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# SURFACE ERROR AND TELESCOPE PERFORMANCE FOR THE LARGE DEPLOYABLE REFLECTOR

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## 1. Introduction

History. In November 1984, a meeting was held in Tucson, AZ, of the LDR Antenna Study Subcommittee. The LDR (Large Deployable Reflector) is meant to be an infrared telescope in Earth orbit, about 10 m diameter, as recommended by the Field Committee in 1980. Since some approaches to a design had already been made based on the concepts of optical telescopes, I was asked for an approach based on the concepts of radio telescopes. I should describe the influence of the reflector's surface errors on the telescope's performance, in order to derive manufacturing specifications meeting observational demands. By performance we mean: sensitivity (gain), resolution (beamwidth), and positional accuracy (pointing error). In addition, I will briefly discuss some more general topics.

Guide Lines. The purpose of the LDR will be to cover the range of wavelelengths between optical and radio astronomy not observable from the ground (25 to 300 μm), and cost increases with accuracy. Thus, I will (for the design) not consider observations below 25 μm.

The LDR will be the telescope above our atmosphere having by far the largest surface (collecting area), and we may consider as its main purpose the detection of weak infrared point sources: detections precede mappings and spectra, and detecting wide sources is better done with small telescopes. Thus, I will maximize the on-axis gain, which then automatically minimizes the beamwidth, too. This approach neglects sidelobes (still 18 db down with uniform illumination for maximum gain), and a wide field of view (not needed for coherent detection with one or a few receivers).

We may assume that the active constraints are financial rather than technical. Thus, I will try to maximize the gain per dollar.

## 2. Definitions

$D$ = telescope diameter, assuming circular aperture;

$\lambda$ = wavelength of any observation,

$\lambda_s$ = design wavelength, to be used for specifications;

$\sigma$ = surface error = rms deviation between the actual reflector surface and its best-fit paraboloid of revolution,

$p$ = rms phase error, of all rays at receiver (dealing with surface errors only, we disregard phase errors from misalignments of secondary and further mirrors),

$A$ = correction factor depending on focal ratio ($A=1$ for $F/D = \infty$ ).

Regarding $\sigma$, both the best-fit procedure and the rms must be weighted with the aperture illumination pattern, mostly strongly tapered at the rim for avoiding spillover. Note: the best-fit paraboloid has all of its six parameters of freedom (three translations each of vertex and focus) only if the receiver can be automatically moved to the best-fit focus location, and if the best-fit axis can be pointed at the source direction. Otherwise, the respective degrees of freedom must be constrained in the best-fit procedure. This can make a lot of difference but is freqently forgotten.

The factor A depends also on the definition of $\sigma$ which on the telescope is mostly measured parallel to the axis, but which must be specified for the panel manufacturer normal to the surface.

$\eta_a = \eta_\infty \times \eta_p$ = aperture efficiency = (power going into receiver) /(power falling on whole aperture),

$\eta_\infty$ = long-wave aperture efficiency = $\eta_a(p=0)$,

$\eta_p$ = phase efficiency, due to phase error only.

Note: $\eta_\infty$ is due to non-uniform aperture illumination if tapered; and due to the shadows of focal equipment and support legs (about twice the geometrical shadow, regarding receiver power). For very long wavelengths, also diffraction losses matter.

For good conventional radio telescopes, $\eta_\infty$ is about 0.6; for a "shaped" two-mirror system with uniform aperture illumination, it can come up to 0.85; and for shaped asymmetric systems, avoiding shadows, it can come close to one.

G = gain, more exactly, effective axial gain = (power going into receiver)/(power which would be received without reflector by a "unidirectional dipole");

ß = beamwidth, more exactly, half-power beamwidth. When scanning across a point source, we call ß the angular distance from the left-hand point with 1/2 the maximum power, over this maximum, to the right-hand 1/2 power point.

## 3. Relations

Regarding phase error and efficiency, we have:

$$p^2 = A(4\pi\sigma/\lambda)^2 \tag{1}$$

$$\eta_p = e^{-p^2} \tag{2}$$

$$\eta_a = \eta_\infty \times \eta_p = \eta_\infty e^{-A(4\pi\sigma/\lambda)^2} \tag{3}$$

where, according to Ruze (1966, Fig.7):

| F/D = | 0.43 | 0.60 | 1.00 | | |
|---|---|---|---|---|---|
| A = | .76 | .85 | .94 | if $\sigma$ parallel to axis, | (4) |
| | .86 | .92 | .97 | $\sigma$ normal to surface. | |

And, if I understood the optical term correctly:

$$\text{Strehl ratio} = \eta_p. \tag{5}$$

Regarding gain and beamwidth, we have:

$$G = (\pi D/\lambda)^2 \eta_a = (\pi D/\lambda)^2 \eta_\infty e^{-p^2} \tag{6}$$

$$\beta = b(\lambda/D)\, e^{+p^2/2}. \tag{7}$$

Or together

$$G = \eta_\infty (\pi b)^2/\beta^2 \tag{8}$$

where

| | | | |
|---|---|---|---|
| b = 1.0 | for uniform illumination, | | (9) |
| 1.2 | most telescopes, with about 15 db taper, | | |
| 1.4 | very strong taper with $(1-r^2)^2$. | | |

Finally, we need the dependence of cost, C, on size and accuracy. In lieu of something better and with hesitation, we will use the relation

$$C = k\, D^n/\sigma^m . \tag{10}$$

Starting with some finished and priced design (yielding k) we may use (10) for some scaling, but certainly by less than a factor of two. Actually, $C(D,\sigma)$ has not only slopes but steps as well: decreasing the wavelength, we start with dipole arrays, stepping to mesh wire reflectors, to honeycomb panels, and to optical mirrors. Increasing the size, we step from longish refractors to more compact reflectors, from polar mounts to alt-azimuth, from fully steerable to fixed primary mirrors.

The guide line here is: to move along a convenient (low-cost) slope, and to push the next limiting step just as far off as possible.

The exponents in (10) depend on details not yet known in our case. But we can give limits. If the surface cost matters strongly, as it will for us, then obviously $n \geq 2$; if the backup structure matters and is large, it will be defined by buckling stability (local and global) and then $n \leq 3$; thus $2 \leq n \leq 3$. If the surface matters at all, then $m \geq 1$; but should the slope increase steeper than m=3 for going to shorter wavelengths, we rather stay put or jump to some cheaper design. From our NRAO design experience, we may use about n=2.5 and m=1.5.

| | n | m | economical telescopes then should be |
|---|---|---|---|
| extremes | 3 | 1 | small and precise |
| | 2 | 3 | large and sloppy |
| use now | 2.5 | 1.5 | |

(11)

4. <u>Gain Curves and their Maxima</u>

Equation (6) shows that the gain of a telescope, as a function of wavelength, first increases with $1/\lambda^2$ if we go from long waves to shorter ones. This is due to the decreasing beamsize, $B \sim \lambda$, wich ist best understood for a transmitting antenna: if all its power is confined to a smaller beam, then the peak of the beam must get higher. The gain reaches a maximum, and then drops very fast for still shorter waves, due to the exponential term.

Fig.1 is an older comparison of various radio telescopes for wavelengths between one millimeter and one meter. It shows clearly that each telescope has a limited useful (competitive) range of wavelengths, less than a factor of ten. Especially to the left of the maximum the exponential drop is so steep, that a much smaller but more accurate telescope becomes much better. Compare, for example, the Haystack 120-ft with the Aerospace 15-ft.

Once a telescope is built, Fig. 2a shows in detail how its gain then varies for the user as a function of the wavelength. The gain maximum is found from (6), by letting $\partial G/\partial\lambda = 0$, as

$$\text{for fixed D and } \sigma,\ \max(G) \text{ at:} \quad q = 4\pi = 12.6 \qquad (12)$$

with a wavelength-surface quotient q defined as

$$q = \frac{\lambda}{\sigma\sqrt{A}} . \qquad (13)$$

At this maximum, gain and efficiency are degraded, below those of a perfect surface, by the phase efficiency $\eta_p$ (Strehl ratio?) of

$$\eta_p = e^{-1} = 0.368, \quad \text{at } q = 4\pi. \qquad (14)$$

For longer wavelengths, the telescope becomes "diffraction limited". For shorter wavelenghts, it is sometimes called a "light bucket", especially when used with a bolometer large enough to catch all distorted rays in the focal plane. Although many telescopes are occasionally used in this mode, a telescope should never be designed as a (cost saving) light bucket, for several reasons:

First, the exponential degradation is extremely steep; for example, going to half the wavelength of the maximum gain, to $q=2\pi$, we have only $\eta_p=e^{-4}=0.018$ which is rather useless. Second, not only do we lose resolution by getting a wider beam, for example wider than the diffraction beam by a factor $e^{+2}=7.4$ for $q=2\pi$, but this is then a "dirty beam, split up into several lobes. Third, this gives not only large pointing errors, but lets them even depend on wavelength.

Fourth, a bolometer may catch all the rays and thus all the power, but then it must be larger than needed for a perfect surface, by a factor $N=e^{p^2}$ in area, increasing its noise by a factor $\sqrt{N}$; thus the signal/noise will be degraded by a factor $e^{-p^2/2}$, or $e^{-2}=0.14$ at $q=2\pi$. And bolometers are not good for spectra.

5. Maximum Gain per Dollar

Whereas Fig. 2a showed the use of an existing telescope, Fig. 2b is meant for the design of a future one. Preparing a design, one must first agree on its design wavelength $\lambda_s$, its shortest wavelength of good observations, to be used for specifications. Second, one must weigh the wanted performance at $\lambda_s$ versus the expected funding, and then one should design for an optimum. This optimization can be defined in several ways:

(a) Guess the available cost C, design for max(G) at $\lambda_s$;
(b) Guess the available cost C, design for min(B) at $\lambda_s$;
(c) Set $G(\lambda_s)$ as wanted, design for min(C);
(d) Set $B(\lambda_s)$ as wanted, design for min(C).

Fortunately, all four definitions yield the same answer. Since guessing the funding C is pure gambling, and since there is not much infrared competition for G, the most realistic definition is (d). Demanding, for example, a beamwidth (or at least a pointing accuracy) of about one arcsecond: for ground-based optical identification of space-detected infrared sources.

Optimization (d) means: we consider $\lambda_s$ and B as fixed, which serves to eliminate D from (10), where C then depends only on $\sigma$. We let $dC/d\sigma=0$ and thus find $\sigma_s$, the surface accuracy to be specified. In detail: from (7) and (1) we find

$$D = (b\,\lambda_s/B)\; e^{+A(4\pi\sigma/\lambda_s)^2} \qquad (15)$$

and we insert this into (10). For simplification, we minimize not C, but a quantity $K = (C/k)^{1/n} B/(b\lambda_s)$ where all except C are constants:

$$K = \sigma^{-m/n}\; e^{+A(4\pi\sigma/\lambda_s)^2} \; . \qquad (16)$$

And letting $dK/d\sigma = 0$ then yields

$$A(4\pi\,\sigma_s/\lambda_s)^2 = m/n \qquad (17)$$

which we again express in terms of a wavelength/surface quotient $q_s$, for specifications:

$$q_s = \frac{\lambda_s}{\sigma_s\sqrt{A}} = 4\pi\sqrt{n/m}\; . \qquad (18)$$

We use both extremes for n and m, and the suggested values of (11):

| | n | m | $q_s$ |
|---|---|---|---|
| extremes | 3 | 1 | 21.8 |
| | 2 | 3 | 10.3 |
| suggested | 2.5 | 1.5 | 16.2 |

(19)

The suggested value of $q_s$=16.2 is used in Fig. 2b. The phase efficiency (Strehl ratio?) for this optimized design, at its design wavelength $\lambda_s$, then is

$$\eta_p = e^{-m/n} = 0.549. \tag{20}$$

The diameter needed to yield the wanted beamwidth ß at the design wavelength follows from (15) with (18) and (19) as

$$D = (b\lambda_s/\beta)\ e^{+m/2n} = 1.35\ b\ \lambda_s/\beta. \tag{21}$$

6. <u>Numerical Example</u>

First, we must fix $\lambda_s$. Since cost matters and depends on accuracy, $\lambda_s$ should not be made smaller than definitely needed, and I would suggest not below 25 µm where ground-based observations begin. But because of the steep exponential gain drop, it cannot be much larger either. For example, selecting various values for $\lambda_s$ yields the following phase efficiencies for observations at 25 µm:

| $\lambda_s$ = | 30 | 35 | 40 µm |
|---|---|---|---|
| $\eta_p$(25µm) = | .421 | .309 | .215 |

(22)

From this I would like to suggest setting

$$\lambda_s = 35\ \mu m. \tag{23}$$

The diameter then follows from (21), with b=1 for uniform illumination:

$$D = \frac{9.73\ [m]}{\beta\ [arcsec]}\ . \tag{24}$$

Which agrees nicely with D=10 m of the Field Report, if ß = 1 arsec for optical identification. The surface error must then be specified

$$\sigma = 2.16\ \mu m\ /\sqrt{A} \tag{25}$$

for manufacturing, measuring, adjusting, residual thermal deformations.

7. Miscellaneous

a) Correlation of Surface Errors can only improve the gain, mainly for short wavelengths far left of the maximum in Fig. 2a (Ruze 1966). Since this improvement depends on the correlation length of the surface errors which is not well known in advance, and in order to be on the safe side, I have neglected this topic and suggest to do so also in the future.

b) Smaller Size. If D=10m cannot be funded, we must relax our resolution demand, but still ask for sufficient pointing accuracy. The intrinsic (phase error dependent) pointing error $\Delta\varphi$ of radio telescopes, for shortest wavelengths, may be adopted at most as

$$\Delta\varphi = \beta/6. \tag{26}$$

Keeping $\lambda_s$=35 µm and equations (24) and (25), we have for various choices of the diameter:

| D = | 10 | 7 | 4 | meter |
|---|---|---|---|---|
| $\beta$ = | .97 | 1.39 | 2.43 | arcsec |
| $\Delta\varphi$ = | .16 | .23 | .41 | arcsec |

(27)

The two smaller diameters need not be deployable. D=4m would fit as a whole into the cargo room of the Space Shuttle, and D=7m into a large container below the fuel tank. Therefore, the cost decrease will be much steeper than $C=D^{2.5}$, by a step function. But the gain will decrease as $G=D^2$. Thus, D=7m may be a reasonable compromize if cost-saving is required.

c) Longish-Surface Telescope. Nick Woolf made an interesting suggestion: to store as a whole a longish, non-circular telescope having still a large surface. As an example, we will adopt the same gain and surface area as the 10m round telescope, and we adopt an elliptical shape for convenient illumination without spillover. For the two storage places we then have:

| | surface size [m] width | surface size [m] length | beam size [arcsec] length | beam size [arcsec] width |
|---|---|---|---|---|
| cargo | 4 | 25.0 | 2.43 | .39 |
| tank | 7 | 14.3 | 1.39 | .68 |

(28)

Since the cargo room is only 12m long, the 7m width is recomended.

d) Suggested Investigations

1. What would be the largest passive monolithic telescope?

2. The largest deployable passive one? Passive means: no measuring or adjusting after erection. Largest size is then defined by thermal deformations, depending on shielding and on the thermal coefficient of the telescope material.

3. What is the largest deployable one, if only misalignments (rigid-body movements, mirrors and receiver) get measured and adjusted?

4. Why do we insist on optical light for measuring? Use infrared or submillimeter radio waves?

5. Why is it better to adjust the pupil rather than the main reflector panels? It takes the same number of motors and the same range of movement, but needs one more mirror. On Earth, the panel weight matters, but not in space.

6. Do we need an extremely wide field of view? If not, a two-mirror system will do, Cassegrain-type but "shaped" for uniform aperture illumination. A radio telescope as the 140-ft at Green Bank, with a magnification factor (diameter ratio of mirrors) of about 14, would allow about 200 good picture elements in the secondary focal plane.

Reference:

Ruze, J. 1966: Proc. IEEE, 54, 633.

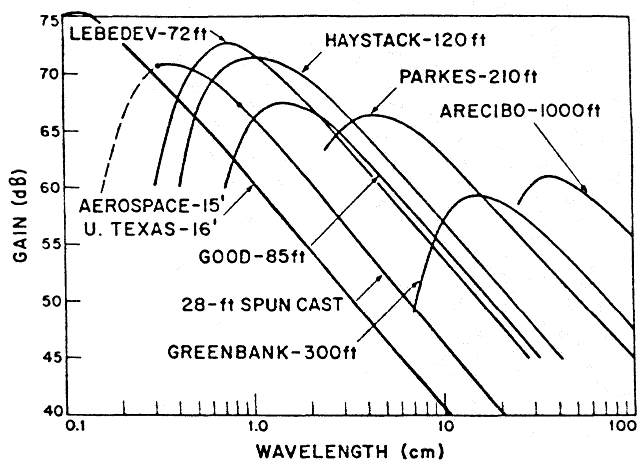


Fig. 1. Comparing the gain, as a function of the wavelength of observation, for various radio telescopes (Ruze 1966).

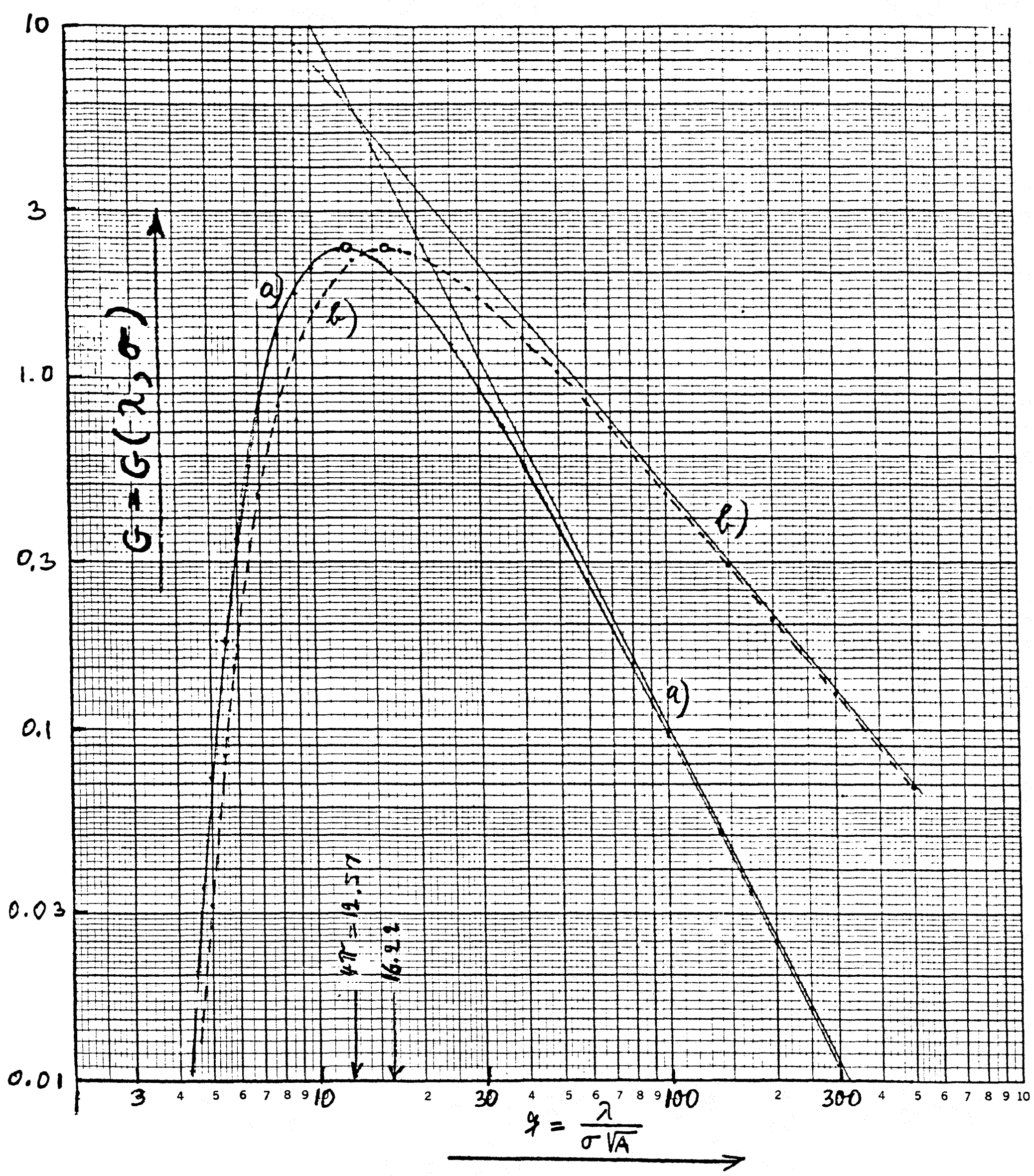


Fig. 2. The gain as a function of wavelength λ and surface error σ. Both curves are (arbitrarily) normalized to equal height.

a) G(λ) for fixed σ and diameter.
This means working at various λ with some given telescope.

b) G(σ) for fixed λ and cost.
This means designing an optimized telescope for a given wavelength, within a given cost ceiling.