developing a conceptual design, and during the approach to the final design, one needs simple approximations depending only on a few variable parameters. The present paper intends to present a set of simple equations which are accurate enough for this purpose. They are checked with experimental measurements or a detailed analysis.

It is also important to find out which fraction of the maximum deformation is to be used as the contribution to the total error budget of the telescope, and how to minimize this contribution. Furthermore, one should know the contribution resulting from corner adjustment errors, for plates of different type.

The "Bending of a 4-Cornered Plate" was treated as a first approach (25-Meter Memo 128, Oct. 1979), was then worked out in detail and verified by actual measurements of twisted plates. These results, and a least-squares procedure for the best-fit adjustment, will be published soon and are only briefly mentioned here. We could not find a description of corner-supported twisted rectangular plates in the standard engineering textbooks [Ref. 1, 2, 3] and thus had to develop our own.

II. THERMAL AND GRAVITATIONAL DEFORMATIONS

A. Center Deformations

Figure 1 shows several simple models which may be used as approximations for surface plates, and we ask for the amount Δz_m of the deformation at the model's center or where the deformation is maximum. We omit all lengthy derivations and just give the results, with only a few explanations. The results are contained in equations (1) through (14), and for an easy comparison with each other they are all combined in Table 1.

Regarding thermal deformations we assume for the truss a constant temperature along each member. For all other models, we assume a difference ΔT between extreme fibers and a linear gradient in-between; this gives a deformation of constant curvature, deforming the straight line of models 1b and 1c into a small part of a large circle, and the plane of models 1d and 1e into a part of a sphere. The maximum deformation Δz_m then occurs at a point P which is at equal distances from all corners (if $b \leq 2a$ for triangles), at the center of the circumscribed circle of radius R, where

$$R = \frac{a}{2}\left[\left\{1+\left(\frac{b_2+b_1}{2a}\right)^2\right\}\left\{1+\left(\frac{b_2-b_1}{2a}\right)^2\right\}\right]^{1/2} \quad \text{for trapezoid,} \qquad (15a)$$

$$R = \frac{a}{2}\left\{1+\left(\frac{b}{2a}\right)^2\right\} \quad \text{for triangle.} \qquad (15b)$$

TABLE 1

EQUATIONS FOR THE DEFORMATIONS OF THE MODELS OF FIGURE 1.

Δz_m = maximum deformation, at center; C_{th} = coefficient of thermal expansion; ΔT = temperature difference between upper and lower fiber; ρ = density; E = modulus of elasticity; $s = A_{skin}/A_{total}$ (A = area of cross section); R = radius of circumscribed circle, equation (15).

Model	Thermal deformation	Equ.	Gravitational deformation	Equ.	Notes
truss	$\Delta z_m = \frac{1}{4} C_{th} \Delta T \frac{a^2}{t}$	(1)	$\Delta z_m = \frac{1}{8} \frac{\rho}{E} \frac{a^4}{t^2}$	(9)	$\frac{1}{8} = 0.1250$
beam, or honeycomb	$\Delta z_m = \frac{1}{8} C_{th} \Delta T \frac{a^2}{t}$	(2)	$\Delta z_m = \frac{5}{32} \frac{\rho}{E} \frac{a^4}{t^2}$	(10)	$\frac{5}{32} = 0.1562$
skin + rib	$\Delta z_m = \frac{1}{8} C_{th} \Delta T \frac{a^2}{t}$	(3)	$\Delta z_m = \frac{5}{32} \frac{\rho}{E} \frac{a^4}{t^2} \frac{1}{1+2s-3s^2}$	(11)	min for s=1/3
			$\min(\Delta z_m) = \frac{15}{128} \frac{\rho}{E} \frac{a^4}{t^2}$	(12)	$\frac{15}{128} = 0.1172$
trapezoidal plate	$\Delta z_m = \frac{1}{8} C_{th} \Delta T \frac{(2R)^2}{t}$	(4)			
rectangle	$\Delta z_m = \frac{1}{8} C_{th} \Delta T \frac{a^2+b^2}{t}$	(5)	$\Delta z_m = 0.156 \frac{\rho}{E} \frac{a^4+b^4}{t^2}$	(13)	correct for a>>b, 8% high for a=b
square	$\Delta z_m = \frac{1}{4} C_{th} \Delta T \frac{a^2}{t}$	(6)	$\Delta z_m = 0.289 \frac{\rho}{E} \frac{a^4}{t^2}$	(14)	
triangular plate	$\Delta z_m = \frac{1}{8} C_{th} \Delta T \frac{(2R)^2}{t}$	(7)			not available
equilateral	$\Delta z_m = \frac{2}{9} C_{th} \Delta T \frac{a^2}{t}$	(8)			

Equations (1) through (8) of Table 1 give rather similar deformations Δz_m for the different models of Figure 1 (for equal ΔT); for example, a triangle with a = b is 22% better than a square of same length but covers only half the area. The largest difference occurs for the plate construction: structures similar to the truss give twice the deformation of a beam, honeycomb, or skin-rib system, again for equal ΔT.

The temperature difference ΔT must be determined experimentally for a given case. It will depend on the environment: night or sun, exposed or radome. It also depends on the type of construction: it will be smallest for skin-and-rib because of good thermal conduction between extreme fibers. Finally, especially for skin-and-rib, ΔT increases somewhat with the thickness, t. The difference ΔT has two different causes: radiation and thermal lag. From experiments done for our telescope designs at NRAO, we found for exposed telescopes and a truss-like construction $\Delta T = 6.7$ °C for full sunshine at noon, and 1.1 °C during clear nights [4]. For telescopes inside a ventilated dome and skin-rib constructions (t = 7.5 cm), we found $\Delta T = 0.29$ °C for sunny days and 0.19 °C during clear nights [5]. These values may be considered fairly representative. For aluminum, $C_{th} = 23 \times 10^{-6}/°C$, and $12 \times 10^{-6}/°C$ for steel.

Regarding gravitational deformations of surface plates under their own weight, equations (10) and (14) are taken from reference [1], and equation (13) is just a simple interpolation between them. The other equations result from our own derivations. We could not find any information about corner-supported triangular plates. The governing ratio $\rho/E = 38 \times 10^{-10}$ cm^{-1} is the same for both aluminum and steel. It is interesting to see that the truss is gravitationally slightly better than the beam, while it deforms twice as much thermally. The <u>smallest</u> gravitational deformation is obtained for a skin-rib construction if the skin contributes 1/3 of the total weight.

Detailed measurements are available for the two experimental trapezoidal surface plates which are described in [4] and [5], and a structural analysis (498 elements) for the latter. Regarding thermal and gravitational deformations, the difference between expected and measured values of Δz_m is 37% rms for the estimates of Table 1, and 23% rms for the analysis.

B. Averages

The telescope performance is not determined by the maximum deformation, Δz_m, but by the average and the rms of Δz over the whole plate. Thus, the shape of the deformation, $\Delta z(x,y)$, must be known or approximated. The only gravitational case available in the literature is the beam [1]; if we normalize one support to be at $x = -1$ and the other at $x = +1$, we derive for the beam: $\Delta z_b(x)/\Delta z_m = 1 - \frac{6}{5}x^2 + \frac{1}{5}x^4$. The simplest approximation is a parabola: $\Delta z_p(x)/\Delta z_m = 1 - x^2$, and for the difference between the gravitationally deformed beam and the parabola we obtain

$$\begin{aligned} \overline{\Delta z_p} - \overline{\Delta z_b} &= 2.7\% \text{ of } \Delta z_m; \\ \mathrm{rms}(\Delta z_p) - \mathrm{rms}(\Delta z_b) &= 2.1\% \text{ of } \Delta z_m. \end{aligned} \tag{16}$$

We consider this small enough to be negligible, and we assume that the same will hold for two dimensions, where we shall use $\Delta z(x,y) = \Delta z_m[1 - (x^2+y^2)/R^2]$ with $x = y = 0$ at P. This is also used for thermal deformations. They have actually a spherical shape, and it can be shown that the difference between this sphere and a paraboloid is completely negligible for all practical purposes, being of the order of $(\Delta z_m/a)^2\Delta z_m$. Under these assumptions, the values given in Tables 2 and 3 have been calculated. The last column of Table 2 will be discussed later in Section IV.

TABLE 2

MEAN AND RMS SURFACE DEFORMATIONS

Δz_m = maximum (center) value of thermal or gravitational deformation;

ε = rms corner adjustment error.

	$\overline{\Delta z} / \Delta z_m$	$\mathrm{rms}(\Delta z) / \Delta z_m$	$\mathrm{rms}(\Delta z - \overline{\Delta z}) / \Delta z_m$	$\mathrm{rms}(\Delta z) / \varepsilon$
beam	$\frac{2}{3} = 0.6667$	$\sqrt{\frac{8}{15}} = 0.7303$	$\frac{2}{3\sqrt{5}} = 0.2981$	$\sqrt{\frac{2}{3}} = 0.8165$
rectangular plate	$\frac{2}{3}$	$\sqrt{\frac{8}{15}}\, K_{r1}$	$\frac{2}{3\sqrt{5}}\, K_{r2}$	$\frac{2}{3} = 0.6667$
triangular plate	$\frac{2}{3}\, K_{to}$	$\sqrt{\frac{8}{15}}\, K_{t1}$	$\frac{2}{3\sqrt{5}}\, K_{t2}$	$\frac{1}{\sqrt{2}} = 0.7071$

TABLE 3

THE COEFFICIENTS OF TABLE 2, FOR VARIOUS SURFACE PLATES

(The equilateral triangle has $b/a = 2/\sqrt{3}$)

Shape	Rectangles		Triangles		
b/a	K_{r1}	K_{r2}	K_{to}	K_{t1}	K_{t2}
0.0	1.000	1.000	1.000	1.000	1.000
0.2	0.994	0.962	1.010	1.003	0.971
.4	.980	.873	1.036	1.013	.894
.5	.973	.825	1.052	1.020	.845
.6	.967	.781	1.069	1.028	.795
.7	.963	.747	1.085	1.037	.749
.8	.960	.724	1.100	1.045	.709
.9	.958	.711	1.112	1.052	.679
1.0	.957	.707	1.120	1.057	.660
$2/\sqrt{3} = 1.155$	.958	.714	1.125	1.061	.650
1.5	.964	.758	1.101	1.042	.676
2.0	.973	.825	1.000	0.957	.707

Thermal and gravitational deformations were measured at 60 points each of the two experimental plates mentioned. The difference between expected and measured values of $\overline{\Delta z}/\Delta z_m$, $\mathrm{rms}(\Delta z)/\Delta z_m$, and $\mathrm{rms}(\Delta z-\overline{\Delta z})/\Delta z_m$ is 17% rms for the estimates from Tables 2 and 3, and 7% for the structural analysis.

III. CONTRIBUTIONS TO ERROR BUDGET

If the Δz_m were the same for <u>all</u> surface plates, then also the $\overline{\Delta z}$ would be the same for all plates. This, regarded by itself, was just a parallel shift of the average surface, which, as a homologous deformation, does not affect the performance. What matters, then, is the deviation from this average, $\mathrm{rms}(\Delta z-\overline{\Delta z})$, as given in Table 2. Furthermore, if the deformations were not the same but would follow a parabolic shape, $\Delta z(r)/\Delta z(o) = 1 - Ar^2$, with some constant A and axial distance r, then the addition of this parabolic term to the original telescope paraboloid yields still only a homologous deformation, and again it is $\mathrm{rms}(\Delta z-\overline{\Delta z})$ which is to be used for the error budget. Obviously, $\Delta z(r)$ will not be constant; but how close to a parabolic shape will it be?

A. Gravitational Deformations

Regarding gravitational deformations, the worst case for surface plates is gravity in z-direction (axial). Calling α the surface slope, with $\tan\alpha = dz/dr = r/2F$, the gravitational deformations are proportional to $\cos\alpha$. The exact value and its first Taylor approximation are

$$\Delta z(r)/\Delta z(o) = \cos\alpha = \{1 + (r/2F)^2\}^{-1/2} \approx 1 - \tfrac{1}{2}(r/2F)^2. \qquad (17)$$

Neglecting illumination taper, the worst case is at the rim, $r = D/2$. Calling the focal ratio $F/D = \Phi$, we have at the rim with gravity in z-direction:

$$\Delta z(\mathrm{rim})/\Delta z(o) = \{1 + (4\Phi)^{-2}\}^{-1/2} \approx 1 - 1/(32\,\Phi^2). \qquad (18)$$

For example, for 60° illumination angle, $\Phi = 0.43$, the exact value and its parabolic approximation are

$$\Delta z(\text{rim})/\Delta z(o) \;=\; 0.865 \;\approx\; 0.831. \tag{19}$$

The difference between the two is only 3.4% of the central value, and less than half of that if averaged over the whole surface with some illumination taper. We call this negligible. The approximation of (17) then is satisfactory, which means that the deformation caused by $\overline{\Delta z}$ is parabolic and homologous, and only the deviations from it matter. The gravitational contribution to the error budget thus is rms$(\Delta z - \overline{\Delta z})$ from Table 2, with the coefficients from Table 3.

B. Error Budget

Which direction of gravity should be used for the error budget? The central deformations Δz_m from Table 1 assume gravity perpendicular to the surface. If nothing else is specified, the plates are manufactured to the desired parabolic shape in horizontal position, thus Δz_m of Table 1 is their central deformation in vertical position, which means when the telescope would point at the horizon. At other elevations E, the central deformation is only $\Delta z_E = (1 - \sin E)\Delta z_m$. Since observations at <u>short</u> wavelengths are limited by the Earth's atmosphere, they are seldom done below elevations of 30° and almost never below 20°. Considering these lower limits as the worst case, the central deformations from Table 1 are then reduced by the factor

$$\Delta z_E/\Delta z_m \;=\; 1 - \sin E \;=\; \begin{array}{ll} 0.500, & \text{at } E = 30° \\ 0.577 & 25° \\ 0.658 & 20° \end{array} \tag{17}$$

The optimum procedure, however, would be to order from the manufacturer the plates with a certain "gravitational offset", such that they have the desired parabolic shape at some specified angle E_o from the horizontal. The shape to be manufactured in horizontal position can easily be calculated; it is only slightly different from a paraboloid but not any more difficult to produce. This, actually, is just the same as adjusting the whole telescope surface not for zenith pointing, as it was mostly done in the past, but for an adjustment angle of about 45°, as it is mostly done or planned nowadays. If a plate is shaped for an elevation E_o, the central deviation from the telescope paraboloid then is for any other elevation E, with Δz_m from Table 1,

$$\Delta z_E = (\sin E_o - \sin E)\ \Delta z_m. \tag{18}$$

Let us demand that the deteriorations are equal for zenith (E = 90°), and for a lower atmospheric limit of E = 20°, as the worst cases. The adjustment angle E_o then follows from (18) as E_o = 42.1°, and the reduction is quite considerable, being for the worst cases

$$\Delta z_{20} = \Delta z_{90} = 0.330\ \Delta z_m. \tag{19}$$

As an example, we consider a rectangular plate with b/a = 1/2. From Table 2 we have $\mathrm{rms}(\Delta z-\overline{\Delta z})/\Delta z_m = 0.298\ K_{r2}$, and from Table 3 we find K_{r2} = 0.825, with 0.298 x 0.825 = 0.246. If the plate was manufactured for E_o = 42°, we have from (19) a reduction factor of 0.330, with 0.246 x 0.330 = 0.081. At the worst pointings, at zenith and at 20° elevation, we thus have for the error budget

$$\text{gravitational contribution} = 0.330\ \mathrm{rms}(\Delta z-\overline{\Delta z}) = 0.081\ \Delta z_m. \tag{20}$$

For a practical application, one must add some safety margins. We suggest to add to (20) about 20% if $\mathrm{rms}(\Delta z-\overline{\Delta z})$ was measured (in horizontal position), or to add 30% if only Δz_m was measured, or to add 50% if Δz_m was estimated from Table 1.

C. Thermal Contribution

Regarding thermal deformations, we consider two extreme cases. First, sunshine or some other radiation in z-direction, parallel to the axis. Its intensity as projected onto the plates then is proportional to cos α, and so is the deformation. Thus all conclusions are the same as they were for the gravitational deformations, and it is again rms$(\Delta z-\overline{\Delta z})$ which is to be used. Second, one might think that the worst case could be a radiation coming at such an angle that half of the surface lies in its own shadow. This lack of over-all symmetry indeed increases the relative rms, but the Δz_m of the illuminated part is so much reduced by the skew illumination angle that the total deterioration of the whole telescope is about 30% smaller than in the first case of axial radiation, which thus is the worst one.

Since radiation may work both ways (sky being warm in daytime and cold at night), we cannot manufacture "thermally offset" plates, and there is no equivalent to (19). For our example of a rectangular plate with b/a = 1/2, we thus have from axial radiation as the worst case for the error budget

$$\text{thermal contribution} \;=\; \text{rms}(\Delta z-\overline{\Delta z}) \;=\; 0.246\ \Delta z_m, \tag{21}$$

and we suggest to add the same safety margins as described for the gravitational case.

IV. CORNER ADJUSTMENT ERROR

Adjusting or readjusting a telescope surface will be done with errors from two causes: from the surface measurement errors, and from the actual turning and setting of the adjustment nuts on their bolts. For the total of both, we call ε_o the rms adjustment error of a single corner of a plate, assuming all corner errors to be uncorrelated. Regarding the whole plate, a single corner error ε_o will introduce a deviation $\Delta z(x,y)$, and if this shape is known we can integrate Δz^2 and obtain rms(Δz). If there are n corners, then the total rms deviation ε resulting from n independent errors is

$$\varepsilon = \text{rms}(\Delta z)\sqrt{n}. \tag{22}$$

For comparison we start with the simplest case, a beam (n = 2) of length b. Here, $\Delta z(x) = (x/b)\,\varepsilon_o$, yielding rms($\Delta z$) = $\varepsilon_o/\sqrt{3}$, and multiplied by $\sqrt{n}$ we have $\varepsilon/\varepsilon_o = \sqrt{2/3}$ as entered in the last column of Table 2.

For a triangle (n = 3) with the two corners of its base at $\Delta z = 0$ on the x-axis, and the third corner at y = a, lifted by $\Delta z = \varepsilon_o$, we have $\Delta z(x,y) = (y/a)\,\varepsilon_o$, with rms($\Delta z$) = $\varepsilon_o/\sqrt{6}$ as can be shown. Multiplication by $\sqrt{n}$ then gives

$$\varepsilon/\varepsilon_o = 1/\sqrt{2}, \text{ for triangles.} \tag{23}$$

Next, we consider a rectangle (n = 4), two sides coinciding with the coordinate axes and their three corners at $\Delta z = 0$ at coordinates (0, 0), (0, a) and (b, 0); the fourth corner (b, a) at height $\Delta z = \varepsilon_o$. The simplest possible twisted shape, which turned out to be already quite satisfactory, is

$$\Delta z(x,y) = (x/b)\,(y/a)\,\varepsilon_o;$$

an integration of Δz^2 yields rms(Δz) = $\varepsilon_o/3$, and multiplied by $\sqrt{n}$ we obtain

$$\varepsilon/\varepsilon_o = 2/3, \text{ for rectangles.} \tag{25}$$

Finally, one should avoid cantilevering plates. The effect of adjustment errors is smallest for corner adjustments, and for most telescopes the adjustment errors are an important contribution to the total error budget (difficulty of good surface measurements). For example, consider an equilateral triangular plate. If it is supported and adjusted at its three corners, then $\varepsilon/\varepsilon_o = 1/\sqrt{2}$ from (23). But if the same plate is supported and adjusted at its three side centers, then $\varepsilon/\varepsilon_o = 1$, as can be shown. This means that the rms surface error has increased by a factor of $\sqrt{2}$, or by 41%. And this result is completely independent of the size or the shape (b/a) of the plates.

V. TRIANGULAR VERSUS RECTANGULAR PLATES

The type of surface plate most frequently used on telescopes is the trapezoidal plate in a polar grid, supported at its four corners. The simplest alternative is the triangular plate, again corner supported. The following will give a comparison between the two. The trapezoid is replaced by a rectangle for simplicity, but we consider both triangles and rectangles of various shape ratios, b/a, see Fig. 1.

Regarding thermal deformations, for each of the two plate types we ask for that size, a, which yields the same thermal error contribution, $\mathrm{rms}(\Delta z-\overline{\Delta z})$, assuming the same temperature difference ΔT, the same slenderness ratio a/t, and the same construction (using the solid plate equations in the following). The shape ratio b/a is considered a variable parameter, but the same for both types, triangle and rectangle.

We call C the combination of all constants,

$$C = \frac{1}{8} C_{th} \Delta T \frac{a}{t} \frac{2}{3\sqrt{5}} \tag{26}$$

and we call, for rectangle and triangles,

$$\left.\begin{aligned} H_r &= [1 + (b/a)^2]\, K_{r2} \\ H_t &= [1 + (b/2a)^2] K_{t2} \end{aligned}\right\} \quad Q = H_t/H_r. \tag{27}$$

The thermal error contribution then is, for a plate of size a,

$$\mathrm{rms}(\Delta z-\overline{\Delta z}) = C\, a\, H,$$

and for a given error contribution, the size then is

$$a \sim 1/H. \qquad (29)$$

Various comparison criteria are summarized in Table 4, where a number smaller than one means that the triangle is of advantage, while the rectangle is better if the number is larger than one. Assuming a polar grid in all cases, the number of rings needed in the backup structure (for supporting the plates) equals the telescope radius divided by the plate size; thus the number of such rings needed for triangular plates equals Q times the number of rectangular plates.

TABLE 4

COMPARISON BETWEEN TRIANGLES AND RECTANGLES

All figures given are the ratio: (triangular value)/(rectangular value). Figures larger than one mean that the rectangle is of advantage.

Shape	Number of				rms surface error resulting from	
b/a	Rings Q	Molds 2Q	Plates $2Q^2$	Adjustments $(3/2)\,Q^2$	adjustments	bumpiness
0.0	1.000	2.000	2.000	1.500	↑	↑
0.2	.980	1.960	1.922	1.441		
.4	.918	1.836	1.686	1.264		
.5	.871	1.741	1.516	1.137	1.061	1.038
.6	.816	1.632	1.331	.998		
.8	.693	1.385	.960	.720		
1.0	.584	1.167	.681	.511	↓	↓

We see from Table 4 that triangles are advantageous regarding the backup structure for all $b/a > 0$. But if identical plates are manufactured using the same mold, the cost increases with the number of molds needed, and rectangular plates are cheaper, needing fewer molds, because in each ring there is only one kind of rectangular plate but two kinds of triangles. Because the area of a rectangle is ab, that of a triangle only ab/2, the total number of triangular plates is $2Q^2$ times the number of rectangular plates, which means we need fewer rectangular plates as long as $b/a < 0.8$. Since a triangle needs three corners to be adjusted while a rectangle needs four, the ratio triangle/rectangle is 3/4 times the number ratio, or $(3/4)\ 2Q^2 = (3/2)\ Q^2$, where rectangles are better as long as $b/a < 0.6$.

In summary, for equal thermal error contributions, triangles are better if the simplicity of the backup structure is most important regarding costs. But if cost-saving is more important in manufacturing, supporting and adjusting the plates, then the rectangles are better, at least for the shape ratios mostly used.

Thermal errors are not the only ones. Unfortunately, we cannot make a similar comparison for the gravitational deformations because they are not known for corner-supported triangular plates, but we expect similar results. The two remaining error contributions can be compared. Regarding the rms(Δz) resulting from adjustment errors, we find from (23) and (25) that triangles are worse by a factor $(1/\sqrt{2})/(2/3) = 1.0607$ for all b/a, as entered in Table 4. Finally, regarding the internal bumpiness of the plates, the triangle has three degrees of freedom for a best-fit adjustment (a lift and two rotations), whereas the rectangle has four (internal twist in addition). Measurements with several experimental surface plates at NRAO gave the result that using the best-fit twist improved the surface accuracy by 3.8% in the average (which will depend on the type of bumpiness but not on b/a). This is also entered in Table 4. Thus, if adjustment error and manufacturing tolerance of the surface plates are major contributions to the error budget, as it is frequently the case, then the rectangular plates are of advantage for all shapes and sizes.

REFERENCES

[1] Raymond J. Roark, "Formulas for Stress and Strain"; McGraw-Hill, 1965.

[2] Timoshenko and Woinowsky-Krieger, "Theory of Plates and Shells"; McGraw-Hill, 1959.

[3] "Manual of Steel Construction"; American Institute of Steel Construction, 1965.

[4] J. W. Findlay and S. von Hoerner, "A 65-Meter Telescope for Millimeter Wavelengths", National Radio Astronomy Observatory, 1972.

[5] S. von Hoerner, "Thermal Measurements of Panel in Radome-Tent", 25-m Telescope Memo 86, NRAO, 1977.

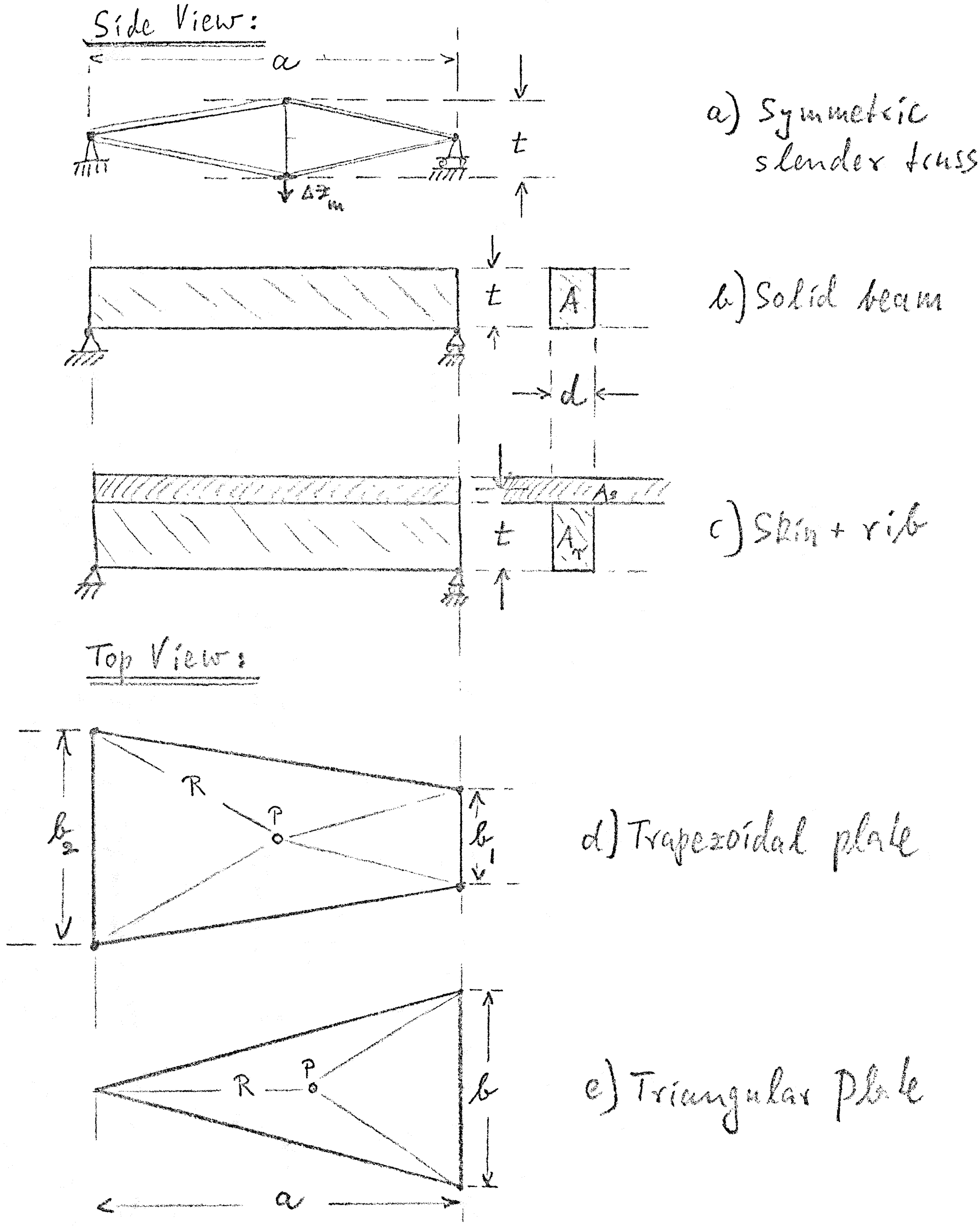

Fig. 1. Simple models for surface plates.

t = thickness

A = area of cross section

Δz_m = maximum (center) deformation