ENGINEERING MEMO #119

140-ft Memo
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140-ft Beam Shape at Short Wavelength

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In his Memo of Oct. 28, 1977, Barry Turner gives 11 figures of beam mapping, obtained at λ = 1.35 cm at various pointing directions; 5 maps show a splitting into 3 beams of more or less comparable height, 3 maps split into 2 beams, and the remaining 3 maps show shoulders. Unfortunately, no optimization of the focal adjustment was done. In the following, we try to derive a tentative understanding of these disturbing results, and we make some suggestions for further observations.

1. The Data

We have reduced the size of the 11 beam maps, and have put them in Fig. 1 at their proper place in an (H,D)-plane of the available sky. In each map, we calculate the direction of gravity according to

$$\sin\gamma \;=\; \cos L\,\sin H\,/\cos E \qquad (1)$$

where L = 38.4°, H = hour angle, D = declination, and E = elevation. Coordinates and γ are given in Table 1. In Fig. 1, the direction of gravity is not perpendicular on the line of constant elevation (as one might expect), because the large (H,D)-plane is distorted, whereas the small (HcosD, D)-maps of Turner are undistorted such that a round beam appears round for all declinations.

Table 1 gives the height of the single lobes (shoulders), and their sum derived as

$$S \;=\; \Sigma\text{lobes} + \tfrac{1}{2}\,\Sigma\text{shoulders}. \qquad (2)$$

As a fast but very crude estimate of the observed gain loss ℓ_o (no direct

__Table 1.__ The 11 beam maps of Turner, their positions, and lobe heights. D = Dec, H = hour angle, E = elevation, γ = angle between gravity and declination in (HcosD, d)-plane.

		position			height of lobes (shoulders)				S =	ℓ_o	gravity
Map	Source	D	H	E	a	b	c	d	Σ h	(S-a)/S	γ
1	W3 OH	61.6°	-4.83^h	40°	40	14	2		56	.29	-77°
2			-1.92	60	48	(10)	2		55	.13	-49
3			0.00	67	48	11	(5)		61.5	.22	0
4			+2.17	59	32	16	8	(7)	59.5	.46	+55
5	DR21 OH	42.2	+0.42	84	9	3	(2)		13	.31	+55
6			+2.75	59	4	2	2		8	.50	+90
7	W51	14.4	+1.08	60	54	(5)	(10)		61.5	.12	+26
8			+3.50	37	30	(12)	(5)	(2)	39.5	.24	+51
9	W Hya	-28.1	-2.33	16	22	14	10	(2)	47	.53	-28
10			+0.25	24	13	12	10	(2)	36	.64	+ 3
11			+2.25	16	16	7	6	(4)	31	.48	+27

data available), we use the highest lobe and the sum*:

$$\ell_o = (S - a) / S. \quad (3)$$

Fig. 2 shows ℓ_o as a function of sky position. Especially uncertain is ℓ_o for the weak source of Maps 5 and 6.

2. Astigmatism and Beam Shape

We know that the gravitational deformations of the 140-ft show a strong astigmatism (surface parabolic in zx and zy-planes, but with difference ΔF of focal lengths). The beam shapes to be expected from astigmatism are shown in Fig. 3,a. Since the astigmatic direction is defined by the telescope backup structure (stiff EW declination axis, flexible NS axis), the beam can be elongated only along the D-direction or the H-direction (but not along γ-direction), and which one depends on the sign of ΔF and on the feed location. The 140-ft surface was adjusted in 1972 for 60° elevation, and since ΔF depends only on elevation, we expect $\Delta F = 0$ for all pointings with $E = 60°$ (measurements in December 1976 actually gave $E_o = 53.4°$ for $\Delta F = 0$). Close to zenith, we have $\Delta F < 0$, and $\Delta F > 0$ toward horizon. We expect about the same astigmatic beam shapes for prime focus and a perfect Cassegrain.

In the observed maps of Fig. 1 we see, first, indeed a strong elongation in H-direction for all maps of low elevation, and maybe a D-elongation for the weak source at zenith. This would fit the expectations if in all cases the focal adjustment was below the best focus.

Second, the observations show not only an elongation, but a splitting into three beams. This fact can perhaps be explained with Fig. 3,b if the

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* The present flux of variable sources is not known. Instead of using the sum, one should integrate the volume under the beam (height x area), and should use the height of an undeformed beam of same volume.

surface deforms not truly parabolic but more U-shaped or V-shaped, yielding three separate lobes.

Third, the astigmatism as discussed so far cannot explain the following facts:

1. Asymmetric heights of left and right lobes in H-direction;
2. Vertical offset of center lobe;
3. Any strong beam distortions close to $E = 53°$.

These facts call for some additional gravitational deformations, either of the surface, or of the Cassegrain mounting (lateral shaft and rotation).

3. Astigmatism and Gain Loss

In a recent paper (IEEE Transact. AP, March 1978) we have measured the gravitational astigmatism of the 140-ft. Calling $\Delta F = F(EW) - F(NS)$ the difference in focal length between the two telescope planes of structural symmetry, A the maximum rim deviation from the best-fit paraboloid (plus is up at N and S rim), Δz the rms surface deviation weighed with an illumination of 15 db taper, B a structural constant, and E_o the telescope adjustment elevation, we derived in general

$$\Delta F = 32(F/D)^2 A = 5.862 A \tag{4}$$

$$\Delta z = 0.329 |A| \tag{5}$$

$$A(E) = B(\sin E_o - \sin E) \tag{6}$$

The observations with an elongated feed horn gave the numerical values

$$E_o = 53.4° \tag{7}$$

$$\left.\begin{array}{l}\Delta F(\text{zenith}) = -10 \text{ mm} \\ \Delta F(\text{horizon}) = +41 \text{ mm}\end{array}\right\} \text{range } (\Delta F) = 51 \text{ mm} \tag{8}$$

$$\left.\begin{array}{l}A(\text{zenith}) = -1.71 \text{ mm} \\ A(\text{horizon}) = +6.99 \text{ mm}\end{array}\right\} \text{range}(A) = 8.70 \text{ mm} \tag{9}$$

$$\Delta z = 2.86 \text{ mm } |\sin E_o - \sin E| \; . \tag{10}$$

This is in fairly good agreement with an intended adjustment at $E_o = 60°$, and with a structural analysis yielding range(A) = 6.67 mm.

From (10) we calculate the expected astigmatic gain loss ℓ_a. Usually, one would apply

$$\ell_a = 1 - e^{-(4\pi\ \Delta z/\lambda)^2} \tag{11}$$

but this holds only for small uncorrelated Δz, and for large F/D ratios. Both can be corrected according to Ruze (IEEE Proc. 54, 633, 1966). First, for F/D = 0.43 the exponent should read: $-0.76(4\pi\ \Delta z/\lambda)^2$. Second, with a correlation length c, the gain loss is

$$\ell_a = 1 - e^{-a}\left[1 + \frac{1}{\eta}\left(\frac{2c}{D}\right)^2 \sum_{n=1}^{\infty} \frac{a^n}{n\ n!}\right] \tag{12}$$

where $\eta = 0.57$ = error-free aperture efficiency; and, for the 140-ft and with $\lambda = 1.35$ cm,

$$a = 0.76\ (4\pi\ \Delta z/\lambda)^2 = 5.38\ (\sin E_o - \sin E)^2. \tag{13}$$

The calculated values ℓ_a from the astigmatism are given in Table 2, to be compared with the observed losses ℓ_o. We see that the astigmatism of the 140-ft can probably explain the observed large losses far south (Maps 9, 10, 11), but certainly not the medium and large losses at other pointings (Maps 1, 3, 4, 5, 6, 8). These again call for some additional deformations.

Table 2. Observed and calculated gain loss.

ℓ_o = observed loss, rough estimate

ℓ_a = loss calculated from measured astigmatism

ℓ_r = additional loss = $(\ell_o - \ell_a)/(1-\ell_a)$

θ = angular distance from point of best efficiency

map	ℓ_o	ℓ_a	ℓ_r	θ
1	.29	.074	.233	49°
2	.13	.012	.119	37
3	.22	.040	.188	40
4	.46	.055	.429	52
5	(.31)	.105	.229	29
6	(.50)	.010	.495	54
7	.12	.012	.109	38
8	.24	.114	.142	71
9	.53	.551	-.047	54
10	.64	.370	.428	59
11	.48	.551	-.158	76

4. Additional Deformations

We do not know the physical cause of the additional deformations. If they are surface deformations (not Cassegrain mounting), they should somehow increase with the angular distance θ between the present pointing (H, D) and that pointing (H_o, D_o) where the 140-ft shows its highest efficiency (best adjustment). From several observers I got a very crude estimate of $H_o = 1.5^h$ East, and $D_o = +25°$. The distance θ then is found from

$$\cos\theta = \sin D_o \sin D + \cos D_o \cos D \cos(H - H_o). \qquad (14)$$

Table 2 gives θ for the 11 beam maps, together with the needed additional loss ℓ_r to be found from $(1 - \ell_o) = (1 - \ell_a)(1 - \ell_r)$ as

$$\ell_r = (\ell_o - \ell_a) / (1 - \ell_a). \tag{15}$$

We see in Table 2 that ℓ_r does not increase too well with θ. First of all this means that our crude estimates of ℓ_o are not good enough and should be replaced by better ones. It also could mean that deformations of the Cassegrain mounting play a role.

This is also indicated by the following. Structural analysis and prime focus observations have shown (IEEE Transact. AP 23, 689, 1975) that the backup structure is about five time stiffer in x and y-direction than it is in z-direction, and that the deformations in z-direction are mostly astigmatic. This means that any additional deformations must be caused mostly by the x and y-components of gravity, which can only give smaller gain losses because of the larger stiffness. Thus, it seems rather difficult to explain the large losses of Maps 1, 4 and 6 by surface deformations only.

Finally, we must ask whether large gain losses can be explained by deformations of the Cassegrain mounting. Two things could go wrong: the deformations needed for the loss could lead to unreasonably large pointing errors, and they could be larger than structurally possible. Using a paper by J. Ruze (unpublished, 1969), and several crude simplifications, we find that the needed loss ℓ_r calls for the following deformations:

$$\left.\begin{array}{lll} \text{either lateral shift} & \Delta x \stackrel{\sim}{\sim} & 3.9 \text{ mm} \\ \text{or rotation} & \Delta\alpha \stackrel{\sim}{\sim} & 5.1 \text{ arcmin} \end{array}\right\} \text{ for } \ell_r = 0.40 \,. \tag{16}$$

These deformations then would cause pointing errors (m = magnification factor = 13):

$$\begin{aligned} &\text{shift} && \Delta\phi \approx \frac{\Delta x}{F} = 0.7 \text{ arcmin} \\ &\text{rotation} && \Delta\phi \approx \frac{2}{m}\,\Delta\alpha = 0.8 \text{ arcmin} \end{aligned} \tag{17}$$

which do not look too large (but should be checked in future observations). The structural analysis yields, between zenith and horizon,

$$\begin{aligned} &\text{shift} && \Delta x = 8.9 \text{ mm} \\ &\text{rotation} && \Delta\alpha \approx 16 \text{ arcmin} \end{aligned} \tag{18}$$

where both numbers should be reduced by about a factor two because the best-fit paraboloid of the deformed primary surface also shows some (rather uncertain) focus shift and axial rotation. Anyway, the needed deformations (16) are not too large as compared to (18) and thus seem structurally possible.

In summary, we find that about half of the observed beam degradation is explained by the astigmatism of the primary surface, while the other half is most probably caused by deformations of the Cassegrain mounting. For a confirmation, we would need prime focus and Cassegrain observations at the same (short) wavelength.

The deformable subreflector will certainly correct the astigmatic part, which is a one-parameter deformation (ΔF only). But it has four actuators (motors pushing or pulling), which gives us actually four degrees of freedom for its use, see Fig. 4. It is hoped that at least a good deal of the additional effects can also be corrected. If that should not be the case, a mechanical correction at the Cassegrain mounting seems possible but awkward.

We then would rather recommend new feed legs (and donut structure) of a stiffer design, which has already been suggested and worked out by Woon-Yin Wong.

5. Future Observations

We certainly need further short-wavelength observations before we can claim to understand the beam degradation and suggest detailed corrections. If possible, the following things should be done:

1. Prime and Cassegrain focus observations at the same wavelength.

2. At each pointing, beam maps with different focal adjustments; up and down from the "best" focus F_o by about $\pm$ 5, 10, 15 mm.

3. Better sky distribution: one pointing at $H_o = 1.5^h$ East, $D_o = 25°$ (or wherever the best beam is obtained). And more pointings close to horizon and close to the telescope limits, East and West, see Fig. 1.

4. Give, for each beam map, the pointing of the expected source position; for checking of pointing errors (17) versus gain loss (16).

5. For each map, give the volume under the complete beam; for estimating the gain loss, see footnote to equation (3).

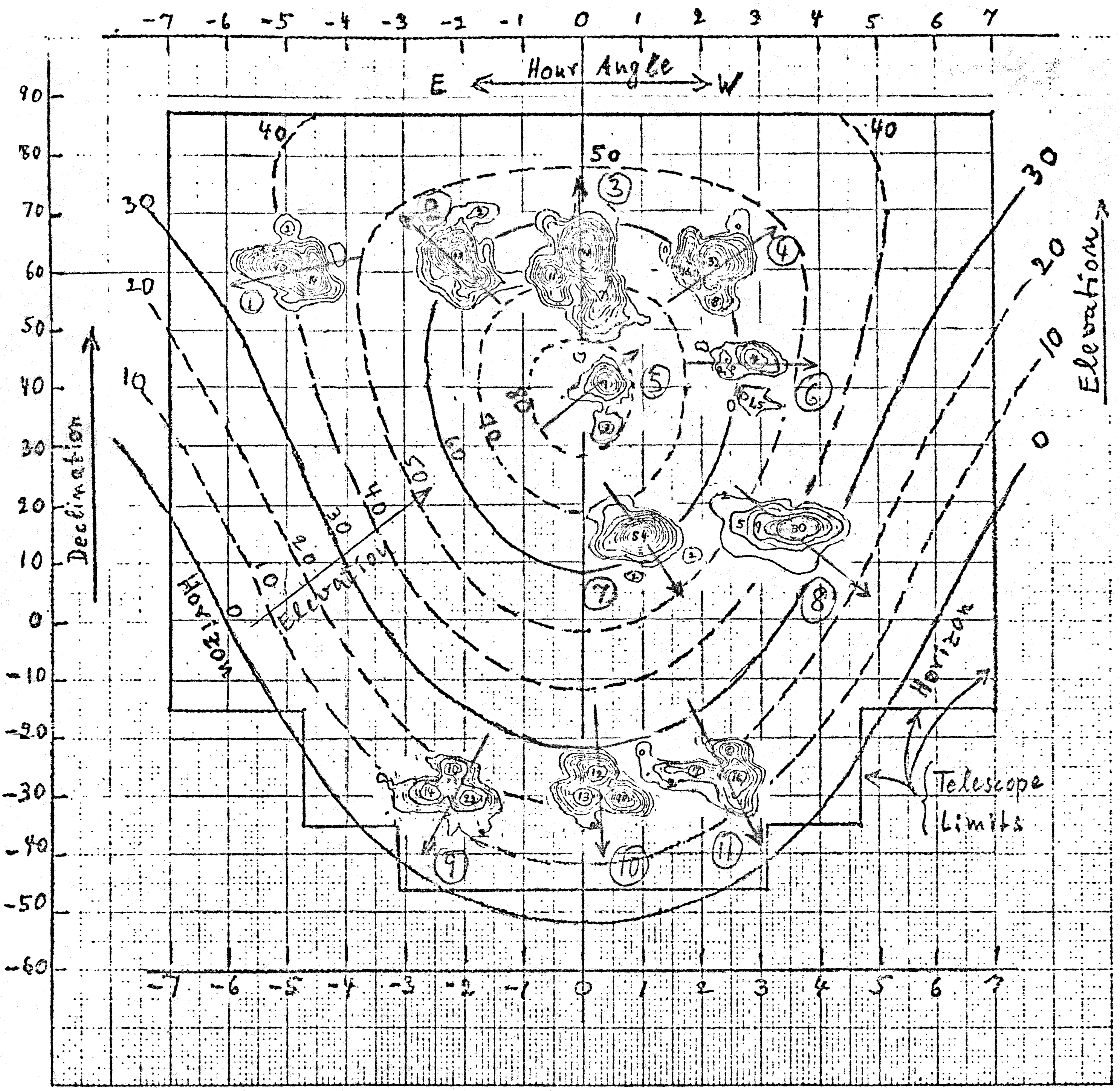

Fig. 1. Beam shape as a function of sky position.

Small numbers are heights of lobes. Large numbers in circles are Turner's map numbers. Arrows give direction of gravity in beam maps.

Scale of beam maps: 0 2 4 6 8 arcmin

The error-free beamwidth (14 db taper) is $b_o = 1.17\ \lambda/D = 1.27$ arcmin.

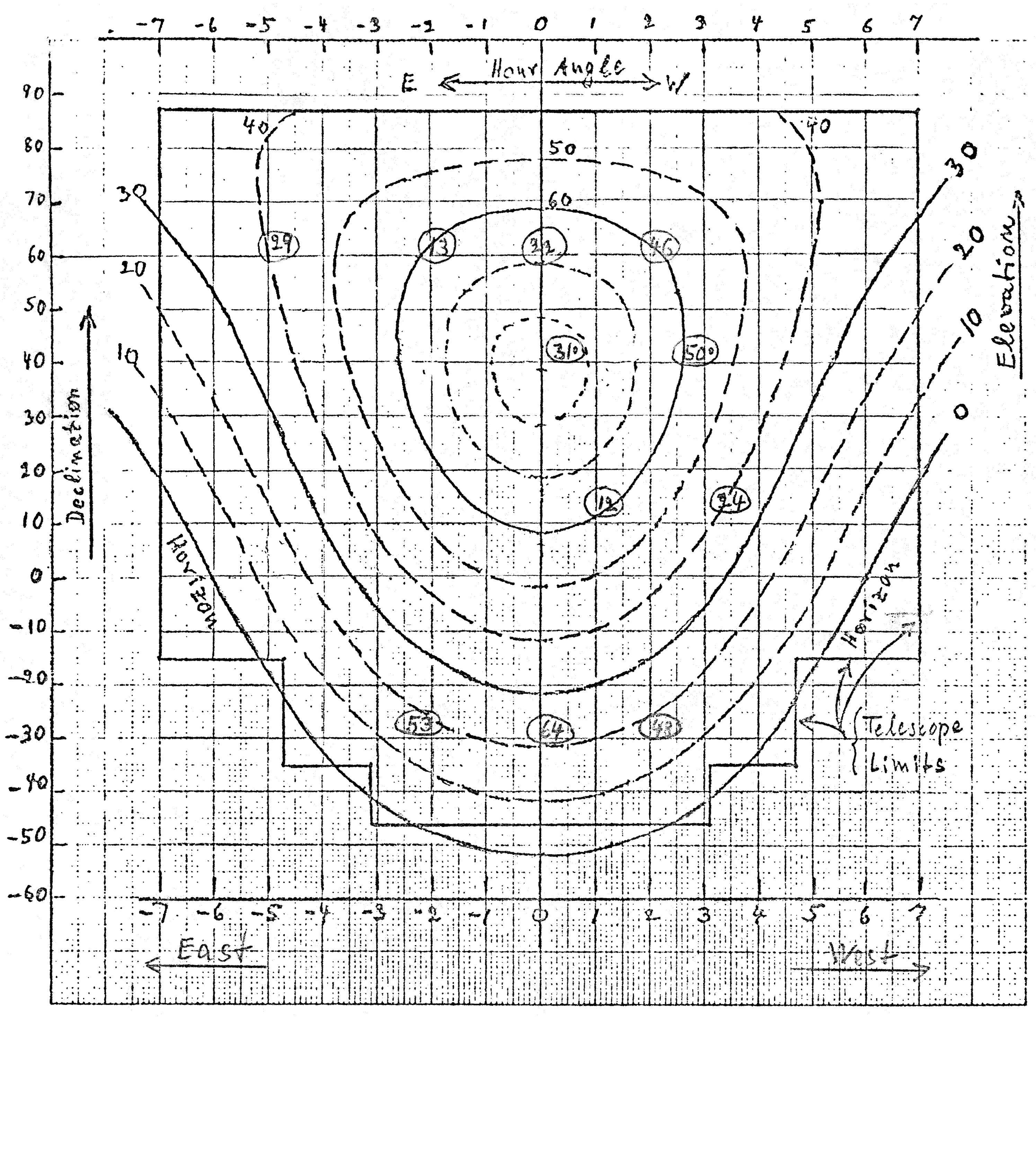

Fig. 2. Crude estimates of gain loss (in %) according to equation (2), as a function of sky position.

The two dots mean "weak source".

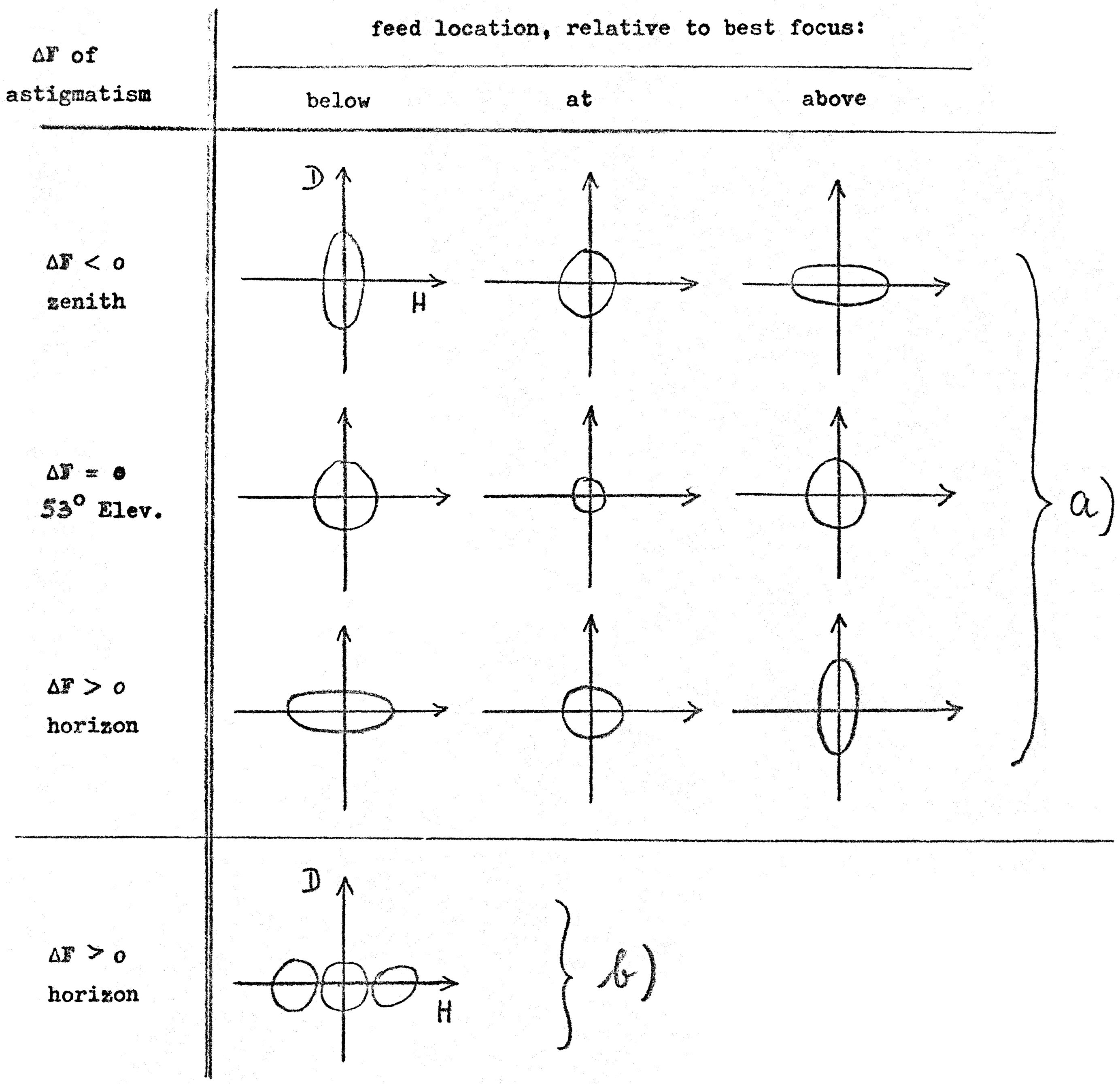

__Fig. 3.__ Beam shape expected from astigmatism, at prime focus.

a) Proper astigmatism, parabolic along each diameter;

b) Stiffer center, deforming outer parts.
(Or V-shaped deformation along each diameter.)

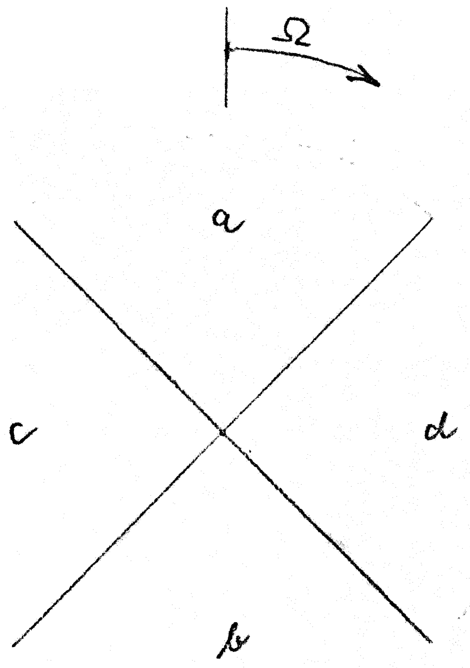

Fig. 4. The deformable subreflector, with two stiff diagonals, and four actuators pushing or pulling by amaounts a, b, c and d. This yields four degrees of freedom for its use, to be called:

$$p_1 = \left[(a+b) - (c+d)\right]\frac{1}{4} \quad \text{astigmatism,} \quad \Delta z \sim \cos(2\Omega)$$

$$p_2 = \left[(a+b) + (c+d)\right]\frac{1}{4} \quad \text{next order,} \quad \Delta z \sim \cos(4\Omega)$$

$$\left.\begin{array}{ll} p_3 = (a-b)\frac{1}{2} & \text{vertical} \\ p_4 = (c-d)\frac{1}{2} & \text{horizontal} \end{array}\right\} \text{shift of half-beam}$$

The displacements then are:

$$a = +p_1 + p_2 + p_3$$

$$b = +p_1 + p_2 - p_3$$

$$c = -p_1 + p_2 + p_4$$

$$d = -p_1 + p_2 - p_4$$