NATIONAL RADIO ASTRONOMY OBSERVATORY
GREEN BANK, WEST VIRGINIA

ENGINEERING DIVISION INTERNAL REPORT No. 115

ARECIBO THREE-MIRROR SYSTEMS, III:
REVISED FORCE ESTIMATES FOR SELECTED SYSTEMS

SEBASTIAN VON HOERNER

MAY 1983

25 COPIES

I. INTRODUCTION AND SUMMARY

Engineering Report 112 presented 11 optimized mirror systems; Report 113 added four systems and gave rough estimates for weight and wind forces. Meanwhile, Mike Davis (letter of March 24) suggested to add a system with still larger aperture, and emphasized that survival winds must be taken for the worst direction (not a selected stow position) in case of power failure in veering winds. Bill McGuire (letters of Apr. 6 and 18) mentioned that the replacement of steel by aluminum will not save as much weight as assumed. And at the end of March, I spent a week at Arecibo with many discussions, and attended the meeting about the broken wires of the support cables.

The present report treats four selected systems, each representing a certain type. The amount of spillover area as a function of the vignetting is given in general, and also the resulting gain loss and pickup of ground noise as a function of the illumination taper.

Survival wind forces are estimated for a face-on wind of 80 mph, and observational forces for 10 mph, the third quartile of the wind distribution. The question of aluminum versus steel is discussed in general, and a weight factor of 0.611 is used, under certain assumptions, to obtain the revised weights. Regarding the demands on the platform, the survival wind is mostly two times the weight, and the combination of both forces ranges between 64 and 150 kip (1 kip = 1000 lb).

Additional forces and resulting total stress are calculated for the long support cables holding the platform. Because of the small angle between cable and horizontal, the cable force from weight is about three times larger than that from survival wind, and the combination ranges from 37 to 72 kip per cable, yielding a maximum total stress of 111 ksi (98 at present).

Regarding the structural analysis, several suggestions and time estimates are given. A general treatment of sagging cables, used as structural members, is included.

II. SYSTEM SELECTION

From the 12 systems treated in Report 113, Mike Davis suggested to exclude from further considerations the following:

system #	main reason for exclusion
4, 4a, 9a, 13	feed very far below cabin
3, 8	feed uphill on arm, and arm is short
2, 9	large feed

(1)

This leaves, as acceptable systems,

system #	offset (ft)	aperture (ft)
5, 6, 7	50	700
10, 11	125	700
12	137	726

(2)

For further comparisons, I will keep only one system as a representative in each group, selecting somewhat arbitrarily #5 and #11, because they require probably the simplest support for the tertiary, other things being about equal.

As suggested by Mike Davis, I added one more system, #14, with 75 ft offset and 750 ft aperture, even larger than #12, but with the same amount of vignetting as #5, which might be called a tolerable and cost-effective compromise between the demand for high signal and low ground noise. This system is shown in Fig. 1, and its work sheet in Fig. 2. The four selected systems are described in Table 1.

In general, vignetting (illumination spillover, beyond the telescope rim) has three adverse effects: loss of gain, increase of sidelobes, and pickup of ground radiation. For future low-noise receivers, and if the

Table 1. Parameters and ~~secondary~~ geometry of four selected systems.

All lengths in feet, angles in degrees.

g = offset, s = spillover (vignetting);
d = largest diameter;
z = deepest point (below paraxial focus);
β = angular size of mirror as seen from its focus;
c = clearance for tiedown cables;
I = illumination ratio (to be removed by shaping);
G = Gregorian, C = Cassegrain

system	aperture			parameters			secondary					Tertiary				Feed
#	d_1	g	s	ν	τ	ρ	d_2	z_2	β_2	c	I	Type	d_3	z_3	β_3	d_f
5	700	50	74	8	-19	-11	64.4	-33.8	145	+5	14	G	21	-26	42	2.2
11	700	125	0	8	-23	- 9	88.4	-70.3	146	-3	15	G	30	-28	83	1.1
12	726	137	0	7	-41	-42	105.5	-83	88	-5	26	C	25	-52	40	2.3
14	750	75	74	6	-22	-40	89.1	-59	103	+3	25	C	24	-33	32	2.8

mirrors are shaped with small or no taper for maximum gain, the added noise from the ground will become the most important of the three. It could be avoided by adding a "collar" all around the rim of the telescope (maybe of low-grade wire mesh, with enough support ribs for survival wind). Table 2 shows a few examples. The radial spillover, s, tells how wide the collar would have to be, in order to avoid any pickup of ground radiation. And without such collar, the fraction Q, multiplied by the illumination taper and by 300°K, yields an estimate for the additional noise temperature from the ground. (Furthermore, 2Q times taper is an estimate of the gain loss, with or without collar.) Fraction Q increases approximately with $s^{3/2}$ for small s.

Table 2. Systems with some vignetting.
s = radial spillover beyond rim,
Q = (spillover area)/(full aperture area).

aperture (ft)	700			750		
offset (ft)	50	75	100	50	75	100
s(ft)	74	49	24	99	74	49
Q(%)	7.8	4.6	1.7	10.4	7.4	4.4

Regarding the following treatment of the four selected systems of Table 1, I would like to emphasize that none of these is in any way the final system. They should be considered each representing a certain possible type of system, still allowing changes if wanted. The final evaluation within a type, for example, of #5 versus #6 or 7, should only be done after the shaping procedure, whereas the final selection of the type, regarding aperture and offset, will depend on how much weight and force an improved platform could carry.

III. NEW WIND FORCE ESTIMATES

1. Survival

The estimates of Report 113 need several changes, and the most drastic one concerns the wind force. Regarding survival, I had assumed that a "stow position" could be declared, as is usual for exposed telescopes, where the mirror presents to the wind only its small area of the side view. But Mike Davis pointed out that it is mostly the outer parts of hurricanes causing the strongest winds, which change direction while the hurricane moves by; and we should never rely on the ability to rotate the feed arm during strongest winds and possible power failures. Thus, for survival conditions, we must assume the worst case, meaning the largest projected area, that of the front view.

The new values are shown in Table 3 for the selected systems. The survival forces have increased considerably, by factors between 1.83 for system #5, up to 2.93 for system #12. (But, as mentioned by Mike Davis, the feed arm now projects less area into the wind, which will make the total difference not quite as bad.) In case that these large forces should turn out to be prohibitive for the whole concept, or at least for the larger (more attractive) mirrors, I would like to mention a way out which seems technically possible, though certainly more expensive. Imagine a large secondary consisting of two parts: a fixed-mounted upper half, and a hinge-mounted lower half with its hinge at the uphill end of the carriage, to be rotated uphill by a remote-controlled motor at a storm warning, and rotated back down again when it is over. This would reduce the large area of the front view by roughly a factor of two, and leave the small side view area unchanged. We would have to rely on power (or generator) only before a storm, not any more during it. Not that I like it; but we might keep it in mind as a last resort if ever needed.

2. Observation

Originally it was suggested to specify $v = 17$ mph as the wind velocity up to which the surface and pointing accuracies should stay within their specified tolerances. The observational wind forces of Report 113 were calculated for 17 mph; but I also mentioned that one could relax the specification considerably, because the usual procedure is to take the third quartile of the cumulative wind distribution for this specification (the wind being lower during 3/4 of all time), which at Arecibo is only 9.5 mph.

The forces of Table 3 are now calculated for a round value of $v = 10$ mph. This reduces the forces from Report 113 by a factor 2.89. Since the survival condition now needs considerably more structural improvement than originally anticipated, and if $v = 10$ mph can be agreed upon, then it seems rather certain that wind-induced pointing errors and surface deformations will be completely negligible.

IV. NEW WEIGHT ESTIMATES

1. Aluminum Versus Steel

Accepting Bill McGuire's objection made me rethink the whole problem. Suppose a structure had been designed and optimized with steel, and now we replace it by aluminum in order to save weight; question: by which factor q will the weight be decreased? This depends on the constraining item which defines the minimum sizing of the structural members, and there are three cases. First, if forces are small and deformations not critical, the member is just defined by the maximum permitted slenderness ratio, $K\ell/r = 120$ for main members and 200 for secondary braces, and by the minimum permitted wall thickness, t = 1/30 of the diameter as mostly recommended (and $t \geq 0.1$ inch for welding). In this case the volume stays constant, and the weight changes as the density does:

$$q = \frac{\rho_{al}}{\rho_{st}} = 0.35. \tag{3}$$

Second, if stability against large forces is the active constraint, then also the maximum allowed stress S matters (as a function of $K\ell/r$), because we need just so much more square inches of cross section:

$$q = \frac{\rho_{al}}{\rho_{st}} \frac{S_{st}}{S_{al}} . \tag{4}$$

Assuming type and temper 6061-T6 for the aluminum, and a simple steel with 36 ksi yield, and using for each one the equations for S given in handbooks, I derive:

slenderness, $K\ell/r$	weight-saving, q
$\leq$ 8	0.427
40	0.500
60	0.546
66	0.553
80	0.742
$\geq$ 105	$\geq$ 1.00

(5)

This means that aluminum needs more weight than steel for long members under large forces.

Third, if the active constraint is a specified small deformation under given force, then aluminum is always heavier than steel, for any slenderness, because

$$q = \frac{\rho_{al}}{\rho_{st}} \frac{E_{st}}{E_{al}} = 1.05. \tag{6}$$

I know from several optimized telescope designs that the majority of steel members usually is just defined by the slenderness limit of 120, or is at least rather close to it, and that wind-induced deformations of the backup structure are mostly not important. And since the side view of the secondaries showed only a small surface to the wind, I expected small forces and simply adopted (3). Now, for face-on survival winds, forces are increased up to three times. For the present estimate, before actual designs are done, I will tentatively adopt:

fraction	is defined by	q	
2/3	forces, with $K\ell/r = 80$	0.742	(7)
1/3	slenderness, $K\ell/r = 120$	0.350	average, $q = 0.611$.
0	deformations	1.050	

2. Resulting Weights

The old weight estimates for the secondary mirrors were based on Table 2 of Report 113, with three NRAO designs of 25-m telescopes, using a weight of 12 kip for the aluminum surface, and 100 kip for the steel backup structure. I then applied three changes: (a) The surface was multiplied by 0.8 because for $\lambda = 4$ cm we need less accuracy. Now, I accept Mike Davis' objection, that the additional mirrors ought to be more accurate than the primary, and I leave the 12 kip unchanged. (b) The backup structure was multiplied by 0.65 because we omit now all backup parts below the elevation bearings, and estimating their weight as 0.35 of the total seems to me still being on the safe side, so I will use it again. (c) The aluminum weight saving factor will now be $q = 0.611$ from (7). In total, we have now 52 kip for a circular aluminum secondary of 82 feet diameter.

I think that the estimates for legs and tertiaries, which are minor items anyway, do not need changes. Under these assumptions the weights of Table 3 have been calculated. And the last column is again the total force, where weight and survival wind have been quadratically added, because they are perpendicular to each other.

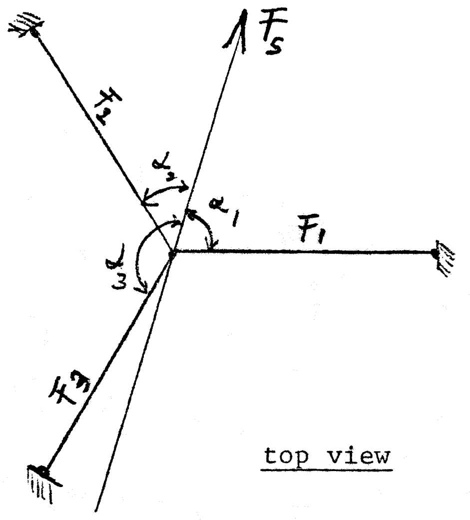
Table 3. Revised estimate for wind force and weight, of secondary, tertiary and legs.

(1 kip = 1000 lb)

System #	Aperture		Wind Force		Weight	Total force
	diameter d_1(ft)	spillover s(ft)	surv. F_s(kip)	observ. F_o(kip)	W(kip)	$F_t=\sqrt{W^2+F_s^2}$
5	700	74	55	.87	32	64
11	700	0	103	1.61	48	114
12	726	0	136	2.12	62	150
14	750	74	94	1.47	48	106

V. STRESS ON SUPPORT CABLES

We calculate the additional force and the resulting total stress for the long support cables, between platform and towers. We consider only the additions from secondary, tertiary and legs, but not yet those from future changes of the platform which, however, can then be treated the same way, after a new design is available.



top view

1. Survival Wind

We have a wind force F_s, in any direction, and three cables 120° apart (actually, three groups of four cables each). First, projected on a horizontal plane (top view), the three forces F_i can be derived as follows, where we assume that any slackening of a cable in case of compressive force direction can be neglected (which was checked to be true):

$$F_i = \frac{\cos \alpha_i}{\cos^2\alpha_1 + \cos^2\alpha_2 + \cos^2\alpha_3} F_s \tag{8}$$

There are two extreme cases, $\alpha_1 = 0$ and $\alpha_1 = 90°$, with

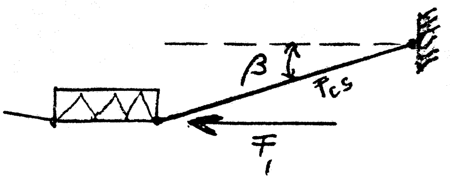
$$\left.\begin{aligned} F_1 &= \frac{2}{3} F_s \\ F_2 &= F_3 = \frac{1}{3} F_s \end{aligned}\right\} \text{for } \alpha_1 = 0; \tag{9}$$

$$\left.\begin{aligned} F_1 &= 0 \\ F_2 = F_3 &= \frac{F_s}{2 \cos 30°} = 0.577\ F_s \end{aligned}\right\} \text{for } \alpha_1 = 90° \tag{10}$$

Thus, the worst case is

$$F_1 = \frac{2}{3} F_s, \text{ for parallel wind.} \tag{11}$$

Second, projected on a vertical plane (side view), we divide by cos β, and to obtain the force in a single cable, we divide by four. With $\beta = 13°$, the additional force from parallel survival wind is, per cable:

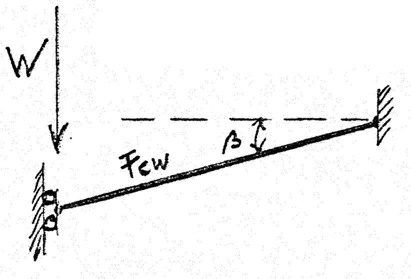


side view

$$F_{cs} = \frac{F_1}{4 \cos \beta} = 0.171\ F_s. \tag{12}$$

Results are shown in Table 4.

2. Weight



side view

In the worst case, the carriage is at the end of the feed arm, and the arm is parallel to one cable group. To be on the safe side, we apply the full weight W of Table 3 on this cable group only, and we neglect a relaxing lateral movement of the platform. Then, in each of the four cables, we have the force:

$$F_{cw} = \frac{W}{4 \sin \beta} = 1.11\ W. \tag{13}$$

3. Together

Force (13) can happen only at the end of the arm, in observing position. We should specify a limiting wind velocity, beyond which the carriage must be brought to the center of the arm. For a suggestion, I will use v = 30 mph. The maximum force on a cable, in observing position at the end of the arm, then will be

$$F_{obs} = F_{cw} + (30/80)^2 F_{cs}\ . \tag{14}$$

For higher winds (up to 80 mph as specified), the carriage will be at the center of arm and platform, and the weight will be equally distributed to all three cable groups; the total then will be

$$F_{surv} = (1/3)\ F_{cw} + F_{cs}. \tag{15}$$

Both these combinations, F_{obs} and F_{surv}, are given in Table 4.

4. Discussion

We should keep in mind that all values of Table 3 and 4 are only rough estimates. We have, for example, neglected: moments, tiedown cables, lateral platform movements, platform tilts.

But still we may conclude from Table 3, that the most crucial addition for platform and feed arm is the survival wind force, which will probably need a new, much wider arm; and its horizontal stiffness must be especially large at its center, where the carriage will be stowed in high winds.

Table 4. Additional force per cable, and total stress, on the support cables from platform to tower, resulting from wind force and weight of Table 3. For comparison, the present force per cable is F = 527 kip, and the present stress is S = 97.6 ksi.

System #	additional forces (kip)				total stress
	max. single force		max. combined force		max. comb.+ present
	surv. wind F_{cs}	weight F_{cw}	arm end F_{obs}	arm center $F_{surv.}$	arm end (ksi) S_{obs}
5	9.4	36	37	21	104
11	17.6	53	56	35	108
12	23.3	69	72	46	111
14	16.1	53	56	34	108

There will be new demands on the platform, too, but not so drastic ones. It carries already the present arm of about 400 kip, and its own weight is about 800 kip. It must now provide for an additional force of at least 63 kip for system #5, or at most 150 kip for #12.

This is different for the long support cables, Table 4, where not the survival wind is the crucial item, but the additional weight at its worst location, at the end of the feed arm when parallel to one cable. In relation to the present stress, the additional stress seems not so large: 6.5% for system #5, and 13.7% for #12. But the problem is that these cables are already highly stressed, with about 98 ksi out of their yield of 220 ksi. And some of their wires have already broken, although maybe at an earlier time, before their present shielding against corrosion was added near the sockets. If 111 ksi from Table 4, half the yield, is acceptable, then even the largest secondary could be used.

Provided, however, that the new feed arm and the strengthening of the platform do not add too much weight. This additional weight W_a will be equally distributed to all three cable groups, and its addition to the cable force will be, per cable,

$$F_{ca} = \frac{W_a}{12 \sin \beta} = 0.370 \, W_a . \tag{16}$$

In case that the total stress (from mirrors and strengthened platform) is too high and dangerous, the question of adding cables or improving their anchoring should be rediscussed. If a new feed arm is designed from aluminum for saving weight, (16) will give a force reduction F_{ca} from a saved weight W_a.

VI. STRUCTURAL ANALYSIS

1. Amount of Work

Page 22 of Report 113 gave a listing of all the input data needed for an analysis of the present structure. The question was raised how much of an effort this would be. I have asked Dr. Lee King, of NRAO, our expert on structural design and analysis. (He designed the astrodome of our 25-m homologous telescope proposal, our present 12-m telescope at Kitt Peak, and works now on the design of a future VLBI telescope.)

I enclose his answer as APPENDIX A. My first questions were concerned with shell stiffness for a different project. My second questions regarded the Arecibo project: as to the preparation of the analysis outlined in Report 113, (a) how long would it take? And (b) which level of expertise is needed? Lee King's answer is: (a) four weeks, and (b) a recently graduated engineer with 1 - 2 years of experience in structural analysis.

He mentions also that a complete job may take 4 - 6 months, if the following were to be included: (i) cable stiffness and reactions as functions of the load, which could more than double the complications; (ii) choosing and defining the right wind loads under various orientations, for maximum forces; (iii) checking and interpretation of the computer results.

2. Wind Loads and Tests

Regarding the wind loads, I would suggest, for the present feasibility and cost estimates, to use a simplified procedure as done on my work sheets, adopting the center of the projected area as the point of attack for obtaining the moments at the carriage. I would suggest to do this for 7 wind angles (face-on, 30°, 60°, ... 180°), and for two carriage locations: center and end of arm. Making 14 computer runs with only two changing input data is only a matter of days (length of waiting line at computer), after the long four-week job of computer-modelling the whole structure has been done once.

Wind direction and resulting force direction will not be the same. I have a large number of older papers, and would probably use: Hirst and McKee, "Wind Forces on Parabolic Antennas", Microwave Journal, Nov. 1965, p. 43-47. It gives useful graphs; but it also shows large differences between various experiments in wind tunnels, which in my opinion are explained by the different backup structures. Which means: for the present state of our project, just use easy approximations; after that, decide on a "semi-final" system type.

At a later state, design a backup structure for it; build a detailed model and go to wind tunnel tests; thereafter, repeat analysis with new wind force data; then do the final sizing, if no problem remains (otherwise, change type). The proceedings of this later state, backup design and wind tunnel tests, will only be needed if and after the present rough estimates have yielded a reliable feasibility, a possible cost, and the selection of a system type.

3. Cable Sag and Stiffness

This question has already been treated in my LFST Report 8, of May 1966, which I enclose here as APPENDIX B. Its equation (7) gives the effective modulus for a long cable, sagging under its own weight, derived for small sag. We write it now as

$$E_s = E \left\{ 1 - \frac{1}{12} \frac{E \rho^2}{S^3} L_p{}^2 \right\}. \tag{17}$$

For our long support cables, we use:

E = 23,500 ksi = material modulus of elasticity, without sag;

E_s = effective modulus, of sagging cable as a structural member;

ρ = 0.294 lb/in^2 = density, including coatings = (lb/ft)/metallic area);

S = 98 ksi = stress, under present loads;

L_p = 576 ft = length of cable as projected on the ground.

With the given material and stress, we have

$$E_s = E \left\{ 1 - (L_p/6214 \text{ ft})^2 \right\} \tag{18}$$

and with the given length, the correction is less than one percent

$$(L_p/6214\ \text{ft})^2 = 0.0086, \tag{19}$$

and finally

$$E_s = 23{,}300\ \text{ksi, under present stress.} \tag{20}$$

This is now to be used in our structural analysis. For a varying stress under varying loads, we use the maximum stress change of ±13.7%, from system #12. Instead of (20), we then obtain

$$E_s = \left\langle \begin{matrix} 23{,}260\ \text{ksi} \\ 23{,}210\ \text{ksi} \end{matrix} \right\} \text{with maximum stress change from \#12.} \tag{21}$$

Fortunately, the difference between (21) and (20) is negligible, thus the inclusion of long sagging cables does <u>not</u> cause any complications in our case.

Other numbers of interest can be derived from APPENDIX B. First, the amount of sag at the middle of the cables, which turns out to be 1.50 ft. Second, the angle of support (reaction orientation) is changed by the sag by 0.60°, and its change from varying loads can again be neglected.

4. Engineering Firms

The question was raised: could the analysis and subsequent improvement, or part of this, be done in-house, with Cornell's own engineering department, or should it be given to a good engineering firm? This is very much a question of general philosophy, and my own is the following. Do in-house, by all means, whatever is new and interesting, whatever requires dedication and inspiration, and whenever a thorough optimization is crucial. Be inventive, optimistic and pushing forward.

But then have the product critically checked by an experienced (neutral) outside firm. Also, leave detailed and more standard-type jobs to firms. Some tasks come in-between, however, and are best done in close cooperation and continuous exchange with a reliable and open-minded engineering consultant. In our own larger engineering efforts, final checks and details were done by Simpson, Gumpertz + Hager in Cambridge, and our consultant was Otto Heine, Systems Development, now in San Diego. We had good experience with both, but this was some years ago.

As to the method of analysis, I would recommend using the STRUDL program because of its very flexible and convenient input arrangement. Before these ready-made software programs became available, I had made in 1965 a general structural analysis program of my own, and used it for several optimization procedures, so I know how much work this requires. I would like to quote our most recent experience with STRUDL: the static analysis of a telescope design, calculating 1/4 structure with 160 joints, 660 members and 20 plates, using an IBM 4341; this took only 6 minutes of CPU-time, 14 minutes total, for one external load condition. Additional load conditions use the same inverted matrix again, and the suggested six load conditions (next section) would take only 9 minutes CPU-time. The dynamic analysis, for the first five lowest modes, took 40 minutes CPU-time, where the total number of 960 degrees of freedom (160 joints, six degrees each) was reduced to about 200 essential degrees.

5. Procedure

At present, the critical question is static stability only; deformations, pointing and dynamics to be investigated at a later state, if and when a stable structure is developed. To start with, analyze three cases, before any re-design:

First, a static analysis of the *present* structure, with the present weight of carriage and feed, but using 80 mph survival wind, instead of 140 mph as it was used by Von Seb in December 1960.

Second, the present structure again, but with weight and wind forces of *system #11* as a useful medium-sized example.

Third, as a last try before designing a new feed arm (which most probably will be necessary as pointed out by Bill McGuire), add 12 *lateral braces* and 6 horizontal members as indicated in Fig. 3, sized for taking up the lateral force from 80 mph wind on the *side* area of #11. Increase sizing of vertical struts at center.

Use the same load conditions for all three cases. Let the feed arm always be parallel to one support cable, with the carriage either at arm center, or at arm end over cable. Apply six load conditions:

1. Arm center	weight only, no wind;
2. Arm end	
3. Side wind	80 mph, plus weight, arm center;
4. Front wind	
5. Side wind	30 mph, plus weight, arm end.
6. Front wind	

The output of the analyses should yield, for each structural member, the actual stress (kip/inch^2) for the present load condition, the maximum allowed stress as a function of its slenderness, and the ratio of both. (Note: the blueprints of Von Seb call "stress" what actually is "load", measured in kip.)

APPENDIX A

Sebastian,

Following are the answers to your Apr.28 questions:

(1) It does surprise me that not too many informations available to such an ordinary geometry. The closest I am able to find are (copies attached):
  - (a) with wrong boundary conditions, Baker, Kovalevsky, & Rish, "Structural Analysis of Shell", pp.44-49.
  - (b) with conc. load at apex, Reissner,"Stresses and Small Displacements of Shallow Spherical Shells. II",J. Math. & Phys. 1946.

(2) To answer your second questions:
  - (a) Four weeks.
  - (b) A recently graduated engineer with 1-2 years of experiences in structural analysis.

However, for a complete analysis, we may need to look into:
  - (i) modeling the cables
    cable stiffness and reaction orientation are functions of load.
  - (ii) generating wind loads
    choose the right wind orientation for max. cable tensions, and str. member forces.
  - (iii) checking and interpretation of the computer results.

If that is an alternative, it would be a 4 to 6 month job under your "supervision". (I may be too conservative. Whenever the cables are involved in the str., complecations are more than doubled.)

Is the original design analysis available? I have given a complete set of 12 meter drawings to John Findlay to be sent to Stanford Uni. for the same project. (??)

LEE
5-5-83

NATIONAL RADIO ASTRONOMY OBSERVATORY
POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE ARBOVALE 456-2011

APPENDIX B

LFST REPORT NO. 8
CONTRACT NO.
PAGE 1 OF 3
DATE May 2, 1966

PROJECT: LFST

SUBJECT: Elasticity of long ropes

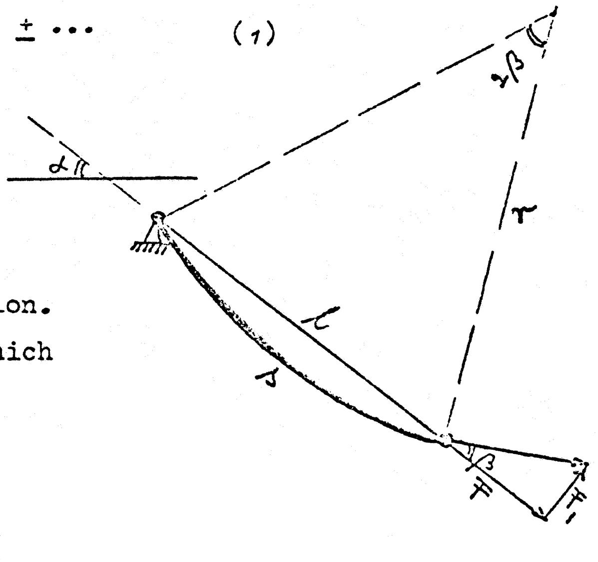
The effective elasticity, E_s, of a long rope, sagging under its own weight

S. von Hoerner

Some structures may use long ropes, for example a guyed tower does. The stiffness of such a structure depends on the modulus of elasticity, E, of the material used, but it also depends on the "sag" of the ropes which decreases the stiffness. We still can treat a long rope in the same way as any solid member, if we define for the rope an "effective modulus of elasticity", $E_s \leq E$, where E is given as the elasticity of the material used, but E_s is a function of the sag. Since I could not find a formula of this type in a few textbooks, and since this question might be important for very large structures, I give the following derivation.

1. We approximate the Catenary of a hanging rope by a circle (assuming a tight rope). Then

$$\left.\begin{array}{l} s = 2r\beta \\ \ell = 2r \sin\beta \end{array}\right\} \quad \frac{s}{\ell} = 1 + \frac{1}{6}\beta^2 \pm \dots \qquad (1)$$



2. In equilibrium, we have

$$F_1 = \frac{1}{2}\rho \ell Q \cos\alpha \qquad (2)$$

with ρ = density of material and Q = cross section. We call ℓ_0 the undeformed length of the rope, which means that

$$\frac{s}{\ell_0} = 1 + \frac{F}{QE} \qquad (3)$$

and we make use of

$$\beta \approx \tan\beta = F_1/F\,. \qquad (4)$$

With (2), (3) and (4), equation (1) can be written as

APPENDIX B

PROJECT:

SUBJECT:

$$\frac{F^3}{QE} = \left(\frac{\ell}{\ell_o} - 1\right) F^2 + \frac{1}{24}\frac{\ell}{\ell_o}(\rho \ell Q \cos \alpha)^2 . \tag{5}$$

3. The effective elasticity we now define as

$$E_s = \frac{\ell}{Q}\frac{dF}{d\ell} . \tag{6}$$

From (5) we find $dF/d\ell$, and after neglecting all terms of higher order we obtain

$$E_s = E\left(1 - \frac{1}{12}\frac{E \rho^2 \ell^2 Q^3}{F^3} \cos^2\alpha\right). \tag{7}$$

4. In order to show more clearly what equation (7) means, we define two quantities:

critical length (material constant) $\ell_c = \dfrac{S^{3/2}}{\rho E^{1/2}}$ (8)

safety factor (free choice) $q = \dfrac{QS}{F} \geq 1$ (9)

where S = maximum allowed stress of material. With these definitions, equation (7) reads finally

$$E_s = E\left\{1 - \frac{1}{12} q^3 (\ell/\ell_c)^2 \cos^2\alpha\right\}. \tag{10}$$

5. Taking, for example, high-strength Bethlehem ropes, we have $E = 23 \times 10^6$ psi and $S = \text{Yield}/1.35 = 81 \times 10^3$ psi, which gives

$$\ell_c = 430 \text{ m} = 1410 \text{ ft} . \tag{11}$$

6. As an example, we assume $\alpha = 45°$, and we allow $q = 2$ (for taking up wind forces). Equation (10) then becomes

$$E_s = E\left\{1 - (\ell/744\text{m})^2\right\}. \tag{12}$$

If we apply this to the guyed tower in Fig.5 of my Flat-Antenna Report (No.7), we find that the modulus of elasticity goes down by only 4%.

LFST

REPORT NO. 8
CONTRACT NO.
PAGE 3 OF 3
DATE

APPENDIX B

PROJECT:

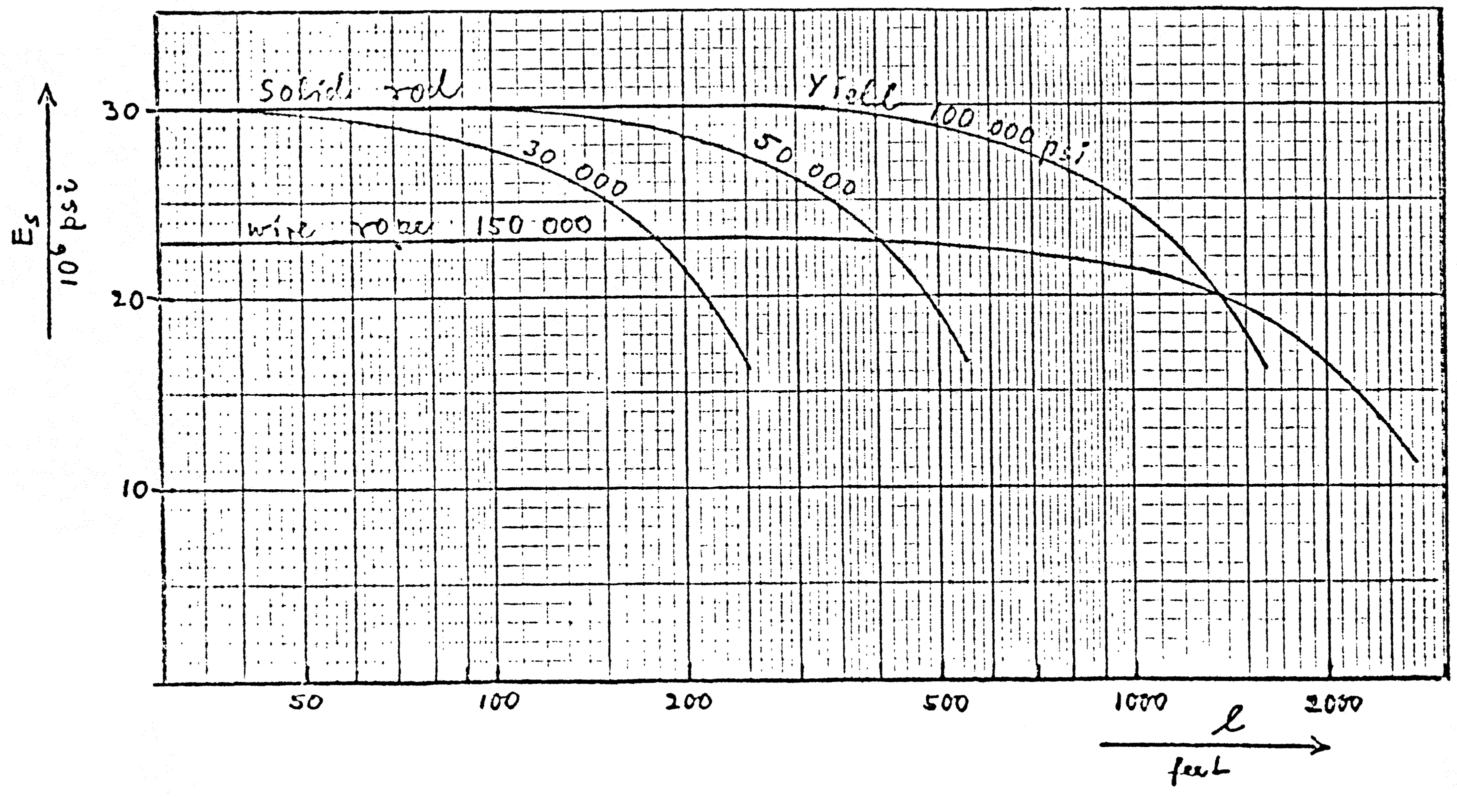
SUBJECT:

7. The critical length, ℓ_c, depends strongly on the maximum allowed stress, S, of the material used. It gets very short for normal steel, which means that high-stress steel should be used for long, solid rods, like tensioned diagonals or long guy rods. A few examples are given in the table below.

material	Yield 10^3 psi	E 10^6 psi	ℓ_c meter	feet
prestreched wire rope	150	23	430	1410
solid steel rod	100	30	268	880
	50	30	94	310
	30	30	44	140

The figure below shows the effective elasticity as a function of length, according to formula (10), for which we adopted the values

$\alpha = 45^\circ$ elevation angle above horizontal
$q = 1.5$ safety factor (stress = $S/1.5$ = Yield/2.77)



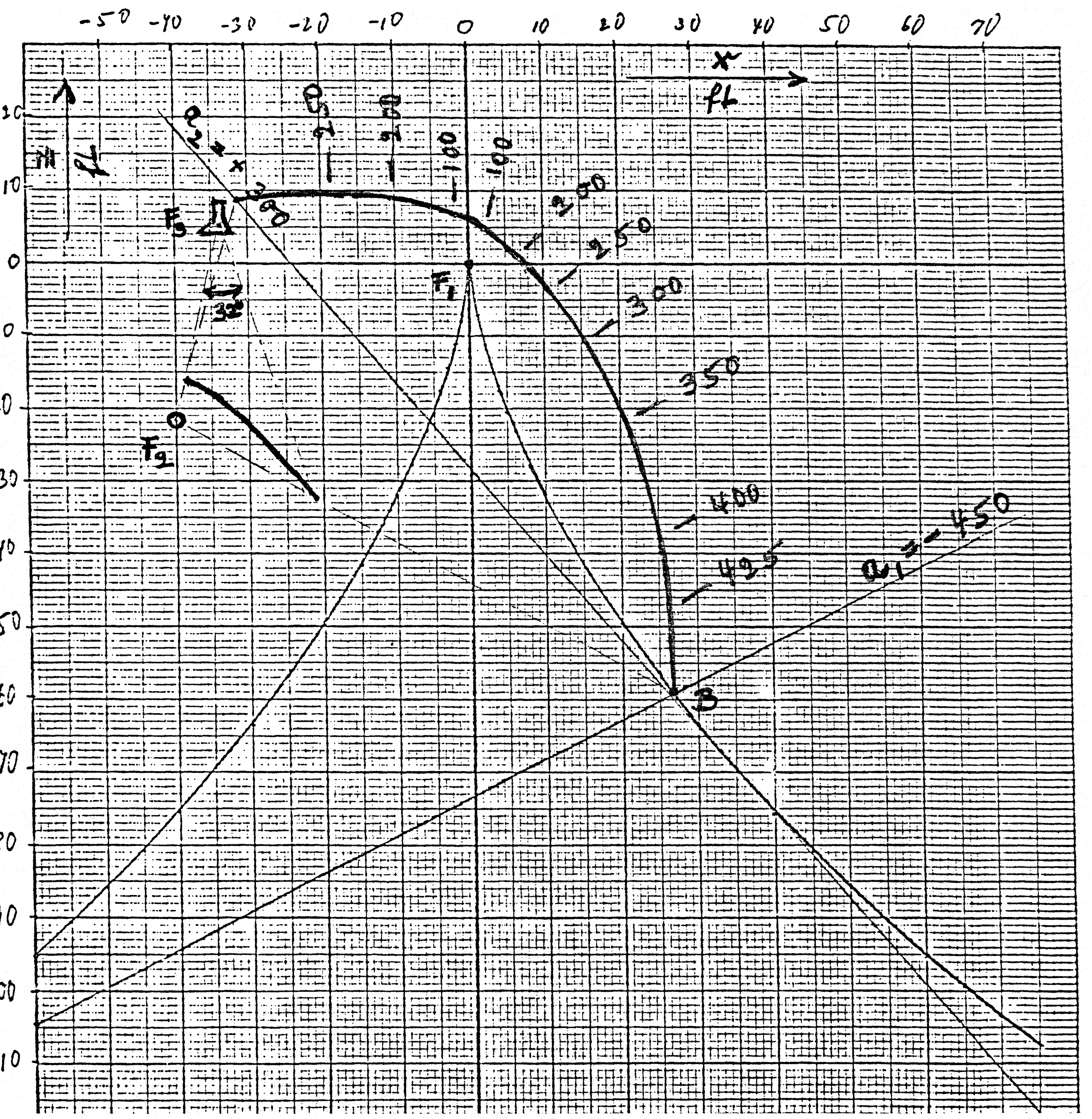
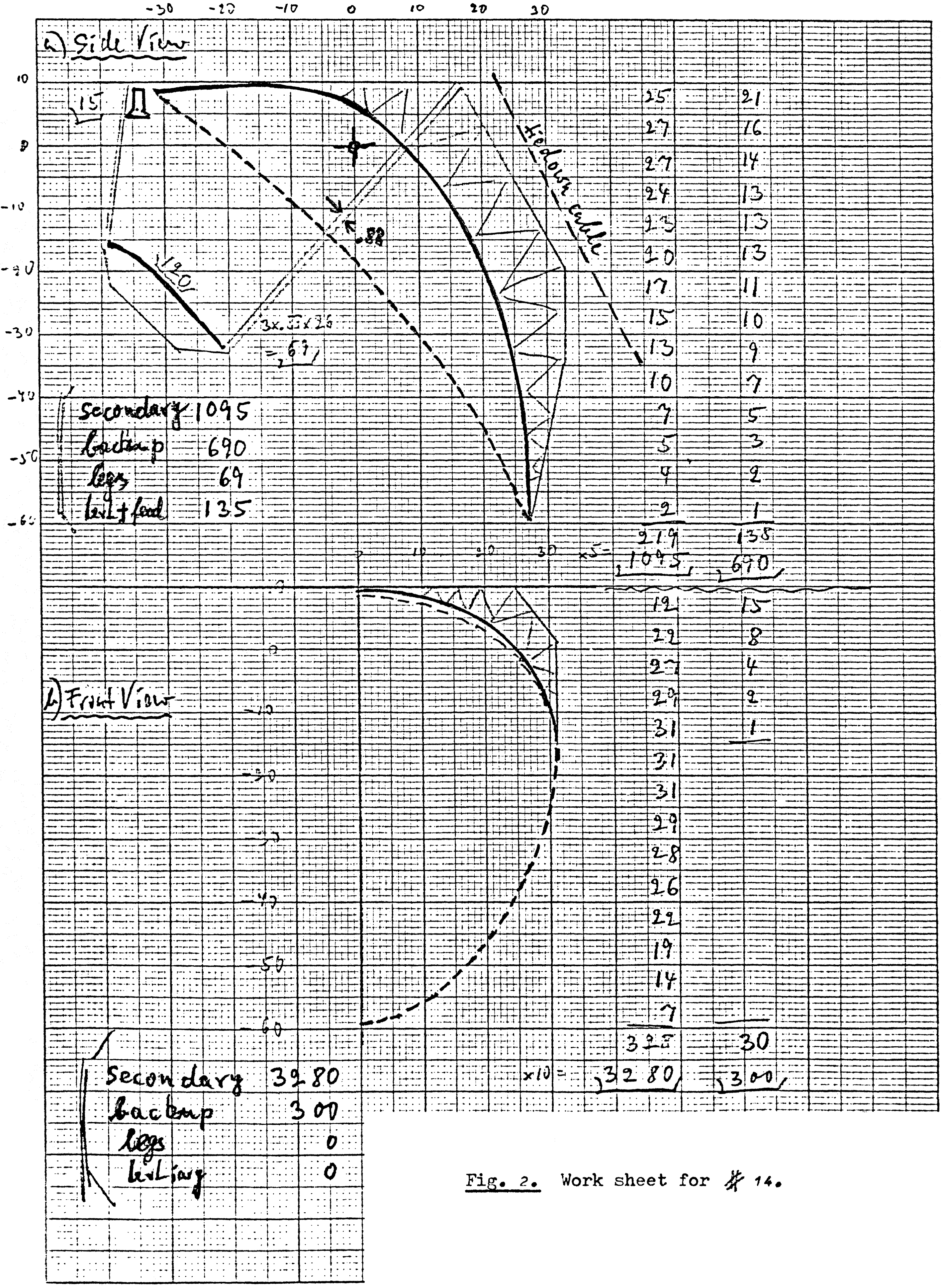


Fig. 1. System # 14: with 75 ft offset and 750 ft aperture (vignetting as # 5).
Feed placed outside of secondary.
Optimized for compactness of secondary, and height of tertiary.



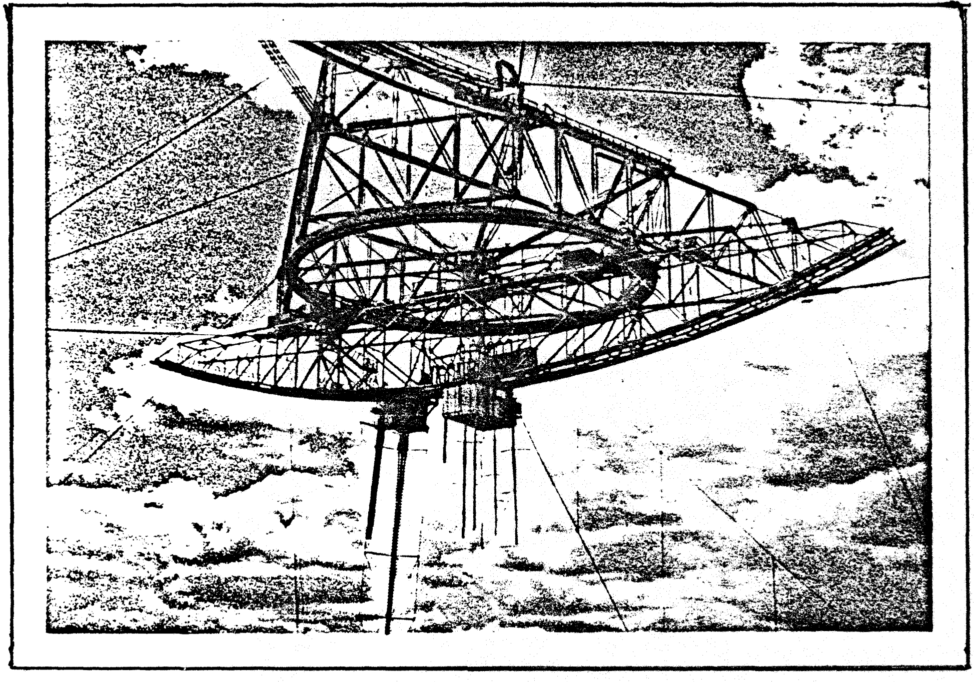
25	21
27	16
27	14
24	13
23	13
20	13
17	11
15	10
13	9
10	7
7	5
5	3
4	2
2	1
219	138
×5= 1095	690

Secondary	1095
backup	690
legs	69
levl + feed	135

12	15
22	8
27	4
29	2
31	1
31	
31	
29	
28	
26	
22	
19	
14	
7	
328	30
×10= 3280	300

Secondary	3280
backup	300
legs	0
levling	0

Fig. 2. Work sheet for # 14.



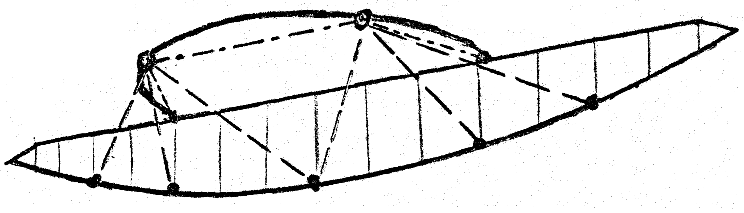


Fig. 3. Lateral bracing of feed arm. (Front half only is drawn)

New pieces, to be added:

o o 4 wheels on azimuth track

—·—·— 6 horizontal members between wheels

— — — 12 lateral braces