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ARECIBO THREE-MIRROR SYSTEMS, V:
APERTURE ILLUMINATION AND SHAPED SURFACES

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APERTURE ILLUMINATION AND SHAPING PROBLEMS

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Summary:

The first study phase has resulted in decisions about many design parameters. The next task will be to develop a shaping procedure which calculates the final surface shapes of the two upper mirrors, for yielding equal pathlength with optimum aperture illumination and without cross polarization.

The present report asks how difficult the problem is which the shaping has to solve, and how this may be eased. We show the illumination properties of the present unshaped systems, and we investigate the dependence of these properties on the design parameters by calculating two series of systems with variable parameters. We find that only one parameter is of direct importance (for illumination as well as polarization): the angular offset between the feed axis and the incoming ray from the aperture center.

This study shows a general problem: we have either very odd illuminations (difficult for the shaping procedure), or very large tertiary mirrors (large expensive domes). Some compromises are suggested. But for the selection of a final system we first must have a shaping program to learn how odd an illumination it can handle.

1. Settled and Open Questions

At the meeting January 1985, several decisions were taken, terminating a first phase of general and preliminary investigations. Since the cost of the secondary will be the main item, about twice the cost of the support cables (P.Stetson), we shall not increase the aperture diameter; and since the ground screen was decided upon, we need not diminish the diameter or the offset. Thus, 700 ft diameter with 50 ft offset were frozen.

Axisymmetric systems shall not be further investigated. But the elliptical aperture of P.S.Kildal was agreed on; maybe with an additional part, "filling-in between the ears" of the Gregorian. The radome suggestion of P.Stetson was accepted.

The longest wavelength shall be $\lambda = 1$ m for sure and $\lambda = 3$ m hopefully (Kildal). The full range needs about 10 receivers, and the largest horn shall be a permanent fixture but acting only as scatter shield for short wavelengths. For changing receivers, the 45°-mirror was abandoned in favor of the rotating turret. The feed range shall be $\beta_3 = 90°$ (size of tertiary as seen by feed).

The most important still open questions call for the development of a shaping procedure and its application to various designs. My suggestion to have a vertical feed axis for avoiding cross polarization was only in analogy to my old two-mirror calculations; this may be actually very different above a spherical primary. And how odd an unshaped aperture illumination can be turned into the desired one by the shaping, we still do not know. But both questions must be answered before we can proceed toward a final design geometry.

2. Aperture Illuminations

For the present purpose we treat only the one-dimensional case, along the aperture diameter. The calculation of the secondary Gregorian was indicated in Report 112 and described in detail in the Appendix of Report 113 (where $\vartheta=0$ for the present case). The coordinates of the tertiary are then found as follows, where we consider only Gregorian tertians, all being ellipses.

We call (x_2 z_2) the coordinates of focus F_2, call (x_3 z_3) those of F_3, and R the distance between F_2 and F_3. The tertiary has one more free parameter, its size, which we fix by the choice of m, the smallest distance of F_3 from the full ellipse of which the tertiary is a part, see Fig.2. The sum of the two distances to F_2 and F_3, from any point of the elliptic tertiary, then is

$$L = R + 2m. \tag{1}$$

We call $c=x_3-x_2$ and $g=z_3-z_2$. Any ray, after reflection at the secondary, has the angle β_s from the z-axis; and along this ray, the tertiary is at distance k from F_2.

We omit the derivation and give the result. The distance k is

$$k = \frac{L^2 - R^2}{2(L + g\cos\beta_s + c\sin\beta_s)}, \tag{2}$$

and the coordinates of the tertiary are

$$x_t = x_2 - k\sin\beta_s, \tag{3}$$

$$z_t = z_2 - k\cos\beta_s. \tag{4}$$

At the feed, the angle beteen the incoming ray and the z-axis is found from

$$\tan \beta_f = (x_t - x_3)/(z_3 - z_t). \tag{5}$$

First, we calculate β_f only for the two extreme rays, a=-400 ft and a=+300 ft, and for the aperture center, a=-50 ft. We obtain the feed range, $\beta_3 = \beta_f(-400)-\beta_f(300)$, and the direction of the feed axis, $\beta_a = [\beta_f(-400)+\beta_f(300)]/2$. The difference $\Delta\beta = \beta_f(-50)-\beta_a$, between the feed axis and the incoming ray from the aperture center, we call the feed offset, and the <u>relative feed offset</u> we call

$$B = \Delta\beta/\beta_3. \tag{6}$$

This we regard as our first measure of the difficulty to be faced by the shaping procedure. If the demand of zero cross polarization means that an orthogonal polar aperture grid must be mapped onto the feed pattern again as an orthogonal polar grid, then the aperture center must be mapped onto the feed axis, or

$$B = 0, \quad \text{after shaping.} \tag{7}$$

Second, we call for any ray its centered feed angle

$$\beta_c = \beta_f - \beta_a. \tag{8}$$

We assume a <u>Gaussian</u> feed pattern

$$p = \exp[-(\beta_c/\beta_t)^2/2] \tag{9}$$

and we adopt a <u>20 db edge taper</u>, which gives the standard deviation of the feed pattern as

$$\beta_t = \beta_3/(4\sqrt{\ln 10}). \tag{10}$$

Third, we go along the aperture diameter, in steps of Δa. After each step, we call $\Delta\beta_s$ and $\Delta\beta_f$ the resulting angular steps at F_2 and F_3, and now we define the aperture illuminations as

$$i_2 = 7.592\ (\Delta\beta_s/\Delta a), \quad \text{from omnidirectional feed at } F_2; \tag{11}$$

$$i_3 = 7.592\ (\Delta\beta_f/\Delta a), \quad \text{from omnidirectional feed at } F_3; \tag{12}$$

$$i_t = p\ i_3, \quad \text{from tapered feed at } F_3. \tag{13}$$

The normalization, of $7.592 = (r/2)(2\pi/360°)$ with r=870 ft, serves to let $i_1=1$, at a=0, from omnidirectional feed at F_1.

After completion of the full aperture diameter, we look for the maxima and minima of i_2, i_3 and i_t, and we call

$$I_k = \max(i_k)/\min(i_k) = \text{illumination ratio}. \tag{14}$$

We regard I_t as another measure of the difficulty faced by the shaping procedure (mostly underestimated since we have used large steps of Δa=5o ft, except for detailed drawings). Not only the magnitude of I_t, but also any strong asymmetry of $i_t(a)$ indicates a difficulty, since we must have

$$i_t(a) = \text{constant, and} \quad I_t = 1, \quad \text{after shaping}, \tag{15}$$

if we want uniform aperture illumination for maximum gain; or we must have a slightly tapered but still symmetrical $i_t(a)$ if we want smaller sidelobes.

3. Our Previous Systems

As a typical example, we show in Fig.1 our system #15 of Report 120. It was defined by a feed range of β_3=90°, a vertical feed axis, and the constraint that the tertiary does not come too close to F_2. We see in Fig.1b that i_2 has the well-known inverse taper of a Gregorian over a spherical primary. The tertiary does give some but not much distortion: i_3 is similar to i_2 but somewhat more asymmetric. However, the shape of $i_t(a)$ is very odd and asymmetric, and it will be very interesting to find out whether or not a shaping procedure can handle a case like that. Also, the feed offset is very large, $\Delta\beta=31.6°$, or B=.351; which can be seen in Fig.1b along the aperture diameter as the large offset of the feed axis ray from the aperture center.

Table 1. Several previous systems.
Ordered with increasing illumination ratio I_t.
(#15 to 18: Report 120; #22: its Addendum; L.Baker: drawing 12-31-84)

name	β_3	$\Delta\beta$	B	R	m	m/R	φ	I_2	I_3	I_t
#18	40	9.71	.243	28.72	4.87	.170	139	23.5	52.8	77
#17	60	16.9	.281	21.17	4.30	.203	121	19.7	57.3	125
L.B.	84	27.6	.328	24.0	6.0	.25	136	12.5	67.4	251
#22	120	40.1	.334	14.0	4.74	.339	180	22.3	77.9	334
#15	90	31.6	.351	17.54	3.15	.180	140	17.4	93.0	346
#16	90	36.4	.404	20.40	1.84	.090	137	19.6	185.5	1978

Table 1 summarizes for six previous systems their design parameters and the resulting illumination ratios, with φ = angle between the x-axis and the direction from F_2 to F_3. The table is ordered with increasing I_t and we look for correlations. There are only two significant ones. First, I_t is well correlated with I_3 (but not with I_2). This means, hopefully, whatever system geometry we select as a good one for this feed, will also be good for other feed patterns.

Second, the only design parameter of direct significance for I_t seems to be the <u>relative feed offset</u>, $B=\Delta\beta/\beta_3$.

4. Two Series of Systems

We found earlier that B should not be too large since it must be zero after shaping, for avoiding cross polarization. And we just found that B again should be small because it (and only it) is strongly correlated with I_t which must be unity after shaping. We now want to investigate: whether I_t really depends only on B, and how to keep B small.

We reduce the number of variables. Since we see no correlation beteen I_t and I_2, it seems that the location of F_2 has no direct influence, so we keep it fixed. We also fix R since it seems of no direct importance. F_3 then lies on a circle of radius R about F_2 as shown in Fig.2. We consider angle φ as our independent variable of a series of systems, and then we have two choices and use both: eiter, we choose a fixed-size tertiary ellipse (allowing only conveniently small tertiary mirrors). Or, we choose a fixed range β_3 (of 90° as selected best for long wavelengths). In the first case, range β_3 is a dependent variable, and in the second case it is the size of the tertiary ellipse.

For the first series, Fig.3 shows the illumination ratios and the feed range. We have chosen R=16 ft, and m=6 ft for the tertiary, which gives R+2m=28 ft for the long diameter of the ellipse and this then is the upper limit for the size of any tertiary.

Angle φ is an important design parameter with strong influence on I_t and β_3. For a convenient location of the feed cabin at Arecibo, we should consider only a limited range of about $60° \leq \varphi \leq 150°$. Fig.3 then demonstrates a problem: angle φ should be small for obtaining a small I_t, but then the range β_3 is too small. And a range of 90° gives I_t=100 which is larger than wanted (but hopefully still managable?) regarding the shaping.

The second series is shown in Fig.4, for a fixed feed range of β_3=90°. We see a similar problem: I_t and B are nice and small, but only for small φ where the tertiary will be large, and vice versa. As it seems, the main problem is not so much the illumination I_t, but B itself for the polarization.

Finally, Fig.5 confirms that the illumination ratio I_t does depend on the offset B, but on not much else. We added a third series of the second type, with fixed β_3=60°, wich also agreed with the two other series. Also the six previous systems from Table 1 fit in, although they have different R, F_2 and secondary mirrors. Thus, for the choice of a future final system, we may just concentrate on getting a small B (within the necessary constraints).

5. Four Examples of the Second Series

Since the feed range of β_3=90° has been agreed on (limited horn size for longest wavelength), we show some details of the second series, selecting four instructive cases.

Fig.6 with φ=140° is the case of a small tertiary, diameter d_3=20 ft only, but with fairly large I_t=139, and rather large B=.288 showing in Fig.6b as the very large offset of the feed axis ray from the aperture center, probably difficult to handle for the shaping.

Fig.7 with φ=120° looks much better with only I_t=22.3 and B=.154, but now the tertiary is rather large with 38 ft diameter. It seems then that a compromise would lie between Figs.6 and 7, at about φ=130°.

Fig.8 with φ=90° serves to show the smallest illumination ratio, I_t =9.24, with B=.074 only, but having a ridiculously large tertiary. Finally, Fig.9 with φ=-10° is the case with B=0 before shaping, but with an impossible tertiary cutting even into the caustic.

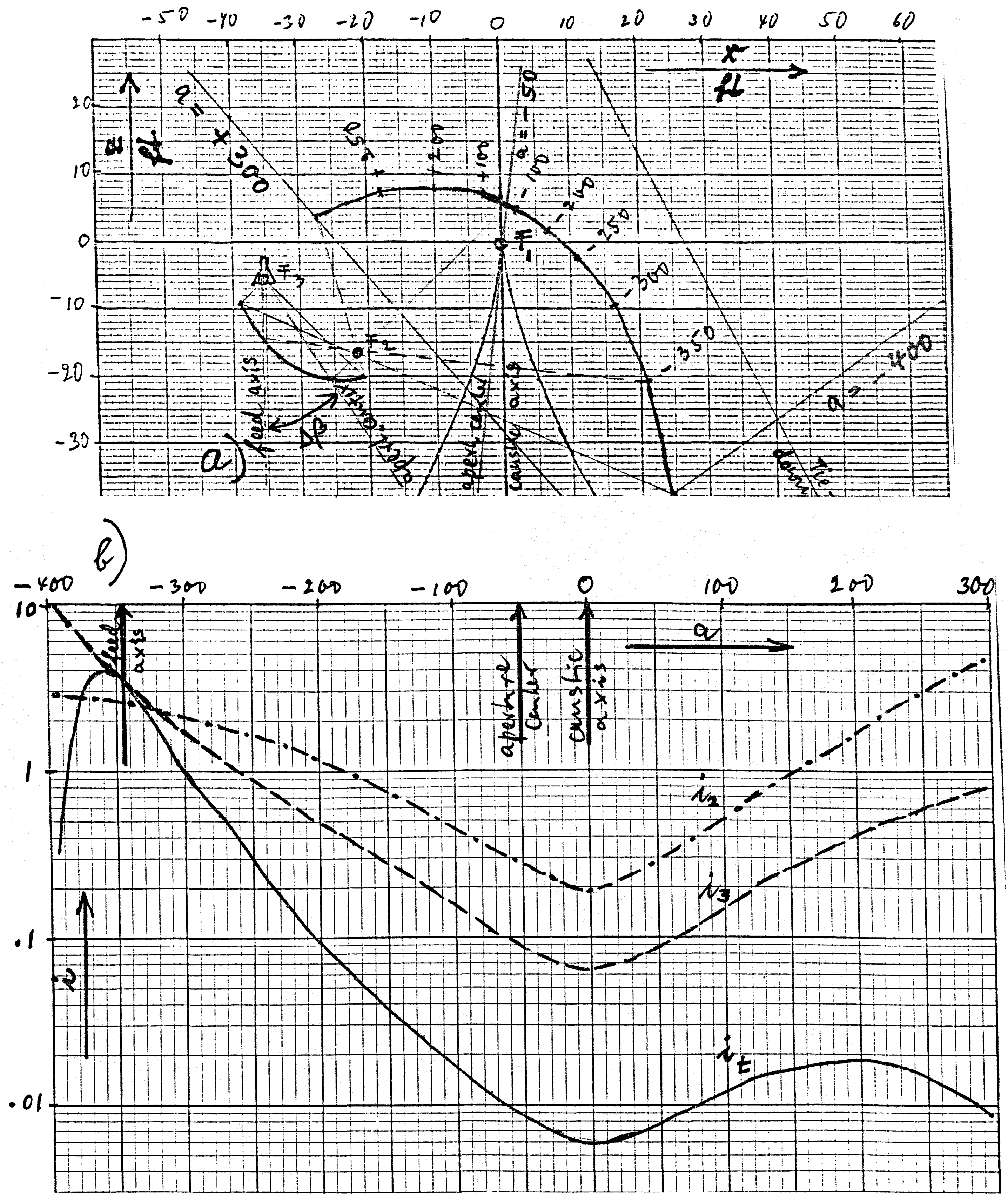


Fig.1. System # 15 of Report 120. a) Its geometry, defined by feed range $\beta_3 = 90^{\circ}$ and a vertival feed axis. b) The illumination across the aperture diameter: i_2 from omnidirectional feed at F_2; i_3 same from F_3; i_t from feed with 20 db edge taper at F_3. Normalisation: i=1 from omnid. feed at F_1 on axis.

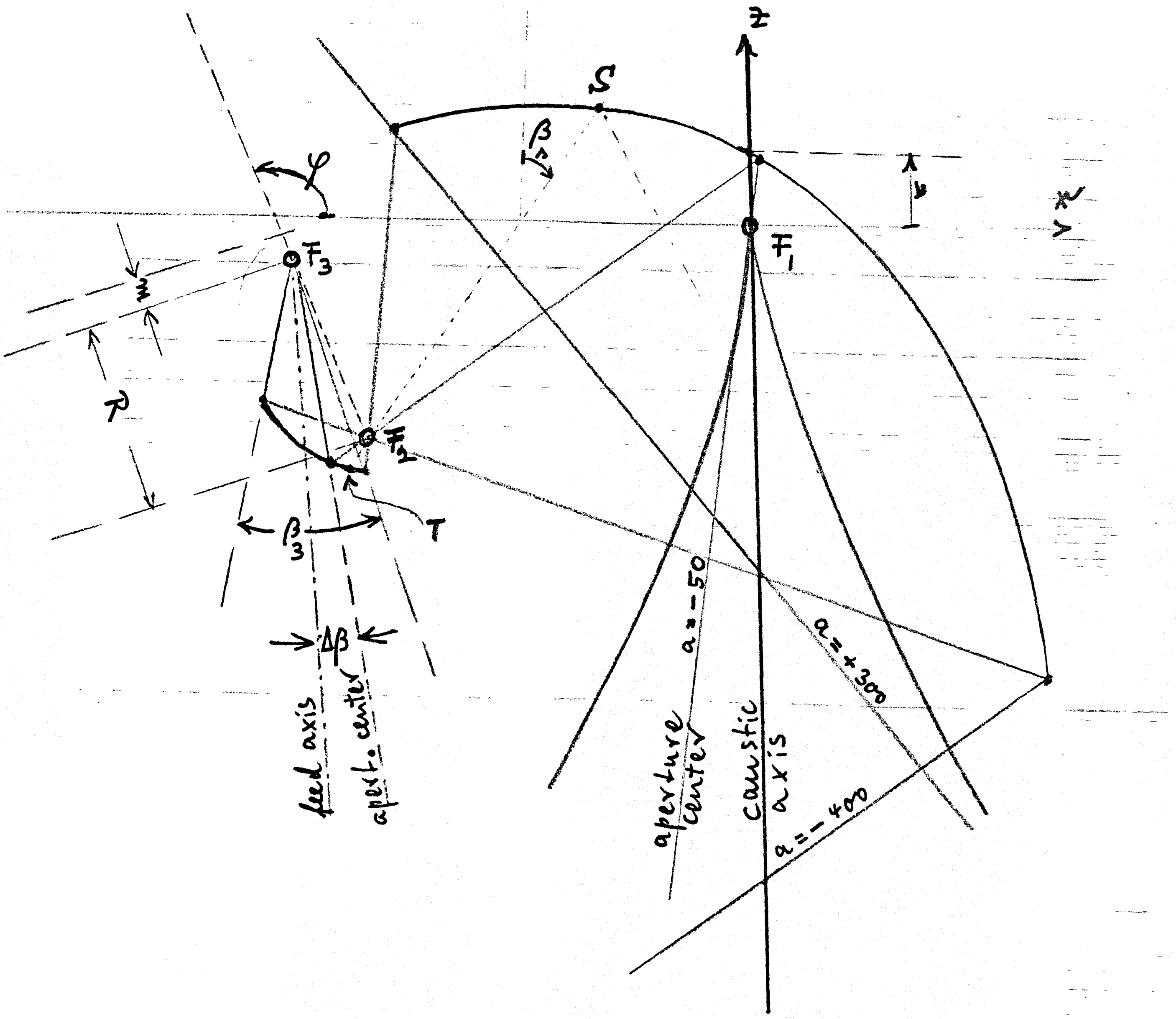


Fig.2. Two series of systems, to find the dependence of the illumination on the design parameters.

All systems have the same secondary mirror, defined by the location of F_2 and the height of v. All systems have F_3 (the feed) on a circle about F_2 with radius $R = 16$ ft, but at various angles φ from the x-axis.

The first series has a fixed-size tertiary ellipse, with $m = 6$ ft.

The second series has a fixed feed range, of $\beta_3 = 90^\circ$.

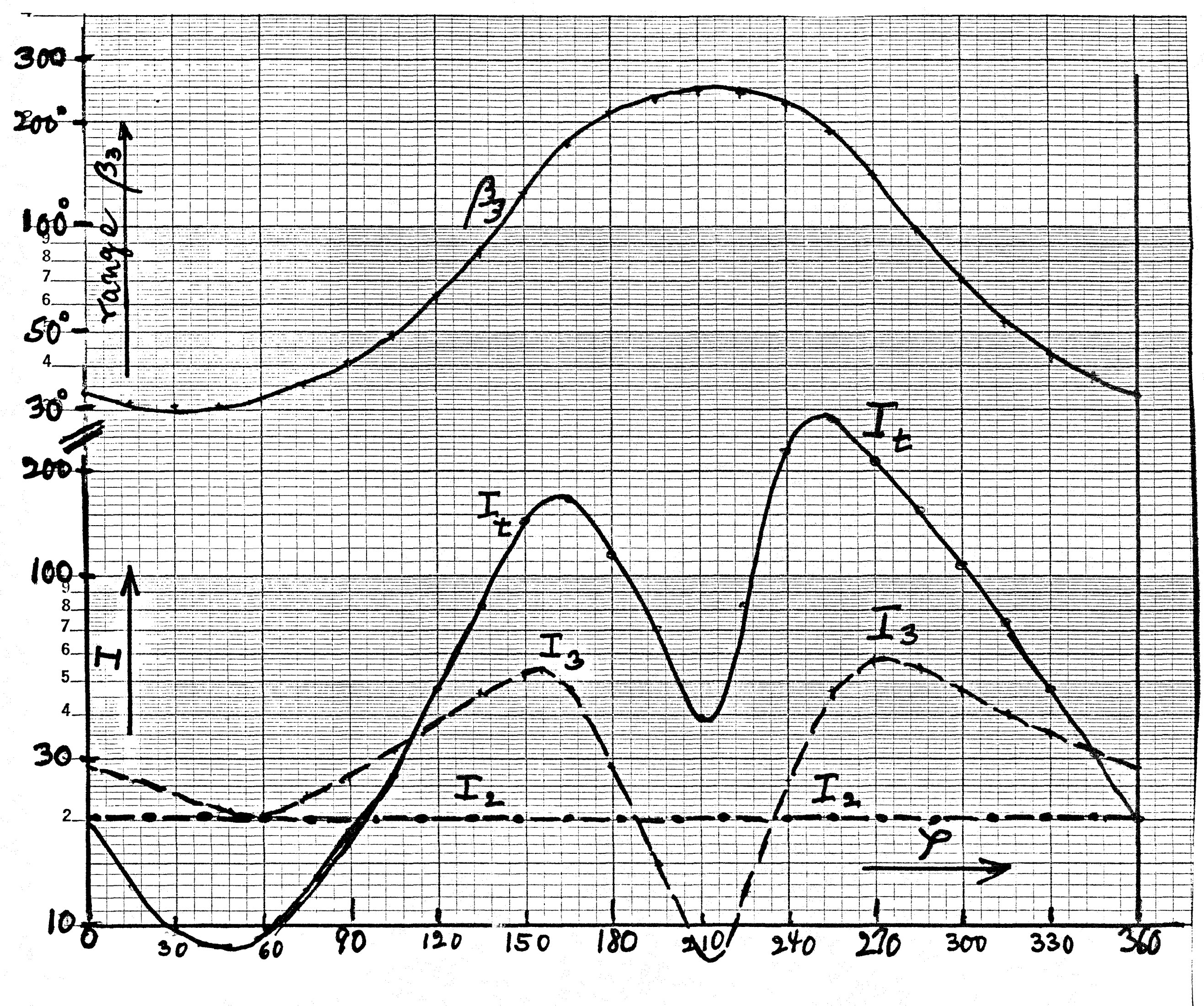


Fig.3. The first series: small tertiaries, with m = 6 ft.

For each angle φ of Fig.2, the illuminations $i_k(a)$ were calculated as defined in Fig.1, and their maxima and minima were noted. The illumination ratio I then is defined as $I_k = \max(i_k)/\min(i_k)$. These ratios show a very strong dependence on φ, and so does the feed range β_3.

Regarding the application to Arecibo, especially the location of the feed cabin, only a small range of φ would be possible, about $60^\circ \leq \varphi \leq 150^\circ$. We may have I_t very nice and small, but then β_3 is too narrow. And choosing $\beta_3 = 90^\circ$ would give a large $I_t = 100$.

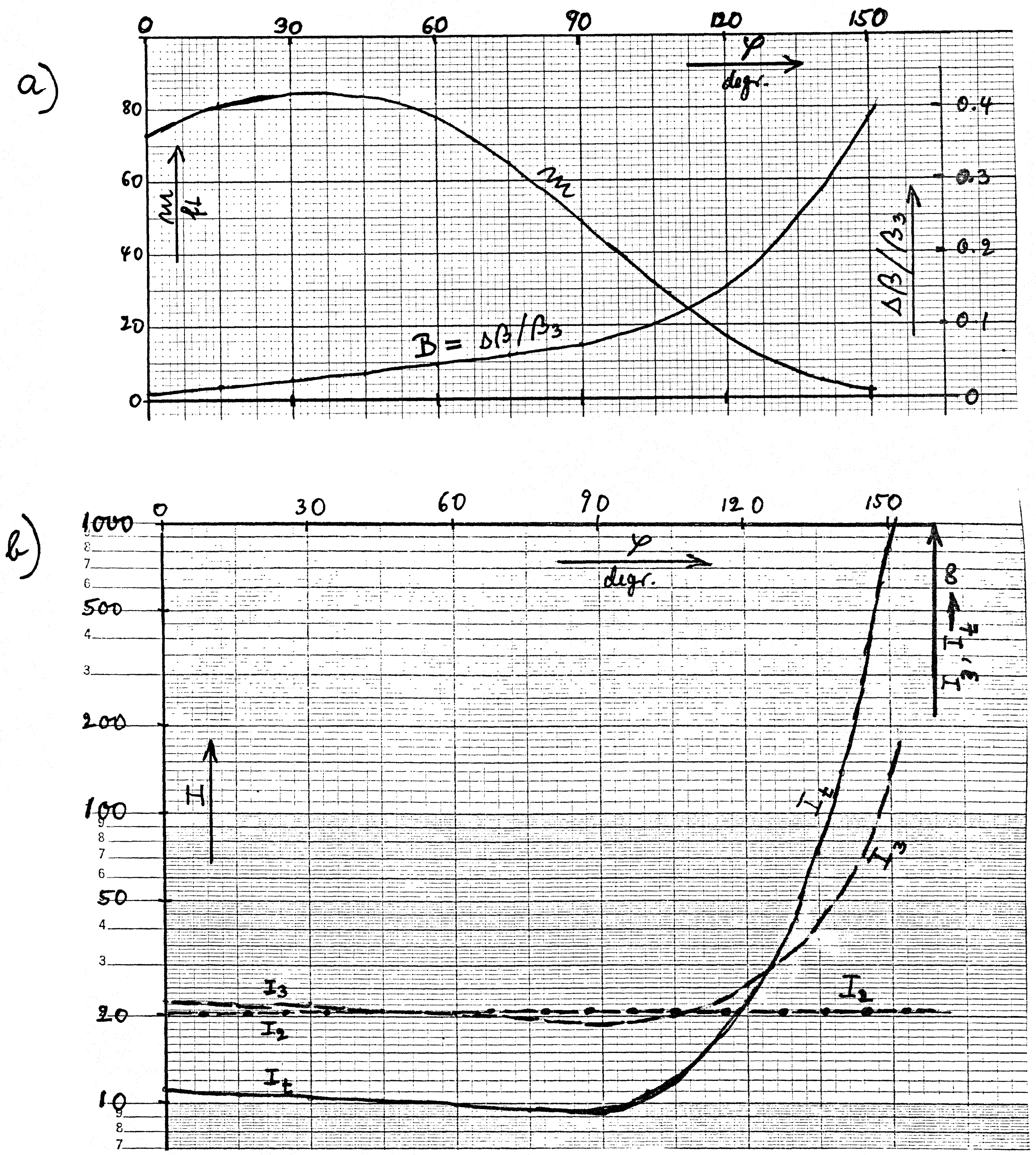


Fig.4. The second series: large feed range, with $\beta_3 = 90^\circ$.

a) Values of m as needed for $\beta_3 = 90^\circ$; and resulting relative feed offset B.

b) Illumination ratios I_k as functions of φ.

Feed offset B and illumination I_t can be made small, but then the tertiary mirror gets large (large m).

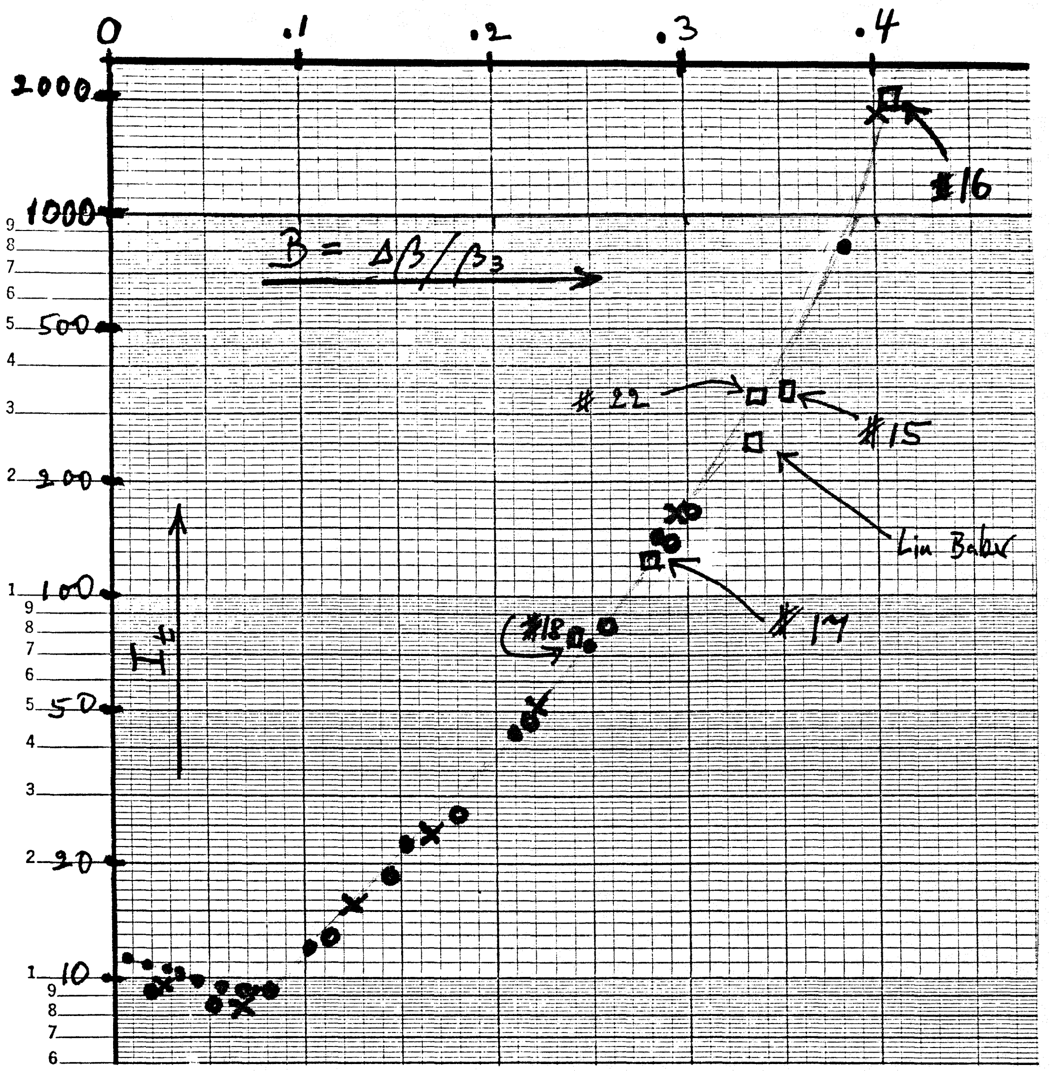
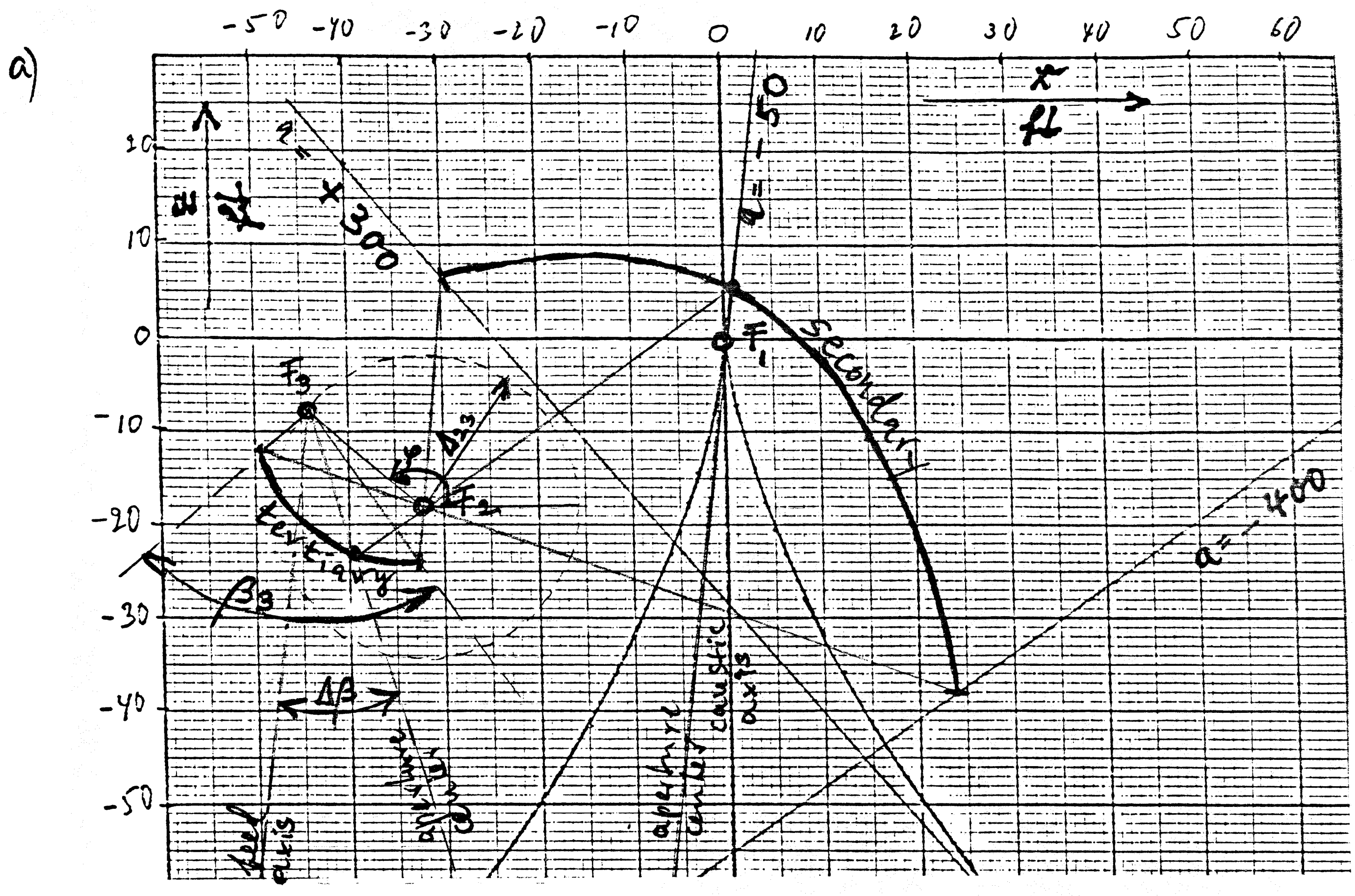


Fig.5. The illumination ratio I_t seems to depend only on the relative feed offset B. A third series with $\beta_3 = 60^{\circ}$ was added and it also agreed. The six previous systems (see Table 1) even have different secondary mirrors

	series	β_3	m/R	φ
o	first	29° ... 124°	0.375	
•	second	90°	.116 ... 5.14	0 ... 150°
×	third	60°	.060 ... 1.20	



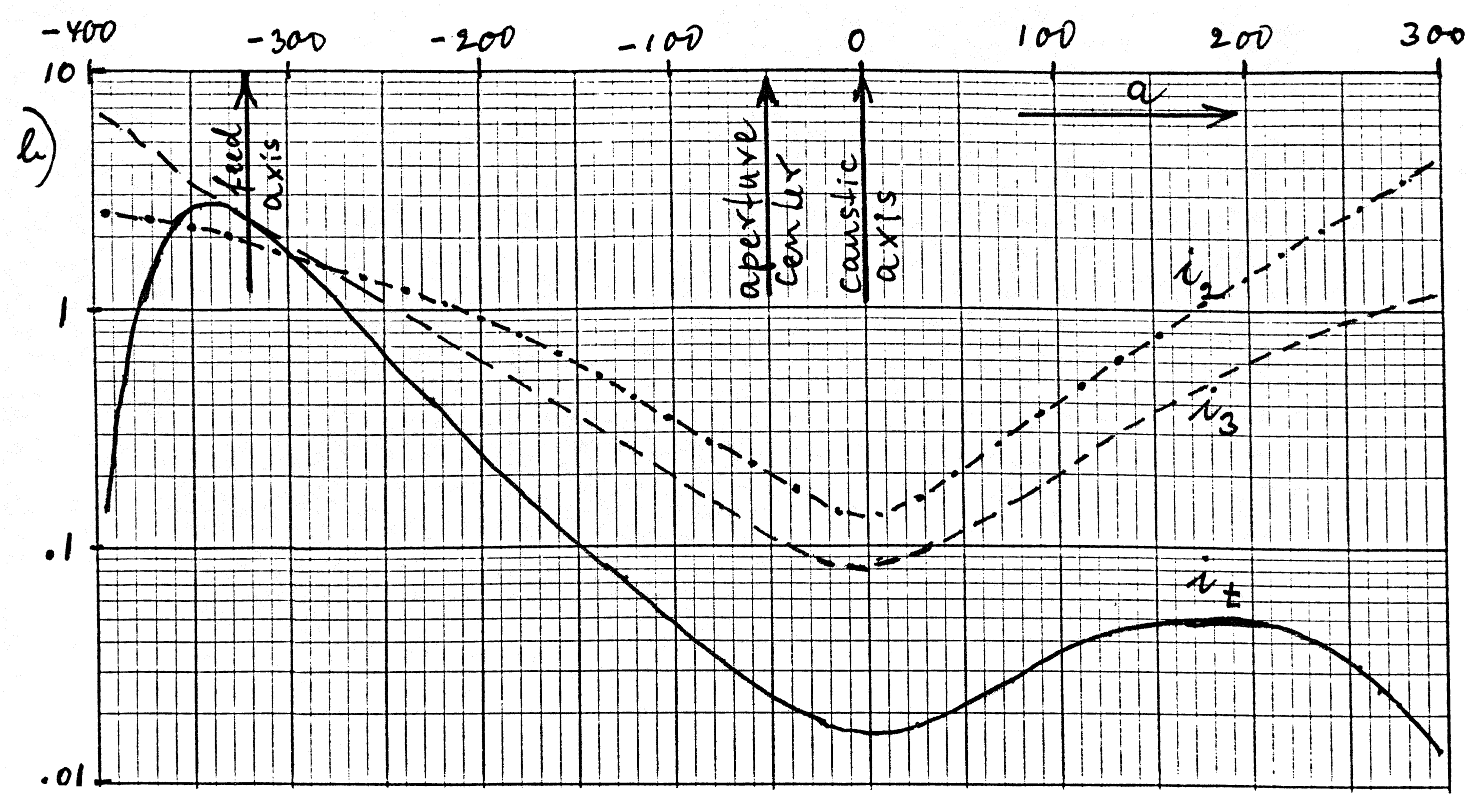


Fig.6. Second series, with $\varphi = 140^{\circ}$. a) Geometry: small tertiary, but large $\Delta\beta$.
b) Aperture illumination: i_t varies very strong and asymmetric.

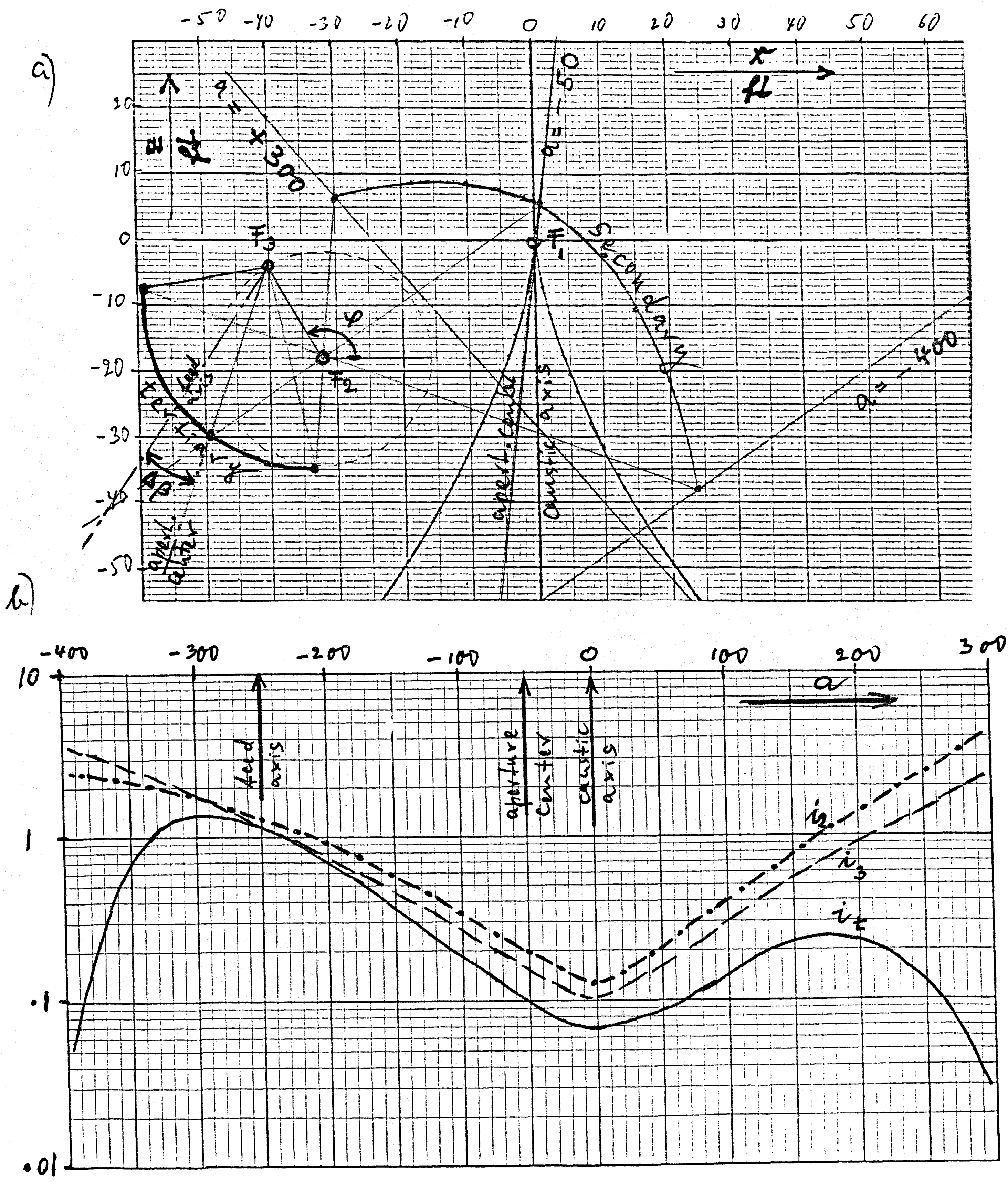


Fig. 7. Same, with $\varphi = 120^\circ$.
Improved i_t; may be useful after some changes (smaller tertiary).

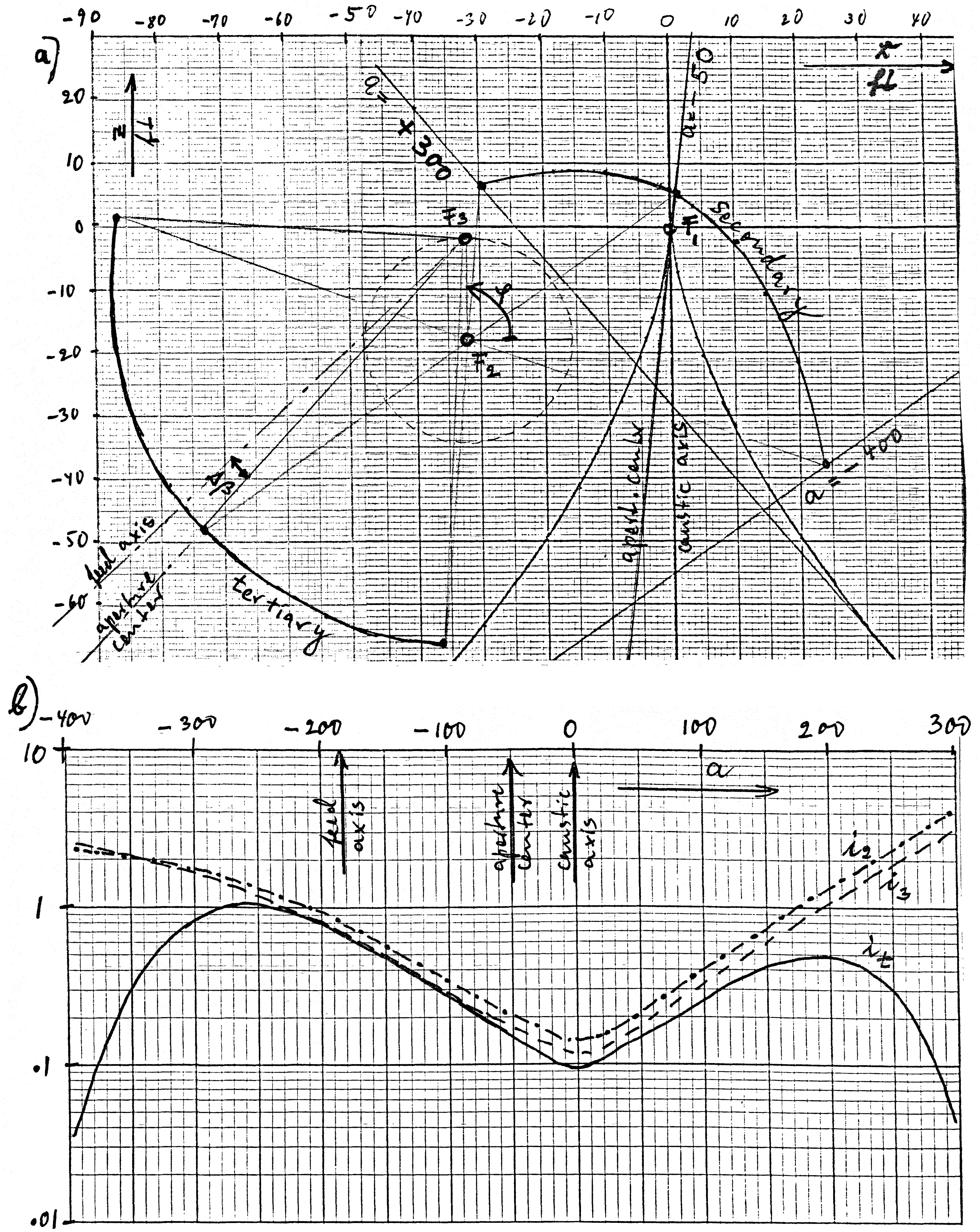


Fig. 8. Second series, with $\varphi = 90°$. Tertiary much too large. Smallest I_t.

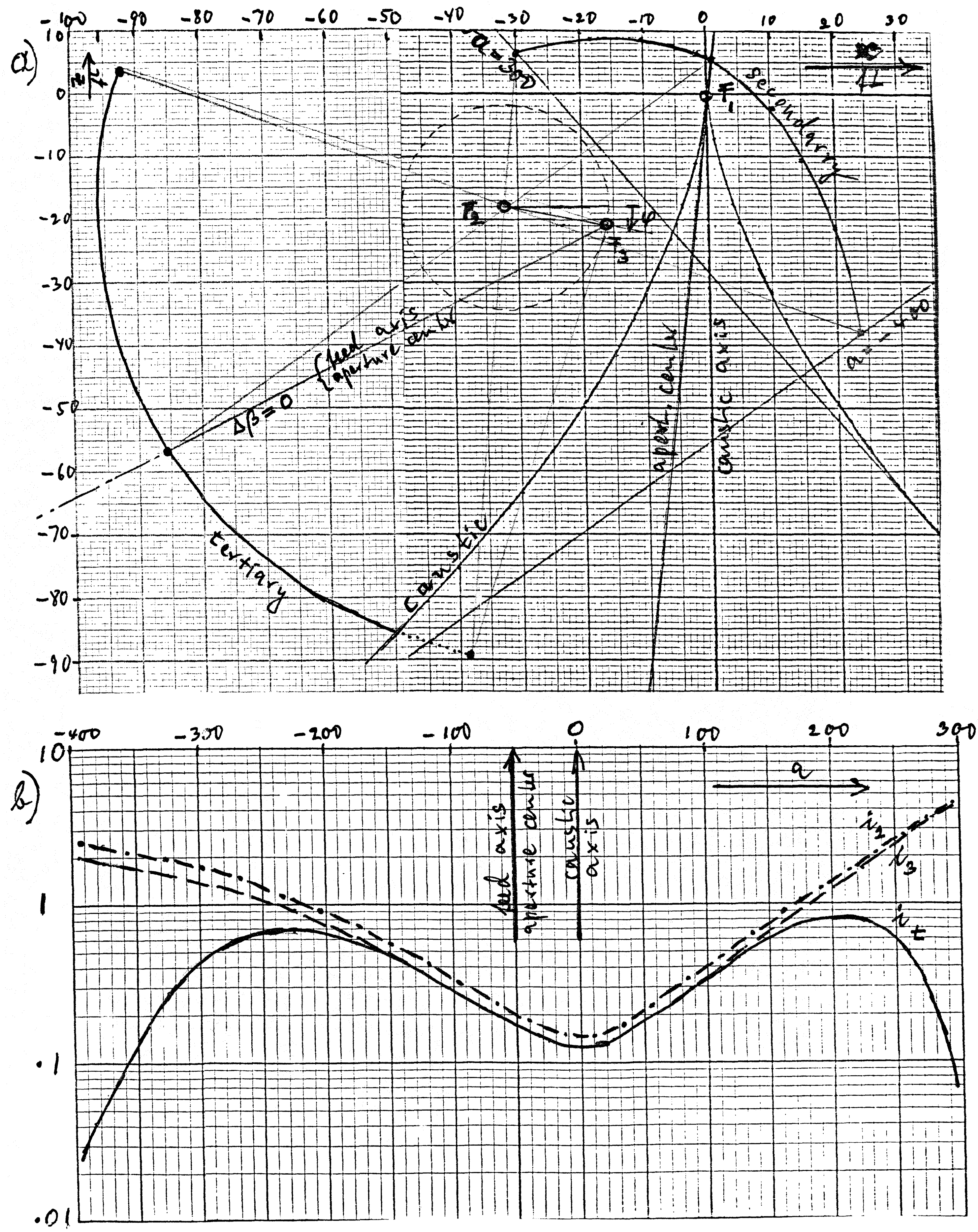


Fig.9. Mapping of aperture center onto feed axis would need $\varphi = -10^\circ$.