NATIONAL RADIO ASTRONOMY OBSERVATORY

ENGINEERING REPORT NO. 120

ARECIBO THREE-MIRROR SYSTEMS, IV:
FEED OUTSIDE SECONDARY, ELLIPTICAL APERTURE, RADOME

S. VON HOERNER

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1. Introduction and Summary

From the previously suggested systems, #5 was selected as a compromise of performance versus cost (Report 112, Fig.8). It has 700 ft aperture diameter with 50 ft offset; it starts vignetting at 15° off zenith, and at 20° its spillover is 74 ft beyond the primary (Report 115, Table 2).

Several changes were suggested. (a) The feed should sit outside the secondary mirror, with enough clearance for a cabin. (b) I still wanted the central feed axis parallel to the telescope beam (probably good for polarization). (c) P.S. Kildal suggested elliptical apertures, and we agreed to use the one which gives the same vignetting as the circular one but with the largest possible aperture area; which means a long axis of 851 ft if we keep the short axis with 700 ft. (d) P. Stetson suggested to enclose seconday plus tertiary mirror in a radome, for aerodynamical reasons: less drag in spite of larger surface, known forces, no flutter even with sluggish platform dynamics; and we agreed to cover the dome with alu-sheets, cut off the lower part (for incoming rays), and close this hole with a translucent but tensionally stiff membrane. (e) For multi-receiver choice, I suggested to use a rotatable 45° mirror before the final focus F_3 (and no rotatable turret).

Since we now approach closer to a final design, I tried to be more systematical: First, feed axis direction and feed illumination angle β_3 could be treated analytically, yielding the coordinates of F_3 and of the tertiary extremes, once β_3 is chosen (and a projected length, P=18 ft, of the tertiary, for keeping the diffraction small at long wavelengths). Second, I listed all constraints which can limit the choice of design variables, and optimized by going to the best intersection of the active constraints.

Data are given for six selected systems, in tables and drawings. Three systems for turret, with feed angles β_3=90° and 60°, and three systems for the 45° mirror, with β_3= 40°. For the last three, the location and radius of the smallest dome is given. And two of these have also been calculated for elliptical apertures.

In the near future we should take a series of final decisions, as to types, sizes, clearances. Some suggestions are made.

2. Location of F_3 and Tertiary

The secondary mirror has three degrees of freedom: the location (x,z) of its focus F_2, and its size which we defined by its height, on the z-axis. Once all this is fixed, the tertiary then has the same three degrees: (x,z) of its focus F_3, and its size.

We want to use up the three degrees by demanding, first, that its length, projected perpendicular to the feed axis (as seen by the feed) is P=18 ft, as in all previous systems (long-wave diffraction loss). Second, the central feed axis shall be parallel to the telescope beam, the z-axis in our case. Third, we select a given value β_3 for the feed illumination angle, under which the tertiary is seen by the feed. There is one further choice regarding type: we have chosen Gregorian tertiaries since they seemed to be a bit smaller than Cassegrains.

Fig.1 gives definitions. Once the secondary is fixed, the locations of points 1, 2, 3 are known. For the tertiary we have six unknowns: the locations (x,z) of points 4, 5, 6. We call t_i the tangens as shown in Fig.1:

$$t_1 = \frac{x_3 - x_1}{z_1 - z_3} \; ; \quad t_2 = \frac{x_2 - x_3}{z_3 - z_2} \; ; \quad t_3 = \tan(\beta_3/2). \qquad (1)$$

We call

$$v_i = \sqrt{1 + t_i^2} \; ; \quad \text{where } v_2 < 0 \text{ if } t_2 < 0. \qquad (2)$$

and

$$g = (v_2 - v_3) \,/\, (v_1 + v_3). \qquad (3)$$

We have six equations for our unknowns:

1) Point 4 is on a straight line through points 1, 3.
2) Point 6 is on a straight line through points 2, 3.
3) Direction 5,4 is given by t_3.
4) Direction 5,6 is given by t_3.
5) Projected length of tertiary is P.
6) For Gregorian ellipse: $d_{43} + d_{45} = d_{63} + d_{65}$.

For brevity we shall omit all details and just give the results:

$$z_6 = z_3 + P/(t_2 + g\, t_1) \qquad (4)$$

$$x_6 = x_3 + t_2(z_3 - z_6) \qquad (5)$$

$$x_4 = x_6 + P \qquad (6)$$

$$z_4 = z_3 + (x_3 - x_4)/t_1 \qquad (t_1 \neq 0) \tag{7}$$

$$z_5 = \frac{1}{2}(z_4 + z_6 + P/t_3) \tag{8}$$

$$x_5 = x_4 + t_3(z_4 - z_5) \tag{9}$$

And the length, unprojected, of the tertiary then is

$$d_3 = \sqrt{P^2 + (z_6 - z_4)^2} \tag{10}$$

3. <u>Constraints and Other Conditions</u>

First, we have some adopted <u>fixed</u> values: aperture 700 ft, offset 50 ft, feed direction parallel z-axis, projected tertiary length 18 ft, and we choose β_3= 90° and 60° for the turret, and 40° for the mirror. (We tried also 120°, but this would have placed the cabin deep down.) P=18 ft means, for example, 26 wavelength at λ = 21 cm, or 18λ at 1 GHz or 10λ at 550 MHz.

All these and the following numerical values are still open for discussion, but should be made fixed in a short while.

Second, the <u>constraints</u> and <explanations> are, see Fig.2:

The secondary shall start on L_1 but right of point B, and shall end on ray R_1 but left of point C <it shall catch all aperture rays, but must not cast shadow on itself> (11)

Secondary must not go above z=8 ft <azimuth arm clearance>. (12)

Secondary must not cross z-axis below y=5 ft <illumination>. (13)

Secondary shall stay ≥8 ft off T <backup, tie-down>. (14)

F_2 (≥5 ft) left of R_1 <tertiary no shadow on itself>. (15)

Tertiary left of R_1 and above C_1 <rays blocking>. (16)

F_3 left of R_2 <cabin, rays blocking>. (17)

$z(F_3) \leq$ 2 ft for turret, or 11 ft for mirror <azim. arm>. (18)

If F_3 is high, then $\Delta x \geq 5$ or 8 ft <cabin, secondary>. (19)

$\Delta F \geq$ 5 ft <strong curvature, diffraction problem>. (20)

Third, <u>optimization for compactness</u> means we must go to the high, right intersection of those constraints which are active for this design.

4. Results

The active constraints and the selection of the best location for F_2 are shown and explained in Figs.3, 5a and 7a.

Since the question of receiver turret versus 45° mirror was discussed but not decided, I have selected three systems each. P.S. Kildal had suggested a feed angle of $\beta_3 = 90°$ to keep the horn size small for long λ, but this either placed the receiver deeper down than wanted, or it made the tertiary too narrow and curved, see Figs.4a and 4b. Thus, $\beta_3 = 60°$ was calculated, and Fig.5b seems a good result if the turret is chosen.

If we prefer the 45° mirror, then β_3 must be smaller, and 40° was adopted as a compromise between feed size versus mirror and cabin size, see Fig.6. We may have receivers in a full circle about the mirror, needing the "two-sided" cabin of 20 ft length in Figs.7b and 8a. Or we have receivers only in a half-circle, with the smaller "one-sided" cabin of 17 ft length in Figs.7c, 7d and 8a.

The diameter of the secondary is the same for circular and elliptical aperture, but rim and dome are different. The dome was always chosen with the smallest diameter, leaving a

$$\text{dome clearance} = 1 \text{ ft} \tag{21}$$

regarding secondary and tertiary mirrors, and the cabin if inside. Three domes for circular aperture are shown in Figs.7b, 7c and 7d. And two domes for elliptical aperture in Figs.8a and 8b. All numerical values are given in the following tables.

Table 1. Six suggested systems: the defining items.
T = turret; M = 45° mirror, M1 one-sided, M2 two-sided.

system #	Fig.	β_3	T,M	secd. hight	active constraints
15	3a	90°	T	$z_m=8$	$\Delta x \geq 8$, $\Delta F \geq 5$
16	3b	90°	T	$z_m=8$	$\Delta x \geq 8$, $z(F_3) \leq 2$
17	4b	60°	T	$z_m=8$	$\Delta x \geq 8$, $z(F_3) \leq 2$
18	6b	40°	M2	$\nu=5$	$\Delta x \geq 8$, $z(F_3) \leq 11$
19	6c	40°	M1	$z_m=8$	$\Delta x \geq 5$, $z(F_3) \leq 11$
20	6d	40°	M1	$\nu=5$	cabin: ray R_2, dome

Table 2. Six systems: the resulting items.
d = (x,z)-diameter, z* = deepest point; I = illumination ratio, see Report 112, equation (2). [All dimensions in feet]

syst. #	secon. height	F_2 x	F_2 z	F_3 x	F_3 z	secondary d_2	secondary I	tertiary d_3	tertiary z*
15	=6.53	-21.70	-16.80	-35.20	- 5.6	66.2	17.4	21.0	-20.8
16	=6.25	-20.02	-11.95	-34.91	+ 2.0	64.6	19.6	22.4	-14.0
17	=6.06	-24.00	-15.57	-35.81	+ 2.0	66.6	19.7	22.0	-20.5
18	z_m=7.20	-24.67	-15.62	-35.44	+11.0	65.7	23.5	22.5	-20.9
19	=6.31	-22.34	-15.51	-32.46	+11.0	66.1	18.6	22.2	-21.2
20	z_m=5.49	-19.00	-31.00	-28.10	- 9.8	65.3	21.1	19.8	-39.0

Table 3. Location and size of dome.
C = circular and E = elliptical aperture; x,z = coordinates of dome center; R = inner dome radius; H = smallest lower hole for incoming rays, h = uper hole for cabin.

#	x	z	R	H	h
18C	-13.3	-29.6	37.9	52	17
18E	-11.7	-31.3	39.7	56	17
19C	-10.3	-28.1	36.6	48	14
20C	- 7.1	-33.0	39.0	57	0
20E	- 5.9	-36.4	42.8	64	0

5. Decisions Needed, and some Opinions

The present six systems are suggestions, for comparing diiferent types, and they are based on dimensions and clearances which still need discussions before final fixing. Many still open questions should be discussed, the result be frozen, and then not be raised again.

1) Is it already a final decision that we need additional support cables, for any of the Gregorian systems?

2) We need cost estimates for these cables, and for at least one of the Gregorian systems. And if the major cost is in the cables and their erection, we might as well chose a larger aperture, for example

system #14 with 750 ft; see Table 4 of Report 115, where 700 or 750 do not make much difference for the cable stress.- Otherwise, 700 ft aperture and 50 ft offset seem a good choice provided we build the ground screen.

3) Is the ground screen a final decision? If not, vignetting would call for more than 50 ft offset, for example system #11 with 700 ft aperture and 125 ft offset and no vignetting.

4) Should we calculate systems with radial symmetry? (All surfaces are figures of rotation about the z-axis, but are asymmetrically cut off.) That shaping programs are available is no strong point: science lives just by doing things which were not yet available. But that all surface panels in a ring have identical shape and are cheaper is indeed a strong point. Only, in order to have still symmetry after shaping, the feed pattern must also be a figure of rotation with an asymmetric sharp cutoff. This seems not well possible. And even if, we then loose one of the great advantages of shaped surfaces: to get a uniform aperture illumination with sharp cut-off rim, from any old simple feed pattern which just must be narrow but does not need edges.

5) Which (if any) of the systems so far presented is the best one? Should different types be calculated? For example: one or two Cassegrain tertiaries for comparison?

6) Is the tertiary large enough, with P=18 ft? What, actually, is our longest wavelength for the Gregorian system, regarding diffraction losses?

7) Turret or 45° mirror? If there are not strong reasons against it, the mirror looks better: less cost and weight. It can also be made rocking for beam switching, which is done with this 45° mirror at the NRAO 12-m dish at Kitt Peak, and with the 10-m diameter Cassegrain at the 140-ft at Green Bank. And the rotation of the mirror, as well as the tilt of the Cassegrain, are both also used for the multi-receiver choice.

8) If the mirror is accepted, is the feed angle of 40° acceptable? The feed horn than needs a size of about 6 λ.

9) Do we agree on the elliptical aperture? I hope, yes.

10) Radome or not? After the explanations from P. Stetson, and after having calculated some examples, I am now much in favour of it, unless it were very expensive. Although it increases the surface, it catches a much smaller wind force especially in the worst case: face-on. Also, it is difficult to estimate the danger of flutter for the large odd-shaped Gregorian supported at a dynamically inert platform, and the dome would el~~#~~iminate this uncertainty.

11) Cabin location at the azimuth arm (#18), or inside the dome (# 20)? My guess is at the arm, but I would let P. Steson decide.

12) Is there a membrane for the dome hole, translucent for our wavelengths, but rigid enough in tension, to give the dome good structural stiffness in spite of its large hole?

13) Maximum deformations should be specified. I am confident that thermal deformations are negligible: the thermal limit for a reasonable 85-ft dish, with white paint in full sunshine, is about λ =5 mm. Gravity is also negligible, with our tilt of only 20°. But wind deformations may call for some extra stiffness. Usually, however, exposed telescopes come automatically close to good pointing accuracy and surface stiffness, once the stiffnes for survival wind is provided.

14) Regarding pointing errors: if our whole system (both mirrors and cabin) makes a rigid-body rotation about the paraxial focus F_1, no pointing error results. A translation in z-direction gives only some axial defocussing which is not so serious. But any lateral (x,y) translation will give a pointing error.

Normally, we specify the pointing error as a small fraction of the half-power beamwidth: $\Delta\varphi \leq B/10$ rms (but mostly have to live with B/6). With D=700 ft aperture and λ=4 cm wavelength, we have $B=D/\lambda$ =39 arcsec. Then $\Delta\varphi \leq B/10=4$ arcsec rms total. We may have n=3 independent comparable contributions: thermal deformations (cables, platform, system), bumpiness of azimuth rails and ring, and wind deformations. For each single one we then demand $\Delta\varphi \leq 4/\sqrt{n}$, or

$$\Delta\varphi(\text{single}) \leq 2.3 \text{ arcsec.} \qquad (22)$$

15) Surface accuracy: normally, we specify $\sigma \leq \lambda/16$ total rms (which I recently could justify as the optimization of "gain per dollar"). Thus, $\sigma \leq 40/16=2.5$ mm rms total. Adopting n=3 independent main parts:

measure and adjust primary mirror, its internal panel accuracy, and the Gregorian system. This gives for each one $\sigma \leq 2.5/\sqrt{3} = 1.44$ mm:

$$\sigma(\text{main part}) \leq 1.4 \text{ mm.} \qquad (23)$$

The upper system may have also n=3 contributions: panel manufacturing, measure and adjust, and deformations (internal and platform); thus, $1.4/\sqrt{3} = 0.8$:

$$\sigma(\text{panel manuf.}) \leq 0.8 \text{ mm.} \qquad (24)$$

This would be for a "balanced" error budget. Usually, however, one tends to demand higher accuracy for smaller parts. Which means getting more accurate from primary to secondary to tertiary.

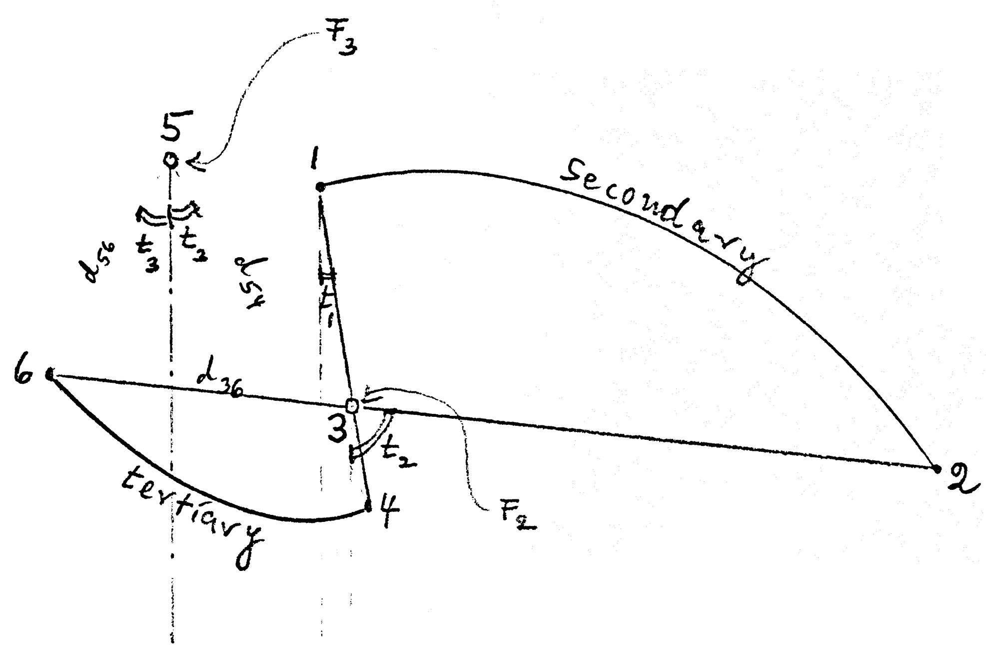
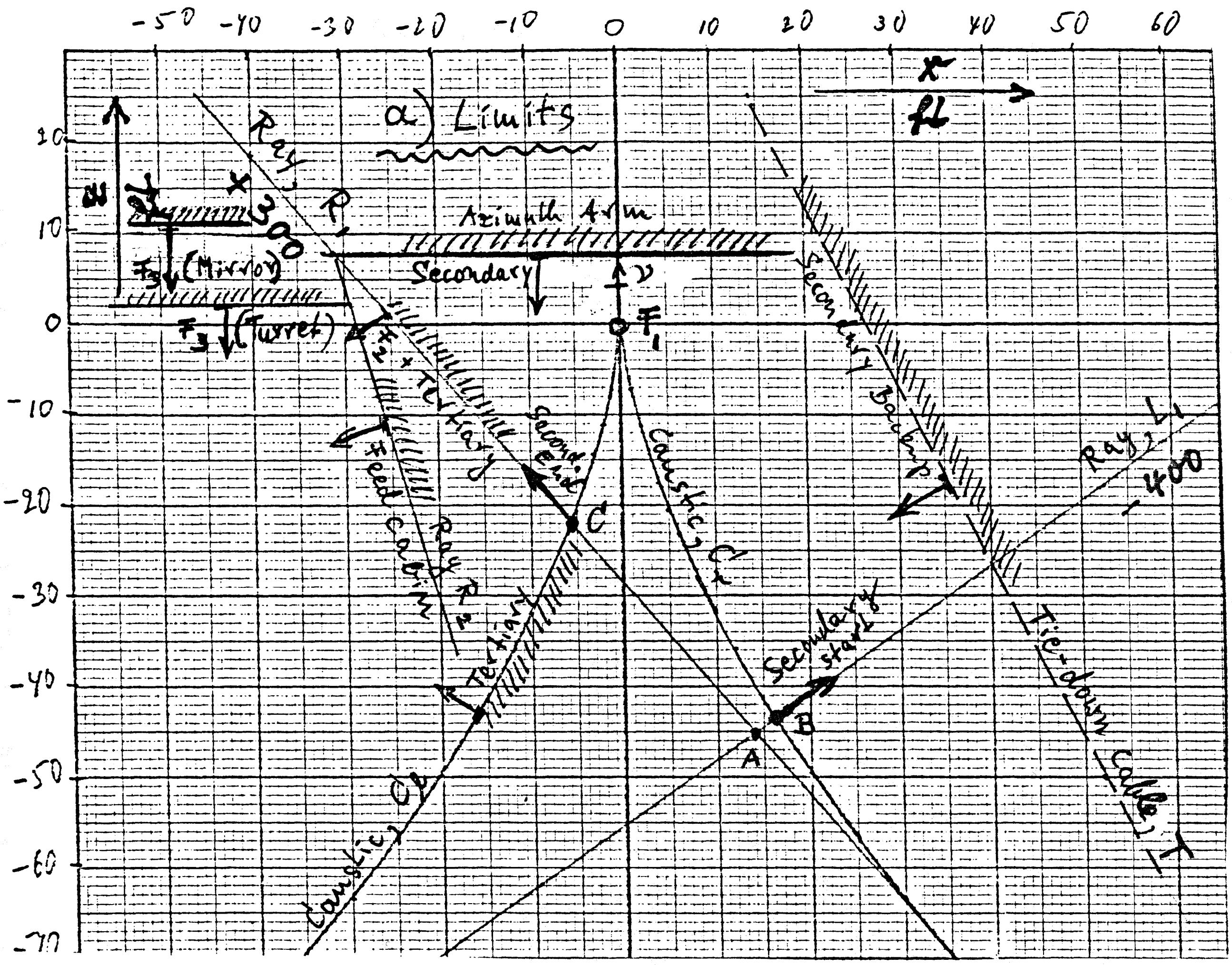


Fig. 1. Definition and numbering of the points to be calculated, equations (1) to (10).

t_i = tangens of the angles shown.



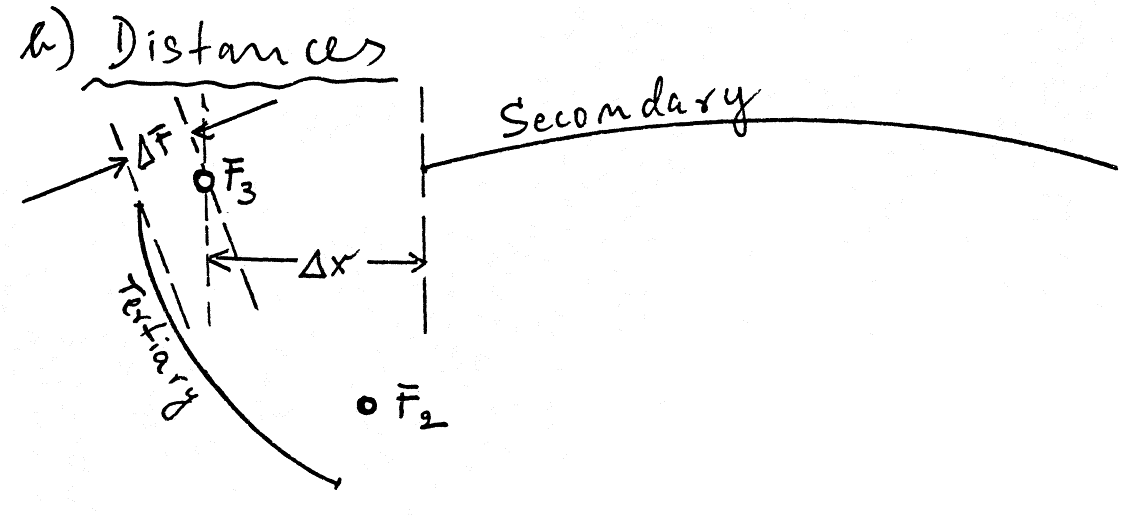


Fig. 2. Definitions of limits and distances, for the conditions required and optimizations wanted. For 700 ft circular aperture, with 50 ft offset.

Ray R_1, L_1: rays of aperture rim, right and left, after one reflection.

Ray R_2: after reflection at secondary mirror.

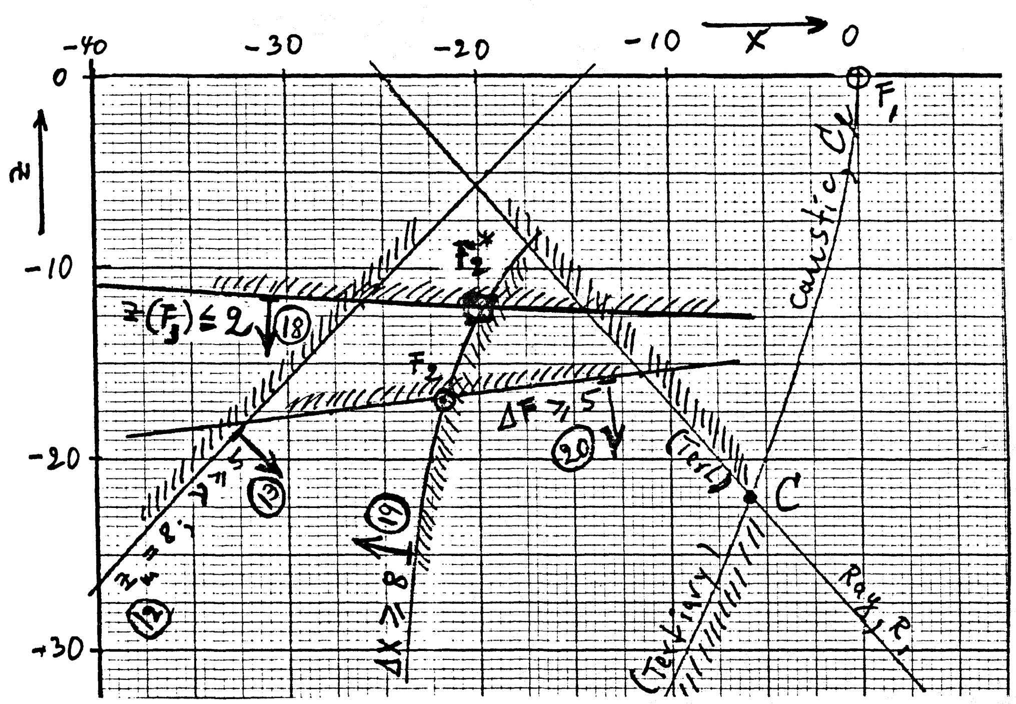


Fig. 3. Selection of coordinates (ρ, τ) for secondary focus F_2. For $\beta_3 = 90^{\circ}$.
All possible locations must be permitted by all constraints;
The optimum (most compact system) wants F_2 to be high and to the right.

constraints, with number (#) from equ. (11) to (20),

permitted direction,

best location of F_2; see system on Fig. 3a,

if (20) were disregarded, then F_2^* were best; see Fig. 3b.

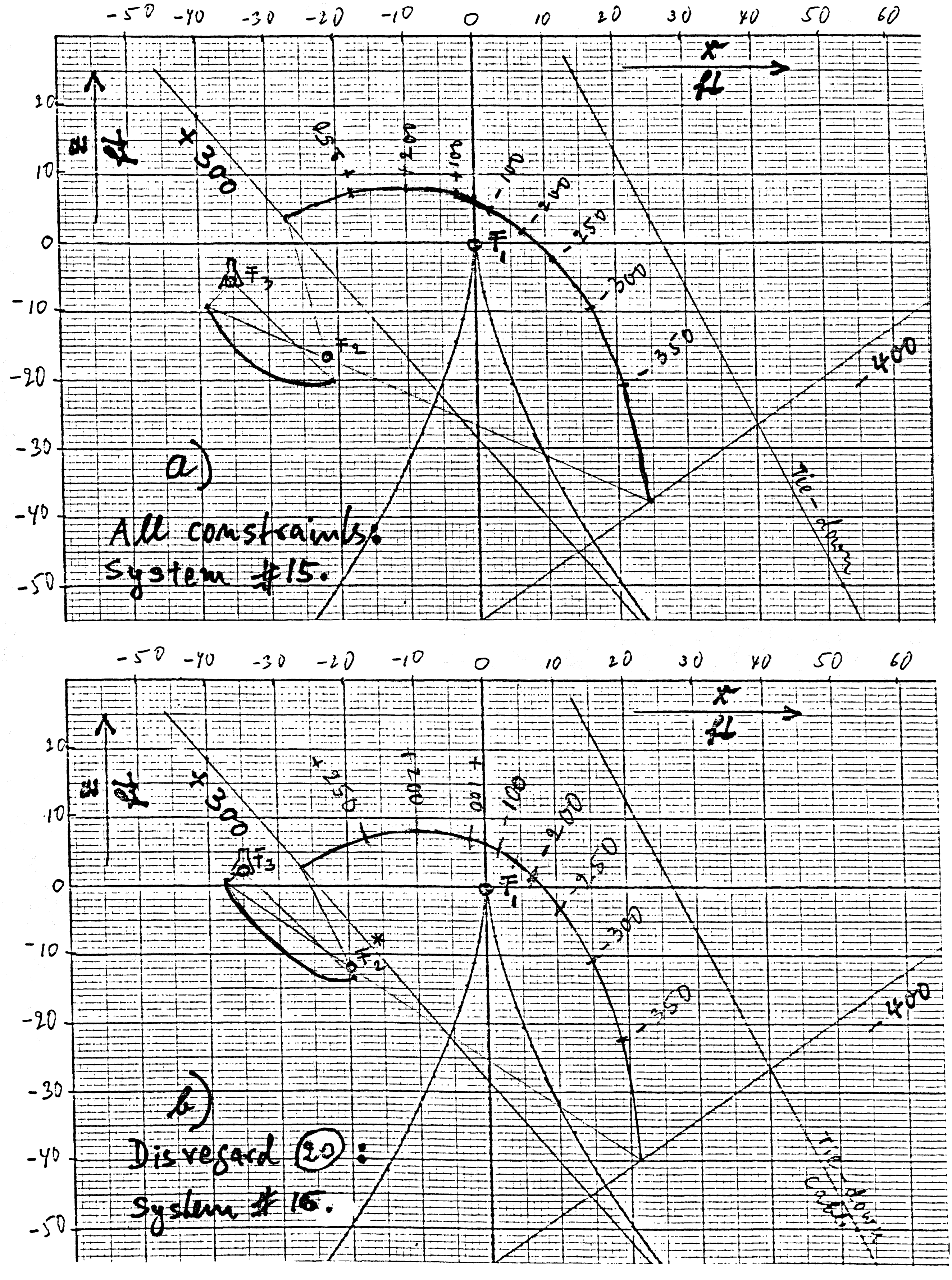
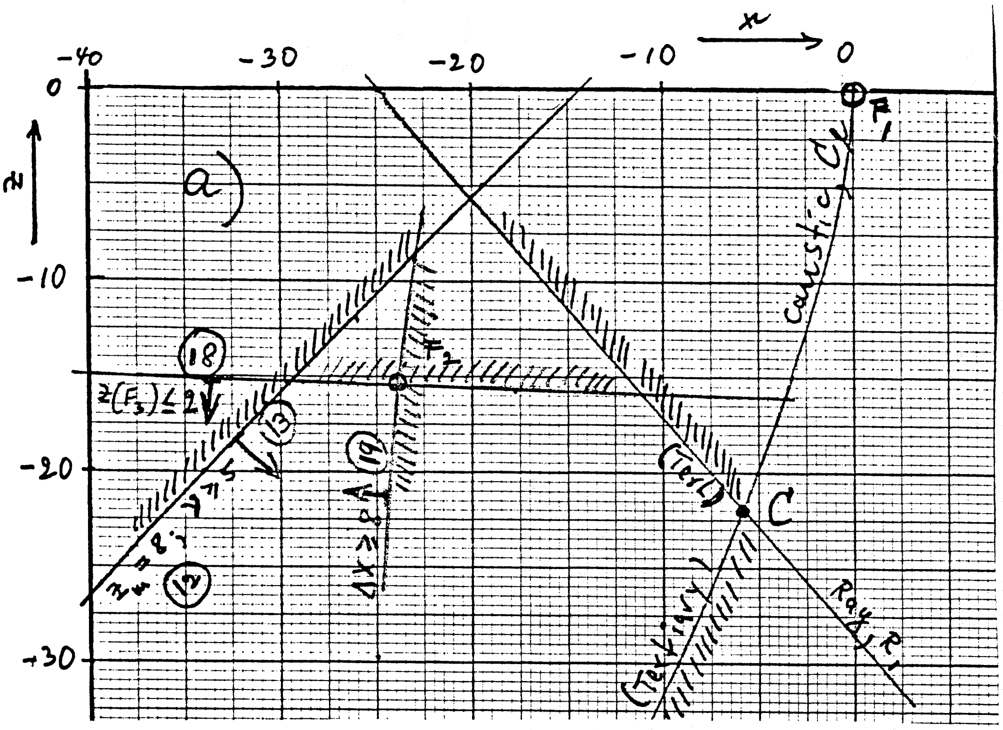


Fig. 4. Optimized system, for feed angle $\beta_3 = 90^{\circ}$.



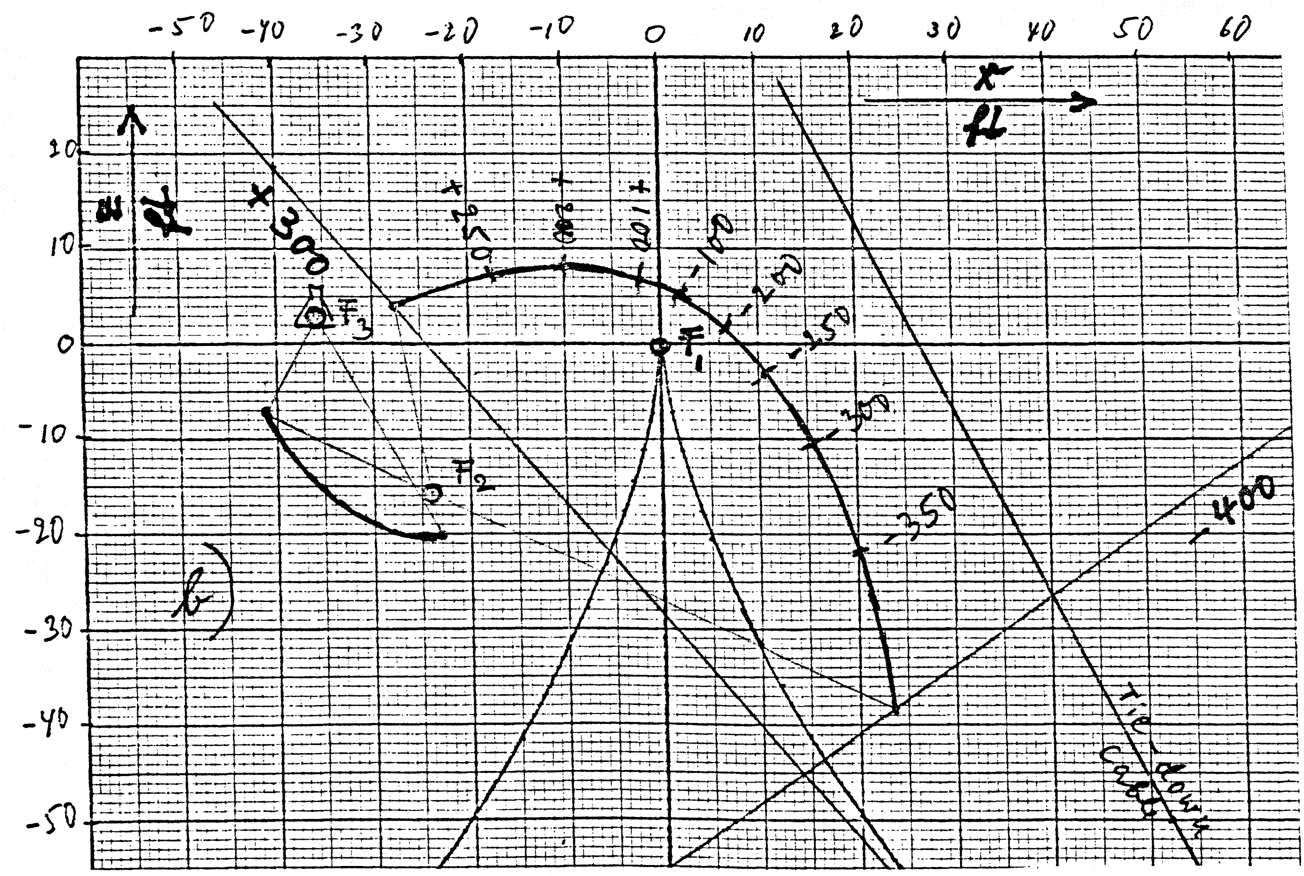


Fig. 5. Optimization for feed angle $\beta_3 = 60°$.

a) Constraints for secondary focus F_2, and selection of best place.

b) System # 17.

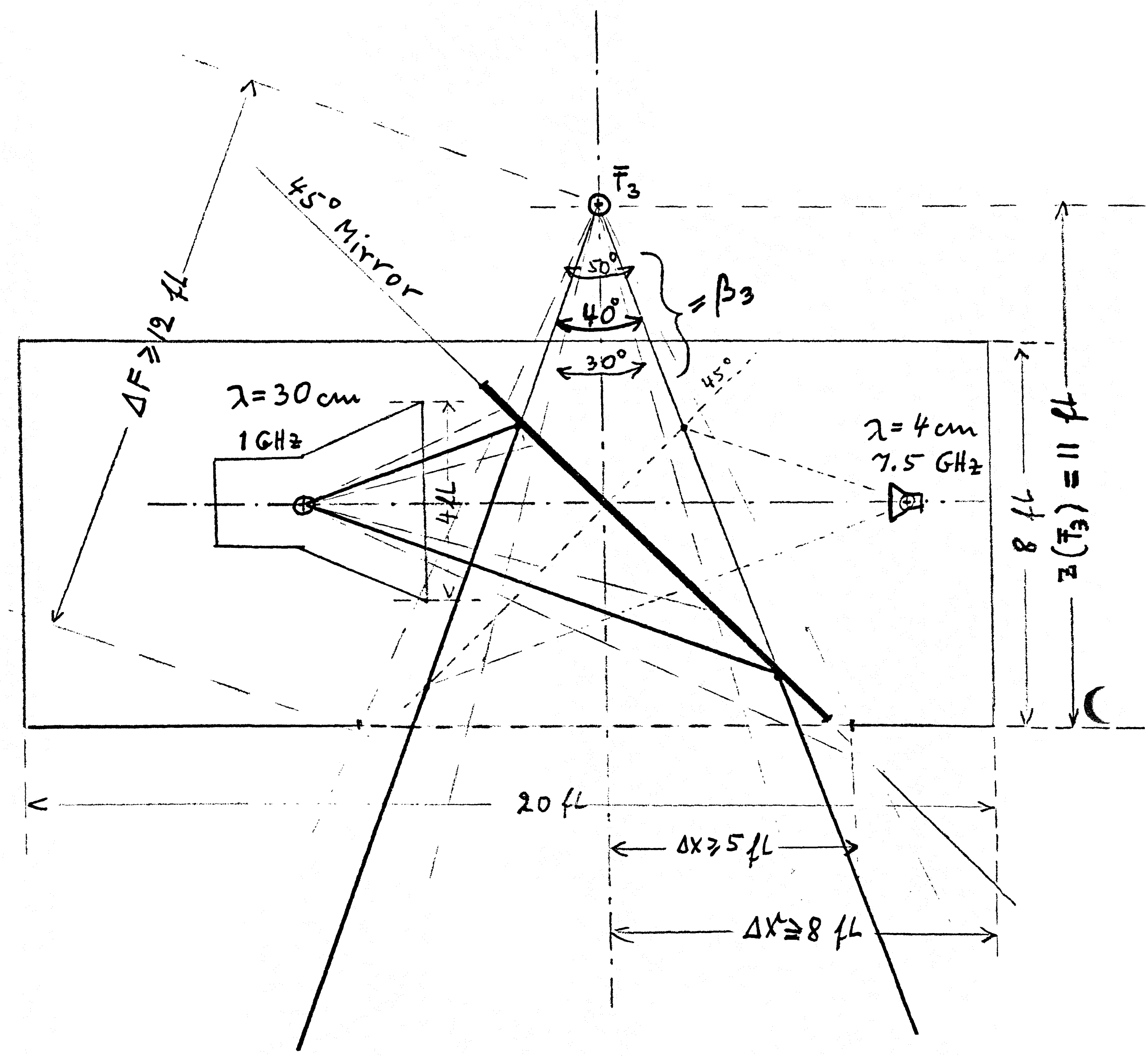


Fig. 6. Selection of the feed illumination angle β_3, to be used with a rotatable 45° mirror and fixed receiver locations in the cabin.

β_3 should be large for avoiding large feeds for long wavelengths, but β_3 should be small for avoiding a large mirror and cabin. We select $\beta_3 = 40^{\circ}$ (and show 30° and 50° for comparison). The mirror is 10 ft long; and the cabin measures 8 x 20 ft, if a short-wave receiver must fit as shown, and 8 x 17 ft if not.

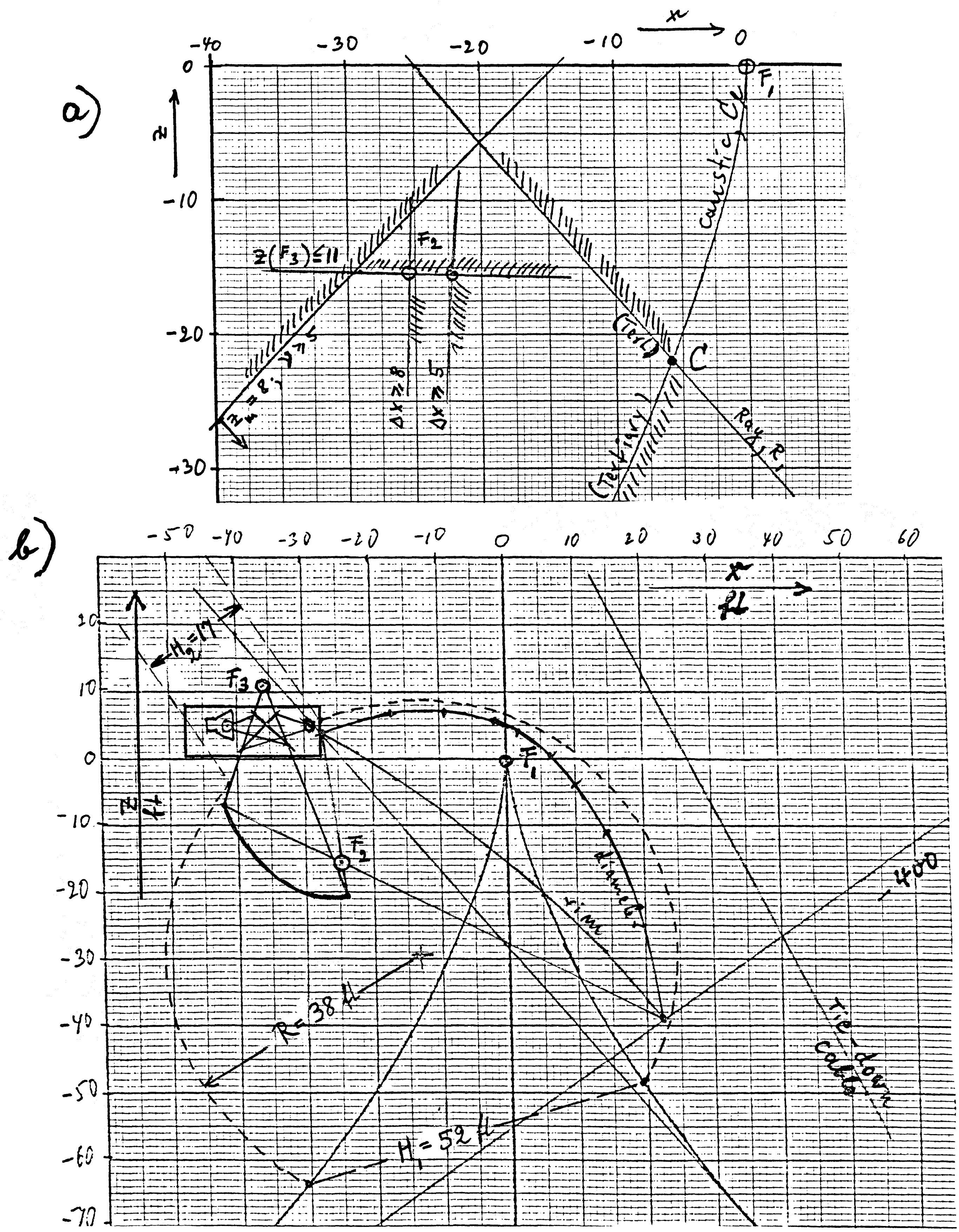


Fig. 7. Optimization for Feed angle $\beta_3 = 40^{\circ}$. With 45° mirror in cabin.

a) Constraints for location of secondary focus F_2.

b) System # 18 with two-sided cabin (Δx = 8 ft).

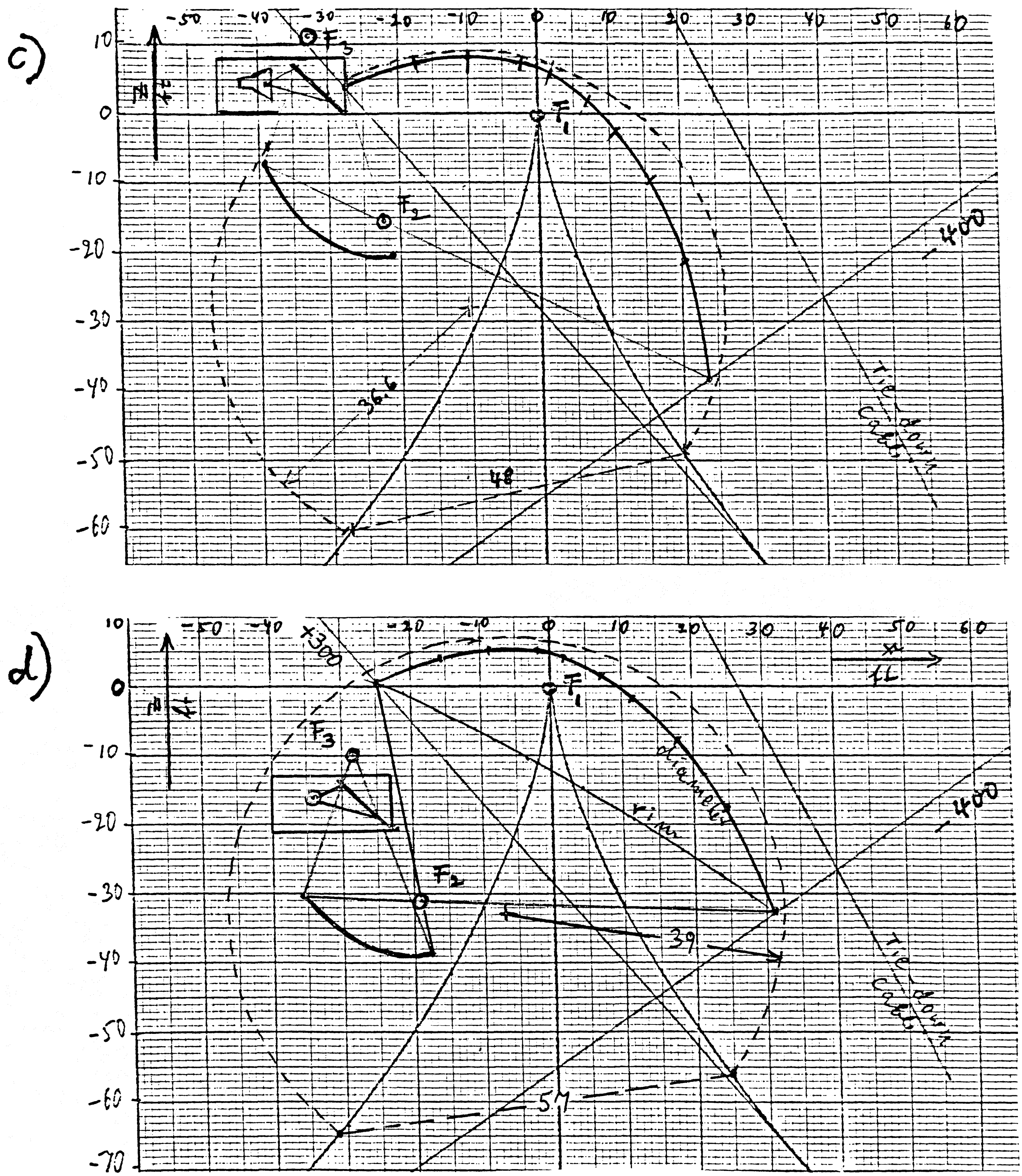
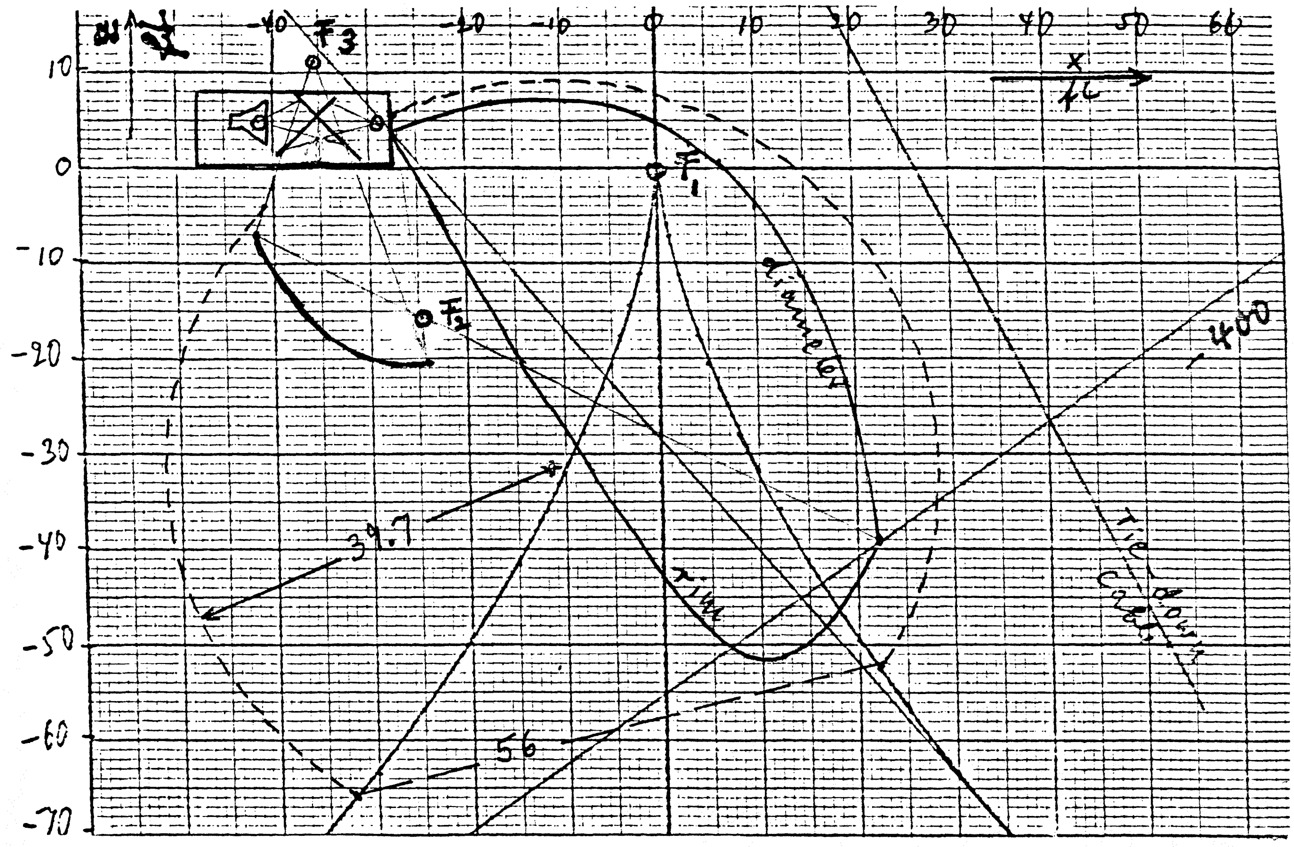


Fig. 7c) System #19 with one-sided cabin (Δx = 5 ft) at azimuth arm.

d) System #20 with one-sided cabin enclosed in radome.

a)



b)

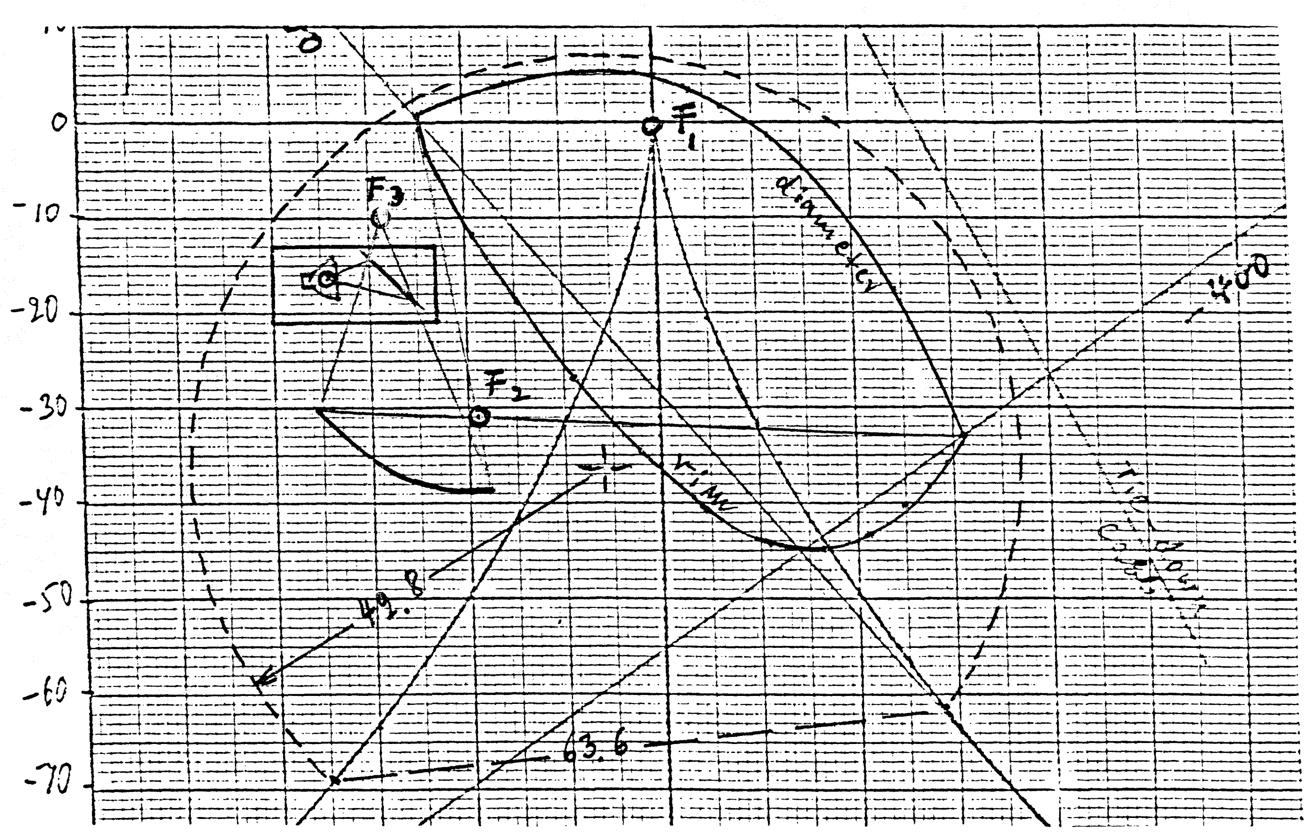


Fig. 8. Two systems with elliptical aperture, 700 and 852 ft diameters. Same vignetting as circular, but 21.6 % more aperture.

a) System #18E, b) System #20E.